International Council for the
Exploration of the Sea

CM.1986/B: 24
Fish Capture Committee

METHODS FOR THE CALCULATION OF FLEXIBLE ROPE AND NET SYSTEMS EXPOSED TO FLOW

by

Harry Stengel

and

Hartmut Leitzke

Wilhelm-Pieck-Universität
Rostock
German Democratic Republic

ABSTRACT

Several possibilities for the calculation of flexible rope and net systems exposed to flow are presented. Their application for the solution of various tasks in fishing technology is discussed. Furthermore it is pointed out to other papers in this field.

RESUME

Dans la présente publication sont exposées plusieurs possibilités pour le calcul de systèmes de filets et de lignes (accordages) lâches flexibles. Leur utilisation pour la résolution des différents problèmes de la technique de pêche est discutée. Les indications sont faites de plus pour les futurs travaux dans le domaine suscité.

1. Introduction

The vast majority of fishing gears applied for fishing may be seen as a flexible three-dimensional system of nets, ropes and various rigging elements being exposed to various external forces. Herewith fishing gears basically have to be separated from most of the engineering equipments. The main difference is that fishing gears slightly change their shape and position in space under the influence of external forces. However, determining those external forces is difficult, because they again depend on the fishing gear's shape and position being unknown and variable. This fact leads to the development of special methods for calculating flexible rope and net systems exposed to flow.

In the following those methods and their calculation for the design and calculation of flexible rope and net systems towed in water and exposed to flow and some ideas on the further development of these methods are presented.

2. Equations of motion of twines and nettings

The equations of motion are formed starting from the conditions of equilibrium that are valid for the differential element of twine or netting. For the general case we are going to suppose the twines and nettings moving in a viscous, infinitely expanded, incompressible fluid and moreover being ideally flexible and inextensible. The extensibility may be taken into account for some methods still to be discussed. That means, no bending moments and transverse forces are transmitted. The tensile forces and coordinates of elements are functions of the arc coordinate or of the coordinate lines and time, respectively, because a non-uniform movement may occur in the general case.

Restricting to the case of a non-uniform motion of a twine element, the following differential equation may be obtained (cf. Figure 1):

$$\frac{dF_s}{ds} + r_{1\tau} = 0 \tag{1}$$

$$\frac{F_s}{\rho^*} + r_{1N} = 0 \tag{2}$$

$$\frac{dx_1^o}{ds} = \frac{z_1^o}{\rho} \tag{3}$$

x_1^o , z_1^o corresponding to unit vectors directed along the axes x_1 and z_1.

Equation (1) characterizes the change of tensile forces of the twine and Equation (2) illustrates the change of sagging.

If with regard to the mentioned conditions netting is considered to be a thin homogeneous skin and to the hypothesis, that all parts of the net are exposed to flow at the undisturbed inflow velocity and the forces acting on the netting can be determined, the following differential equations result for the differential net element being in uniform motion or inflow, respectively, (cf. Figure 2):

$$\frac{\partial(\sqrt{G}\,\sigma_1)}{\partial\alpha_1}\vec{e}_1 + \sigma_1\sqrt{G}\,\frac{\partial\vec{e}_1}{\partial\alpha_1} + \frac{\partial(\sqrt{G}\,\tau_{12})}{\partial\alpha_1}\vec{e}_2 +$$

$$+ \sqrt{G}\,\tau_{12}\,\frac{\partial\vec{e}_2}{\partial\alpha_1} + \frac{\partial(\sqrt{E}\,\sigma_2)}{\partial\alpha_2}\vec{e}_2 + \sigma_2\sqrt{E}\,\frac{\partial\vec{e}_2}{\partial\alpha_2} +$$

$$+ \frac{\partial(\sqrt{E}\,\tau_{21})}{\partial\alpha_2}\vec{e}_2 + \sqrt{E}\,\tau_{21}\,\frac{\partial\vec{e}_1}{\partial\alpha_2} + \left(p_1\vec{e}_1 + p_2\vec{e}_2 +\right.$$

$$\left. + p_3\vec{e}_3\right)\sqrt{EG-F^2} = 0 \tag{4}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{\bar{\sigma}_1}{\bar{\sigma}_2} \left(\frac{\bar{\sigma}_1 + \bar{\sigma}_2 \sin\omega}{\bar{\sigma}_2 + \bar{\sigma}_1 \sin\omega} \right) = \frac{1-u_1^2}{u_1^2} \tag{5}$$

$\bar{\sigma}_1, \bar{\sigma}_2$ normal forces directed along the coordinate lines related to unit length

ω angle between coordinate lines

3. Method for the solution of the equations of motion

3.1. Analytical solutions

At present an analytical solution of Equations (1) to (3) is known for 3 special cases of twine loads in one plane (/1/). Those cases are a twine loaded with the underwater weight only, the twine -imponderable under water- uniformly loaded over the chord and the twine -imponderable under water- , on which the hydrodynamic force is acting in the normal direction of the twine element and that may be presented as

$$r_{1N} = r_N = c_N \frac{\rho v^2}{2} d \sin^2\alpha \tag{6}$$

c_N resistance coefficient of a twine with perpendicular inflow

d diameter of twine

ρ density of water

α angle of attack between twine element and flow

In the first case the solution represents the relations of a catenary and in the second case the relations of a parabola. The relations of the catenary may be applied for practical calculations, if the inflow velocity is so small that the hydrodynamic forces may be neglected. The relations of the parabola may be applied for the calculation of the form of sagging and tensile force of twine, if the density of twine in water is approaching zero and the acting hydrodynamic force may be considered as a load being uniformly distributed over the chord (small deflection). The third case of load for an approximative calculation of twines having a density in water of zero may be assumed with angles of attack of more than 30^{o}, because the tangential force coefficients of the hydrodynamic forces are much smaller than the normal force coefficients.

At present an analytical solution of Equations (4) and (5), i.e. the analytical calculation of shape and tensile forces of a netting is possible for a netting imponderable in water and fixed to 2 hoops and being not exposed to inflow (basket section) only and this is just valid on condition that the number of meshes is constant over the width of the sheet of netting. Elliptical integrals of the first and second kind (/2/) result from the analytical solution. For practical calculations the tables for elliptical integrals may be applied.

3.2. Numerical methods of solution

An analytical solution is available for few special cases only, as it has been shown above. The solution of differential equation systems that are resulting from physical relations of equilibrium and being used for the description of flexible systems (Equations 1 to 5) is complicated. Subdividing them into components in the space-fixed coordinate system we obtain quasi-elliptic partial differential equation systems, that must contain additional equations for the description of the anisotropy of sheets of netting (Baranov formula) (Equation 5) and of tailoring, too. The last ones are not necessary in the treatment of twines. Special complications result from the consideration of unsteady processes of motion of those towing systems, as they are represented by fishing systems in the classical sense.

For the solution of special tasks various strategies of solution are possible and necessary. In general all numerical methods of solution may be considered as discretization methods, because infinitesimally small segments ds, dt are replaced by finite segments Δs, Δt. Nevertheless variations still can be performed here.

3.2.1. Regarding as continuum and numerical solutions

a) Twines in the stationary operational condition

The natural equations of twine (1, 3) may be subdivided into a system of ordinary differential equations of first order by substitutions (cf. /1/, p. 41). Using Runge Kutta methods that are available in various applications in program libraries of modern electronic data processing systems, the resulting set of equations can be solved for a given initial force (magnitude and direction) and known behaviour of the hydrodynamic coefficients. Here the extensibility and flexural rigidity can be taken into account. As a solution the shape and loads of a heavy twine in water exposed to flow is obtained. According to this method a whole series of calculation programs has been developed for different tasks, just to mention the following:

- method for the calculation of the drag rope /1/, /3/
- method for the calculation of net sounder cables /4/
- method for the calculation of shape and forces of drag ropes in tucking /5/.

b) Closed netting areas in stationary working condition

Subdividing the set of equations (equations 4, 5) into three space-fixed coordinates the following combined differential equation system is obtained by transformation

$$\frac{\partial \sigma_1}{\partial \alpha_1} = \Phi\left(\bar{\sigma}_1, \bar{\sigma}_2, \alpha_1, \alpha_2, u_1\right) \tag{7}$$

$$\frac{\partial \sigma_2}{\partial \alpha_2} = \Phi\left(\bar{\sigma}_1, \bar{\sigma}_2, \alpha_1, \alpha_2, u_1\right) \tag{8}$$

$$\frac{1 - u_1^2}{u_1^2} = \frac{\bar{\sigma}_1}{\bar{\sigma}_2} \cdot \frac{\bar{\sigma}_1 + \bar{\sigma}_2 \cdot \sin\omega}{\bar{\sigma}_2 + \bar{\sigma}_1 \cdot \sin\omega} \tag{9}$$

$$\frac{\bar{\sigma}_1}{\rho_1^*} + \frac{\bar{\sigma}_2}{\rho_2^*} - P_N = 0 \tag{10}$$

ρ_1^*, ρ_2^* - radii of curvature of parameter curves α_1 and α_2 = const.

This set of equations can be solved for some special cases and under considerably restricting or simplifying conditions, if the hydrodynamic coefficients are known and the acting forces of gravity are neglected.

- For the calculations of rotary nettings the conditions $\frac{\partial \sigma_2}{\partial \alpha_2} = 0$, $\omega = \frac{\pi}{2}$ are valid.

  For those net shapes several solutions are available, that had been published in the GDR some time ago (/6/,/1/).

- As a special case the calculation of baskets and of rotary net cages can be considered. For these forms we suppose the external loads on the netting being very small and negligible compared to the forces acting in it. From this we have

$$\frac{\partial \sigma_1}{\partial \alpha_1} = 0, \quad \frac{\partial \sigma_2}{\partial \alpha_2} = 0, \quad \omega = \frac{\pi}{2}, \quad \vec{P} = 0.$$

After transformation and solitary integration of the equations left, an elliptic integral is obtained, that may simply be solved numerically or using tables with good approximation to reality /2/. For this a special method of solution using an electronic data processing system is available for rotary net cages (/7/).

- If rotational symmetry is no longer required, the simplifications taken so far have no validity. Giving the shape of net (trawl part) to be realized and assuming that the directions of main tension σ_1 and σ_2 are just the same as those of the parameter curves α_2 =const. and α_1 =const., respectively, one method of solution has been published so far /8/. Resulting from this the following quantities could be calculated: the stress trajectories in the sheet of netting, loads on twine, hanging of netting u_1 and tailoring. At present calculations of complicated shapes of netting are not possible with this approach and will be difficult in future, too.

3.2.2. Considerations as discreticized towing system of twines

For the investigation of dynamical processes of motion as they are normally occurring in reality caused by ship's manoeuvres, seaway, wind drift and other external influences on the fishing system, the system can't be considered as a continuum any more. Exposed to these influences the right sides of the natural equations, e.g. (7, 8) prove to be time-depending quantities. Hence for the twine the following quantities are occurring

$$\frac{dF_S}{dt}, \qquad \frac{dx}{dt} = \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}.$$

For the solution of these problems specific approaches have been developed that are still justified according to the task. For all of the subsequently mentioned methods of solution it is valid, that the twine or the system consisting of single twines having a multiple connection with each other, may be admitted in the points of connection, i.e. the knots with single forces as e.g. otter boards, sinkers, floaters or combined substitutional point masses and the resulting forces (Figure 3 and 4).

The system is presented as sufficiently fine subdivided system of rods and chains. All forces acting in the single rods and external forces acting on them are summarized in the knots as point forces. There are also modifications to be found with the point masses in the centre of rod. However the main principle is not changed by this fact. The other approaches for gaining a useful solution may be subdivided in two basically different methods.

3.2.2.1. Solution using differential equation systems

Due to the motion of the towing system in water, that may be represented by a twine loaded at the end or a combined system of several twines or a net, different forces are acting on the point masses combined in the knots. Those forces are

- tensile forces of twine elements adjacent to the knots (rods)
- hydrodynamic loads united in the knot
- forces of inertia due to accelerations $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ from point mass and hydrodynamical mass to be added.

At each moment t these forces must form an equilibrium. Connecting the knot equilibriums with each other must be carried out using the tensile forces in the single twine elements to both knots adjacent to them.

Unknowns to be solved for each knot are the following quantities $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, x, y, z, $F_s^{(i)}(t)$. For each knot a differential equation system may be formulated with its subdivision into an ordinary differential equation system containing an adequate number of equations and unknowns. For n knots in the twine system the number of ordinary differential equations must be multiplied by the factor n. The set of equations gained in this way and being very extensive in general is solved applying efficient Runge-Kutta methods by means of electronic data processing. Although this method is very expensive, recently it could successfully be used for the solution of very interesting problems in the dynamics of towed systems. One restriction is, that only rod chains without connection, i.e. multiple pendulums, have been used as substitutional systems for much more complicated systems. However, from the theoretical point of view the treatment of systems with multiple connections must be possible, too.

Examples for special solutions found out by GDR specialists in fishing technology are:

- models on heaving and veering of pelagic trawl systems (among others /9/, /10/)
- model on the connection between ship and towed system during steering manoeuvres /9/.

3.2.2.2. Complete discretization of flexible towing systems

As it has already been mentioned in the introduction, towing systems are characterized by their special design forming under the influence of external and inner forces on the one hand and of shape on the other a unique and specified appearance guaranteeing the equilibrium of forces in each point.

a) Steady state calculations

If now simplifying the towing the system as mentioned is discreticized in a way, that it is subdivided sufficiently fine in a number of rigid rods, being in the points of connection, i.e. the knots, momentless connected with each other, then the equilibriums of forces can be formulated for each knot. It is useful to fix the unknown orientation of the single rod elements in advance in a way, that the unit vector $\vec{e}_i$ of i-th rod element still to be calculated is always directed from the neighbouring knot K_I to the following knot K_{I+1}. In the rods i the tensile force T_i is acting, that is unknown, too (Figure 5). This tensile force is introduced into the equilibrium of forces in knot K_I with $+T_i \cdot \vec{e}_i$ and into that in knot K_{I+1} with $-T_i \cdot \vec{e}_i$. On the knot additionally the external loads united to a point force are acting in the range of the knot. The equilibrium of forces for knot K_I in Figure 5 is:

$$-T_{i-n-1} \cdot \vec{e}_{i-n-1} - T_{i-n} \cdot \vec{e}_{i-n} + T_{i-1} \cdot \vec{e}_{i-1} + T_i \vec{e}_i = -\vec{F}_{KI} \qquad (11)$$

According to this for m knots in the discreticized towing system m analogous equations (11) may be formulated. Because as a rule the number of knots is lower than the number of rod elements in the system other relations are lacking, that may be formulated resulting from the geometrical conditions. As an example from Figure 5 only one equation should be mentioned being typical for nets:

$$a \cdot \vec{e}_{i-n} + a \cdot \vec{e}_i - a \cdot \vec{e}_{i+1} - a \cdot \vec{e}_{i-n+1} = \vec{0} \qquad (12)$$

Summarizing all analogous relations according to Equations (11) and (12), a set of equations can be formulated, that may be linearized and which contains the components of unit vectors $\vec{e}_i$ in the 3 dimensions of space x, y, z as unknowns:

$$\vec{e}_i = \begin{Bmatrix} x'_i \\ y'_i \\ z'_i \end{Bmatrix} \text{ with } \sqrt{x'^2_i + y'^2_i + z'^2_i} = 1 \; ; \qquad (13)$$

$$x'_i = \frac{\Delta x}{\Delta s} \ldots$$

During the last a years a special method of successive approximation has been developed by means of which very good approximative solutions have been made possible for the calculation of different towing systems. Initial values are roughly estimated tensile forces T_i. The knot forces $\vec{K}_I$ contain hydrodynamic forces depending on the unknown orientation of unit vectors $\vec{e}_i$, forces of gravity and single forces as e.g. sinkers, floaters and others. For in this way the knot forces are slightly depending on the left side of the set of equations, those have to be calculated newly at each iteration step with the obtained approximations /11/, /12/. The calculation is carried out using a method for the solution of extensive linear equation systems. In 1984 the calculation of a complete trawl net had been presented for the first time /13/ (Figure 6).

b) Calculation of dynamically loaded discreticized towing systems

This case of calculation is connected to the statements in 3.2.2.1. The knot forces of such a towing system are considerably changed under steady inflow to the system compared to the statical load:

- At each knot another inflow velocity is acting which is assumed to be equal for the whole range of action of the knot.
- From the masses and hydrodynamical masses united in the knot forces of inertia are resulting increasing the knot force

Both shares of force may be calculated with the accelerations to be expected being low and with the shape of the system being known.

The expanded method is based upon the method described in the previous passage. At first a method is introduced for the calculation of the local velocities from actual initial values, which allows the calculation of the knot force by the end of a finite step of the duration Δt using the following set-ups:

$$\vec{a}_I(t+\Delta t) = \vec{a}_I(t) + \dot{\vec{a}}_I(t+\Delta t)\cdot\Delta t \tag{14}$$

$$\vec{v}_I(t+\Delta t) = \vec{v}_I(t) + \vec{a}_I(t)\Delta t + \dot{\vec{a}}_I(t+\Delta t)\frac{\Delta t^2}{2} \tag{15}$$

$$\vec{\Delta s}_I(t+\Delta t) = \vec{v}_I(t)\Delta t + \vec{a}_I(t)\frac{\Delta t^2}{2} + \dot{\vec{a}}_I(t+\Delta t)\frac{\Delta t^3}{6} \tag{16}$$

Faulty values for the acceleration increases $\dot{\vec{a}}_I(t+\Delta t)$ lead to the fact, that the knot distances are situated in a way that the rod elements must be turned, extended or cut. Therefore at this stage a special iteration for "system embedding" is inserted guaranteeing the reproduction of the original length of the rod elements. The acceleration increases $\dot{\vec{a}}_I$ still being faulty cause a non-equilibrium of forces in the single knots by the end of time step Δt. The shape obtained from Equation (16) does not correspond to the forces to be calculated with Equation (14) and (15), if they are determined according to the method of discreticized towing systems. The determined "error forces" are now used for an iterative correction of the acceleration increases $\dot{\vec{a}}_I$ until the "error forces" do not exceed certain tolerances anymore. This is done applying Regula falsi. After the first tests this method offers promising set-ups for the calculation of dynamically loaded complicated towing systems as fish catching systems and others. At present net cages in seaway are investigated /14/.

4. Determination of the hydrodynamic coefficients

A sufficiently accurate determination of the hydrodynamic coefficients of twines and nets raises considerable methodical and measuring problems. From the physical point of view the hydrodynamic coefficients represent the Newton number.

$$Ne = \frac{\Delta p}{\frac{\varrho}{2} \cdot v^2} \tag{17}$$

Δp - pressure difference related to area unit.

Their determination by calculation requires the flow pattern for the considered body - i.e. the twine or the net with complicated surface - to be known, this is however not the case. Hence the coefficients must be determined experimentally. It is a pre-condition to determine the functional dependence of the coefficients to be calculated on various physical parameters. It may be described qualitatively with

$$\vec{c}_F = \vec{f}(Re\text{-}d, Sh, \pi_{\gamma^*}, MA, FA, \acute{x}, \acute{y}, \acute{z} \ldots) \tag{18}$$

for the twine and

$$\vec{c}_N = \vec{f}(Re\text{-}d, Sh, \pi_{\gamma^*}, MA, FA, NA, u_1, \frac{d}{a}, \alpha_1, \alpha_2, \ldots) \tag{19}$$

for the netting

Re-d	- Reynolds number related to diameter
Sh	- Strouhal number
π_{γ^*}	- extended Froude number
MA	- kind of material
FA	- kind of twine
NA	- kind of net
u_1	- hanging coefficient
$\acute{x}, \acute{y}, \acute{z}$	- orientation of twine in space
α_1, α_2	- orientation of netting in space
d	- diameter of twine
a	- width of mesh

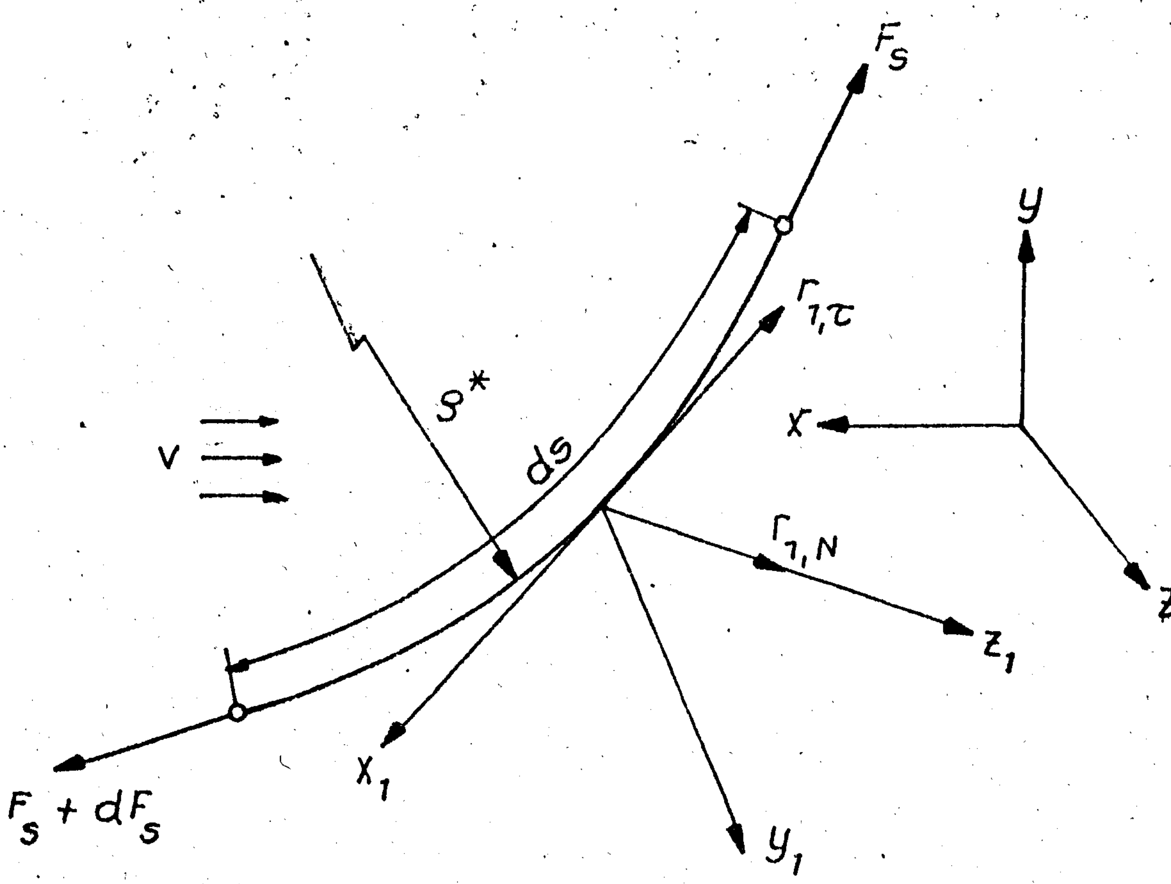
Carrying out measurements on original designs has the consequence, that in the general case the normal component of the hydrodynamic load causes a deflection resulting from which an exact assessment will be much more difficult. Strong prestrainings for the decrease of deflection require clamping devices which cause a considerable influence on flow and with this a falsification of measuring results. However the influence of the kind of material MA may only be shown in this way. Investigations of towing cables are often carried out the cables being towed with free end. Then the obtained towing angle is a function of the $\pi_{\gamma}{}^{*}$ number. From this follows, that the independence of the coefficient functions on the Reynolds number must be supposed.

The investigations of nettings are still more complicated. Here the influence of the hanging coefficient u_1, that can't be taken constant for original designs, must additionally be taken into account. In comparison to the methods mentioned so far the application of rigid models offers the advantage, that the possible deflections may be neglected and the fastening of models may be carried out as fine that the influence on the measuring results is only unimportant. Another advantage results from the fact, that by partly considerable model augmentations the dependence of the coefficients on the Reynolds number is to be realized very well. By means of model augmentation the exact structure of the surface may also be simulated very good. It is however very difficult to simulate the kind of material.

Due to these physical circumstances the determined coefficients are always valid for a strictly restricted range only, i.e. for the actual model used for the measurements. A transmission of the hydrodynamic coefficients to extended fields, as it is usual at present, must necessarily cause errors that could be more or less important. Several own investigations on this problem are available /15/ /16/.

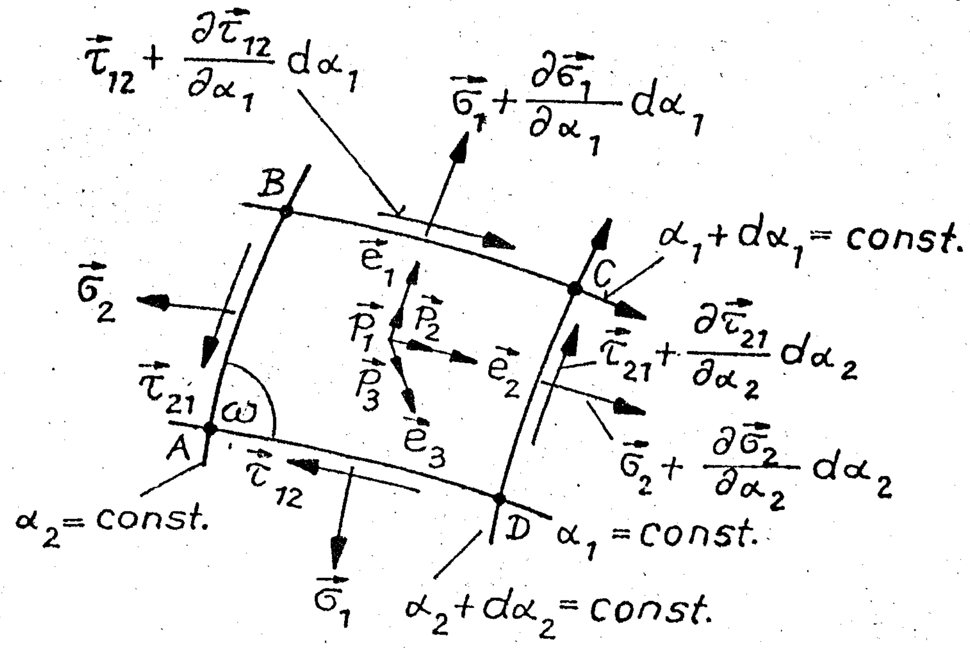
5. Conclusions

Detailed investigations on the dynamics of heavy towing systems (fish catching systems) being towed require the determination of the influence of residual vibrations of rope elements in the towing system on the coefficients, which may be described by the Strouhal number. The improvement of the calculation methods for dynamical and quasi-steady conditions of motion described in this paper is mainly depending on the successful determination of this influence in future. GDR specialists in fishing technology try to make contributions to the solution of these problems.



x_1, y_1, z_1	–	body-fixed system of axes (accompanying triplet)
x, y, z	–	space-fixed coordinate system
s	–	arc coordinate
F_s	–	tensile force
ϱ^*	–	radius of curvature
$r_{1,\tau}$	–	component of the resulting force acting on the length unit of twine – in direction of tangent to twine element
$r_{1,N}$	–	component of the resulting force acting on the length unit of twine - in direction of normal to twine element
v	–	velocity of flow against a body

Fig. 1: Forces acting on twine element



$\vec{\sigma}_1, \vec{\sigma}_2$	-	normal forces related to length unit in the direction of mesh diagonal
τ_{12}, τ_{21}	-	tangential forces related to length unit in the direction of mesh diagonal
α_1, α_2	-	coordinate lines
E, G, F	-	coefficients of the first basic form of surface terms
$\vec{p}$	-	external force acting on a surface unit of netting
$\vec{e}_1, \vec{e}_2, \vec{e}_3$	-	unit vectors

Fig. 2: Forces acting on netting element

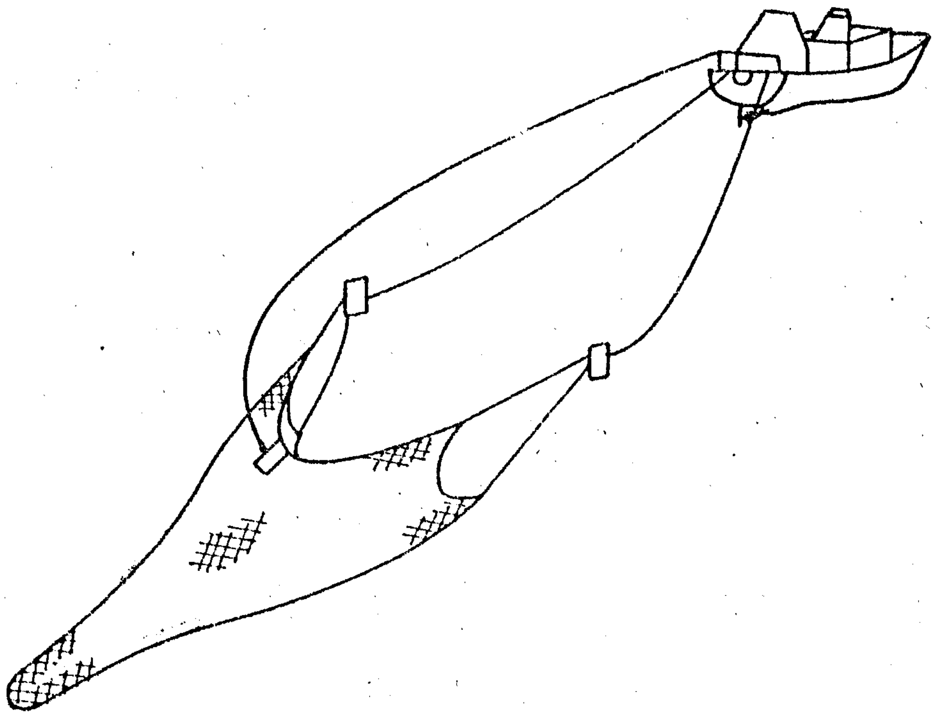


Fig. 3 Real towing system with multiple connection

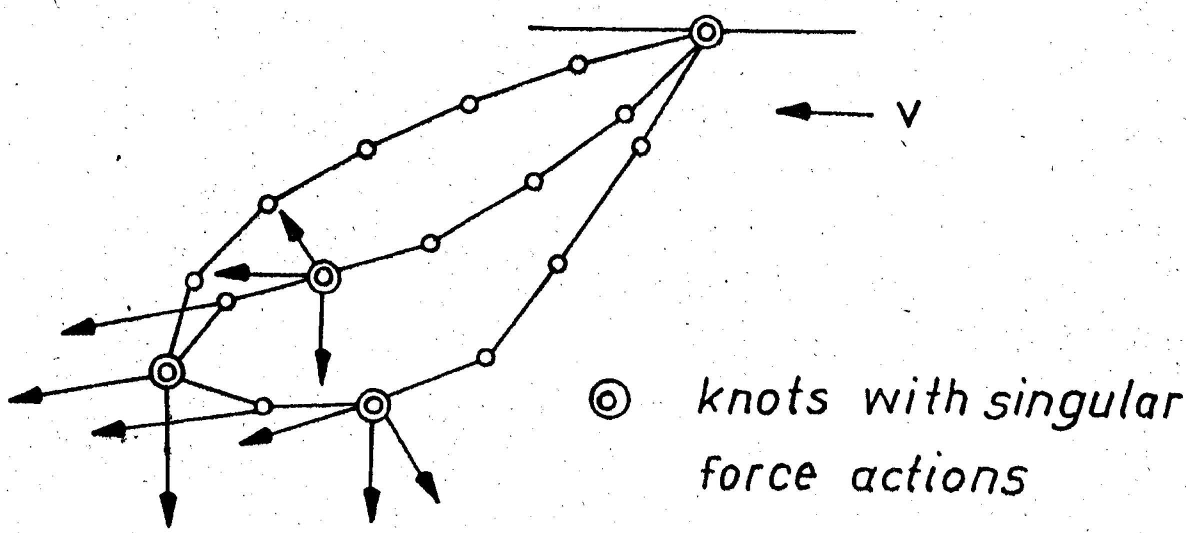
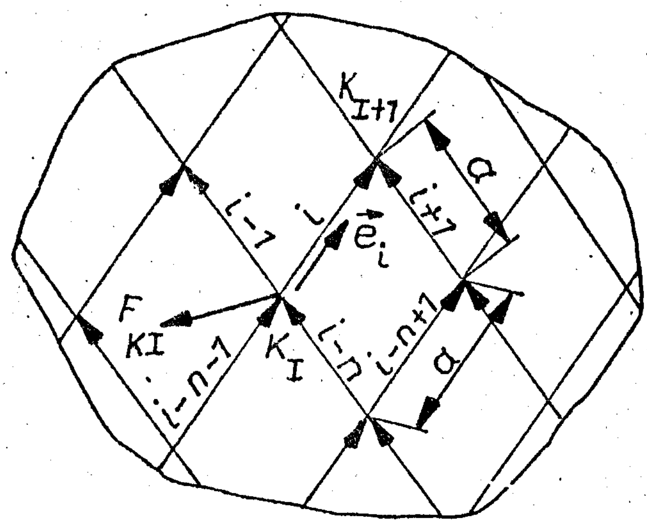


Fig. 4 Possible simplified rod-chain system of the real system



a - length of mesh leg

Fig. 5 Example of a netting segment modelled as rod system

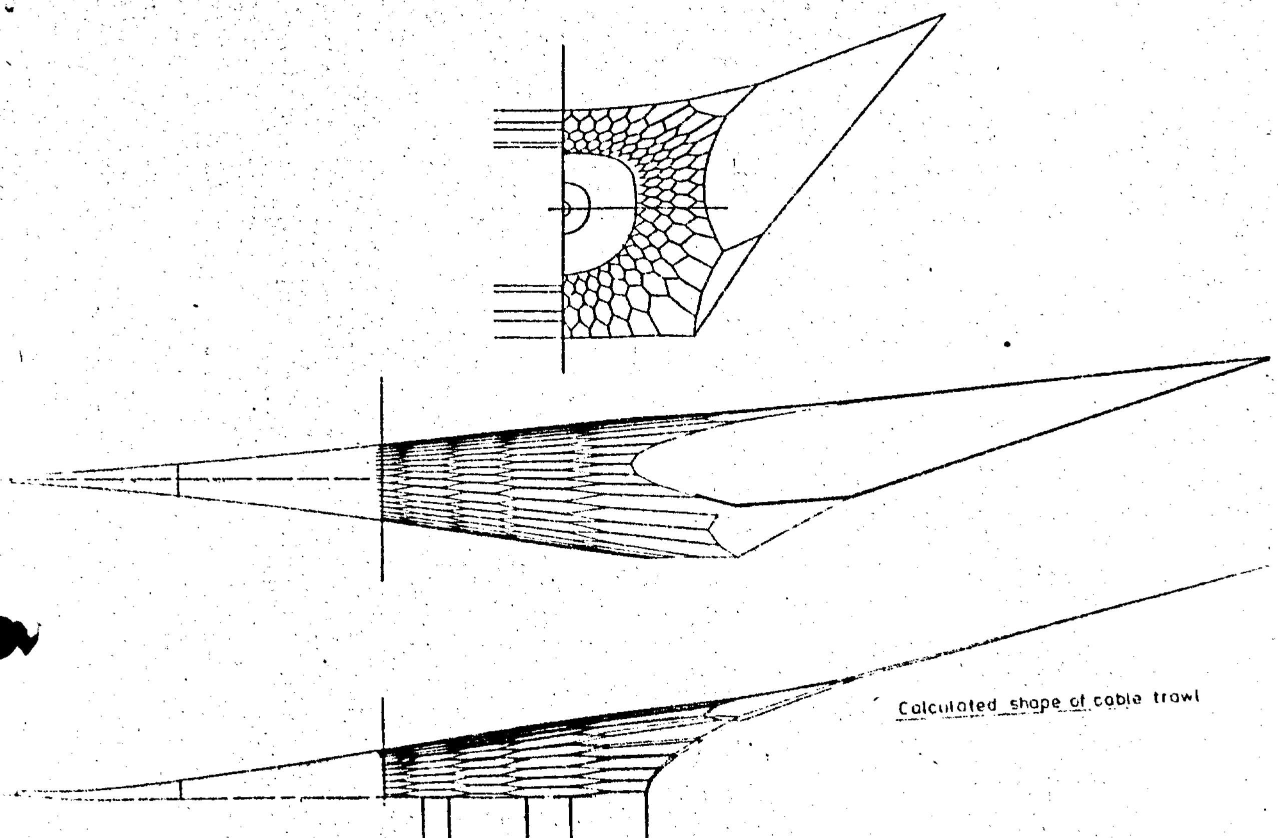


Fig. 6 Calculated shape of cable trawl