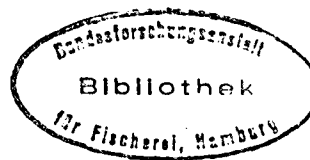


International Council for the Exploration of the Sea
Statistics Committee. ICES CM 1992/D:28
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**Phase-space diagrams, a first approach to chaos modelling
of North American landings of *Homarus americanus*.**

by

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Summary

Over the past years the landings of North American lobster (*Homarus americanus*) have reached a record high for the past century. Although strict enforcement of fishing regulations may be involved, the reasons for these high landings are not fully understood.

Positive correlations of lobster landings with environmental parameters such as annual average water temperatures, or fresh water outflow from the St Lawrence have been invoked, or negative correlations with potential predator stocks such as cod. Such effects tend to be local and do not last for the whole period of observation.

North American landings seem to follow global patterns, irrespective of local differences in fisheries regulations and of local environmental disturbances. A global system reacting non linearly to minute disturbances by positive and negative feedback fulfils the prerequisites for chaos dynamics.

We have conducted a preliminary chaos analysis of 100 years of lobster landings data using tools borrowed from physics. We conclude that lobster landings may behave as a chaotic system gravitating around a triple loop attractor in phase-space. The landings have presently reached a loop which is rarely visited and appears to be less stable than the other loops. Slight disturbances either positive or negative could have a major effect on the evolution of the landings.

A chaotic behaviour does not imply that the fishery cannot be managed. However management approaches may have to be reconsidered if stabilizing becomes a more important goal than maximizing the landings.

Introduction

The landings of North American lobster (*Homarus americanus*) have reached their highest level in a century over the past years, both in Canada and the United States. Although strict enforcement of fishing regulations is probably involved, the reasons for these high landings are not fully understood.

The trends appear to be similar in both countries and no particular stock management method can be identified as having determined these increases in landings. Stock management methods vary considerably between regulation areas or States, and between countries. In Canada a great emphasis is set on regulating effort (limitation of number of fishing licences, of traps per licence, of fishing seasons) as well as on minimal legal fishable size, selectivity of the traps (escape gaps) and prohibition to land berried females. In United states, emphasis is set on regulation of legal fishable sizes (both minimal and maximal in certain locations) and on protection of the females in reproduction (prohibition to land berried females and "V notched" females which have carried eggs). The enforced legal minimal fishable sizes vary greatly between regulation areas.

Authors have attempted to identify environmental factors which might have affected larval recruitment, such as average water temperatures occurrence of storms beaching early lobster stages, or river outflow (Drinkwater *et al.* 1991). Sometimes convincing correlations, and even predictions have been identified between these factors and the landings after a time lag of 6 to 10 years (Flowers and Saila, 1972; Sutcliffe, 1972, 1973; Sheldon *et al.*, 1982; Drinkwater, 1987). However the effects identified have always been limited to specific locations and efficient over a limited period of years.

Since North American lobster landings seem to follow global patterns, it is tempting to treat the population(s) as a complex global system similar to a meteorological system. Local effects can be propagated on the long term between fishing locations. Geographic genetic differences between lobster subgroups have so far been found to be limited. Large scale diffusion of benthic adults, as well as drift of larvae which may remain pelagic for several weeks to months probably explain the long term uniformity in group dynamics.

The global system is evolving as a whole as a function of multiple minute local disturbances. The relationship between the number of individuals or the biomass and population or environmental parameters is generally thought to be non linear. The system reacts to disturbances by positive and negative feed back mechanisms such as density dependence of growth recruitment, natural mortality, or fishing mortality through management regulations. The system may even have a (genetic) memory. Such a system seems to fulfil all prerequisites for chaos dynamics (May *et al.* 1978; Wilson *et al.*, 1991).

We have conducted a preliminary chaos analysis of the available lobster landing data from Canada and the United states. We used statistical tools developed for the analysis of physical phenomena, but which can also be applied to biological data. We attempted to introduce

supplementary variables such as physical environmental factors (average yearly temperatures, outflow of the St Lawrence) and an index of abundance of a reputed predator : records of cod fish landings.

Generalizing from the chaos analysis of lobster landings, we provide general thought on fisheries management. We shall comment on the efficiency of long term predictions and management regulation, as well as on the applicability of current fisheries modelling methods to a chaotic situation.

Material and Methods

Biological data

Cod and lobster have been actively fished for more than two centuries in Eastern Canada, and reliable ancient records of landings are available. Ancient effort information is not available.

Lobster landings *per se* may be a good measure of stock abundance, since it is generally believed that male lobsters are exhaustively caught soon after they reach legal minimal size. The Southern Gulf of St Lawrence the catch is believed to contain no more than two age groups.

Many fishermen believe that cod is a major predator of lobster and that the depletion of cod stocks by fishing has boosted lobster stocks in recent years. We used cod catch as an approximate index of cod abundance in order to check for effects on lobster catch.

Lobster landings

Canadian lobster landings were taken from Williamson (1989), landings for the years 1990 to 1992 were provided by the Industry Development and Program Directorate, Department of Fisheries and Oceans, Canada. Additional information on lobster landings were provided by John Temblay and Mike Eagles, DFO, Scotia Fundy Region. Precise information is available by statistical areas or by county in ancient records.

Cod landings

Cod landings for Atlantic Canada were obtained from the Second Annual Report of the International Commission for the Northwest Atlantic Fisheries (ICNAF) for the years 1951-52, and from the Statistical Bulletins of ICNAF for the years 1953 to 1989.

Physical environment

The Gulf of St Lawrence to the Gulf of Maine inclusive may be considered as an oceanographic system (Sutcliffe *et al.*, 1976). The influence of the outflow of the St Lawrence affects the Scotian Shelf and the Gulf of Maine. Global effects of average temperatures and St Lawrence outflow on the catch of lobsters were investigated.

Surface water temperatures

Surface water temperature should well characterize the habitat of lobster in the Gulf of St Lawrence. In this region, lobsters are found only between 0 and 80 m, the maximal depth of the mixed layer in summer. At the surface the salinity may be as low as 20‰ and the temperature as high as 20 °C in summer. Lobsters apparently never penetrate the intermediate water layer flowing from Labrador (stable -1 to + 1°C and circa 33‰ salinity all year round). Lobster larvae are found mostly within the most superficial centimetres of water.

Similar surface water temperatures should influence all North American Lobster habitat. The influence of the outflow of the St Lawrence affects the Scotian Shelf and the Gulf of Maine (Sutcliffe *et al.*, 1976). Deep concentrations of lobsters are found off the Scotian Shelf and the Gulf of Maine in regions not affected by the Labrador intermediate water. However, Exchanges occur between shallow and deep concentrations at the larval and adult stages.

We used temperature records from Booth Bay Harbour (Maine), Halifax(Nova Scotia), and St Andrews (New Brunswick) provided by John Tremblay and Mike Eagles. In order to compensate for gaps in data the temperatures from the three locations were averaged.

St Lawrence water run off.

The run offs were obtained by summing detailed information available for the St Lawrence river affluents and the Saguenay river from 1923 to 1963. Anon. 1926 to 1966. Water Resource Papers 48 to 107.

Statistical analysis

Linear correlations

A common way of searching for causality effects from biological or environmental factors on stock abundance consists in using simple scatter plots and calculate linear correlation coefficients. Setting a time lag between the measure on the factor and the census on the fishable stock allows to take into account effects on early life history. We used lags of 0 to 10 years for lobster landings vs average temperatures, St Lawrence run off and cod landings.

Chaos analysis

May (1974) introduced the idea of chaos in population biology, and later in the theory of harvesting natural populations (May *et al.* 1978). Intuitively, in purely deterministic world, a population being displaced either negatively or positively by an external factor from its simple equilibrium logistic growth curve should tend to return to equilibrium conditions by the effect of negative density dependent feed back. However, May shows that the "return" to equilibrium may lead to dumped periodic oscillations, stable periodic oscillations, chaotic

oscillations (pseudo random), and eventually to extinction. In a chaotic situation the evolution of a population becomes unpredictable, even in the simple logistic growth conditions, because the slightest initial differences in population abundance or population parameters will generate diverging behaviours. Global patterns or "attractors" can nevertheless be identified. The attractors can lead to a dynamic stability within apparent chaos.

Natural populations are continuously submitted to external disturbances, and their survival must have lead through evolutionary processes to the genesis of complex homeostatic systems. These homeostatic systems may be conceived as attractors. The attractors may be simple in the case of dampened periodic oscillations, or of stable oscillations. The attractors may be of a very complex nature in the case of chaotic oscillations: when oscillations are repeated a great number of times, from apparent randomness organized patterns emerge. Although it is impossible to forecast where the population will be located from one step to the next, it is possible to outline patterns for its trajectory.

Under such conditions identifying the effects of factors on the population using linear correlations becomes an fictitious task. The population may react linearly to the effects of a factor over a number of generations; but its owns homeostatic feed back mechanisms will soon create oscillations which will eventually free the population from the effects of the factor. Linear correlations cannot be used for assessing the effects of a factor over long periods of time in the presence of chaos. Specialized tools are required.

Chaos theory has been documented (Gleick, 1987; Briggs & David Peat; 1990) and successfully used for some time in many sciences such as physics, and meteorology, but has been so far of limited application in fisheries science (Wilson et al. 1991). We have borrowed tools from physical sciences (Sprott and Rowlands, 1992) to analyze the lobster landing data for chaotic behaviour.

Sprott and Rowland warn that, in practice, with a limited amount of data that may be contaminated by noise, success cannot be guaranteed. Time series in fisheries biology are available over much shorter sequences than in physics. The smallest time unit for a lobster population is a year since reproduction is seasonal, but a more logical unit may be a generation time (6 to 10 years). A hundred years of data on lobster landings (Williamson, 1989) is an exceptionally good set of observations. In fisheries, records of landings can be quite precise, but population abundance estimates are provided with no better than 30-50% confidence intervals. Three point running average smoothing was used at times to reduce noise in the time sequences of lobster landings, no attempts were made to use population abundance estimates.

As a guidance for the interpretation of the results of the chaos data analyzer tools, we processed reference data sets containing 100 data points. Typically random, periodic and chaotic data sets were assayed together with the records on lobster landings. References for the tools and standard data sets are provided in Sprott and Rowland. We avoided using tools such as return maps which showed to be unsuitable for small data sets.

Graph of data:

The simplest graphic representation of data is landings vs time. Simple visual inspection may reveal possible cycles or long term trends. Sprott and Rowland recommend to remove long term trends prior to chaos analysis.

Frequency distribution:

Purely random data will give rise to a simple curve. Periodic data gives a simple histogram with sharp edges. The distribution is likely to be fractal for chaotic data, but can also be a simple curve.

Power spectrum:

This is a Fast Fourier transform of the data, a usual procedure for periodic time series analysis. The power (mean square amplitude) is displayed as a function of frequency. A power spectrum with a few dominant frequencies shows that the data could be well approximated by a Fourier series with a few terms. Periodic and quasi periodic data will produce few dominant peaks in the spectrum. Random and chaotic give rise to broad spectra. Power spectra which are straight lines on a log linear scale are thought to be good candidates for chaos.

Dominant frequencies:

The approach is similar to the Power Spectrum. Instead of a fast Fourier transform, the Maximum Entropy method is used. The method can be used for linear predictions of the next values in the time series.

Correlation dimension:

Quantifies the underlying complexity of a chaotic time series a dimension greater than 5 implies random data. As the embedding dimension increases, the correlations dimension should increase, but eventually saturate at the correct value (asymptote).

Phase-space plots:

In a two dimensional plot the derivative is plotted vs the abundance of the population at each time of observation. the derivative is approximated by half the difference of the two data points adjacent to each point. periodic data should appear as a closed curve.

On a three dimensional plot the second derivative is plotted along with the first derivative and the population abundance on three orthogonal axes. The second derivative is the difference between the slopes of the lines connecting each data point with its two nearest neighbours. Some cases which are not obviously periodic in 2 dimensions reveal their periodicity in three dimensions.

Lyapunov exponent:

Measure of the rate at which nearby trajectories in phase space diverge. Chaotic orbits have at least one positive Lyapunov exponent. For periodic orbits, all Lyapunov exponents are negative. Only the most positive exponent is calculated here. The existence of a positive Lyapunov exponent implies a chaotic time series, although noise will also produce a positive exponent.

Correlation matrix:

The approach is similar to a Principal Components Analysis of multivariate data. The matrix analyzed, instead of containing correlations between multivariate observations, contains autocorrelations between observations for variable lags. Attempts are made to represent the data as a finite set of orthogonal functions, the number of significant eigen values of the correlation matrix. At least 3 eigen values are required to create chaos. The number of significant eigenvalues is a measure of the complexity of the system.

One might try correlation matrices with the goal of finding a set of model equations whose solution as evidenced by the phase-space plot is a strange attractor that bears some resemblance to the phase-space plot of the original data. Because of a change of coordinates, these phase-space plots will be distorted images of one another, but because the changes are linear the topological factors such as holes in phase space must be common to both.

Predictions of next values can be made and tested using the principal components as model variables. With chaotic data predictions will be realistic only on very short intervals.

Results

Environmental parameters

Our data sets are still incomplete, we did not succeed yet to assemble information over the full time interval over which lobster landing data is available, but we do have long enough time series to visualize long term effects. An index of outflow from the St Lawrence was calculated from information in Anon. Surface Water Supply of Canada St Lawrence and Southern Hudson Bay Drainage, Ontario and Quebec (1934 to 1966). The outflows from 32 affluents of the St Lawrence were summed (fig. 1). Surface temperature records from Booth Bay Harbour (Maine), St Andrews (New Brunswick), and Halifax (Nova scotia) were in turn processed separately, then averaged to obtain a general index from 1926 to 1989 (fig.2).

Extensive assays were made to attempt to identify trends in lobster landings within wide subareas of the Gulf of St Lawrence and the outflow index from the St Lawrence. No better correlations could be identified than for the landings over the whole Atlantic Canada coast (figs. 3, 4). The correlations are weak and negative. Six years is usually the average time ascribed to a lobster for growing from larval to commercial stages, setting a lag of 6 years between catch and outflow index reduces the correlation in stead of improving it.

There is a correlation between lobster landings in Canada and average temperatures the same year as well as 6 years earlier (figs. 5, 6). We did not detect any peculiarities in local effects. The drastic increases in catch for the last years cannot be explained by a temperature effect only.

Predation

Cod Landings in Easter Canada were obtained from Anon. 1952 and Anon. 1953 to 1990 ICNAF Stat. Bull.. Detailed information is available for geographic subregions. We tried to relate lobster landings to cod landings records available from 1893 to 1989, we incorporated lags of 0, 2 and 5 year lags to account for possible cod predation on early life stages. The correlations are weak but positive instead of negative, and best for a 2 year lag (fig. 7).

Chaos data analysis

Attempts were first made to segregate data into subregions into which sub populations of lobsters may be logically expected to exist. The local analyses did not provide a better definition than the global analysis of the data, and are not reported here. A smoothing of the data using repeatedly a three point running average did appear to erase random variability superimposed on the chaos process (fig. 8). This variability may be simple white noise generated by the poling of part of the data where and when precise information is difficult to obtain.

One hundred data points from typical periodic, white noise, and chaos data are provided for comparison (figs 9, 10, 11). The same data is presented in frequency distribution form in figs 12 to 15, and as phase space diagrams in figures 16 to 19.

Using the criteria defined by Sprott and Rowland, lobster time sequences are best described as chaos data. Particularly, the phase space plot (fig. 16), without having the regularity of periodic data plots (fig. 17) clearly differs from random data plots (fig 18), and displays irregular tracts as in the chaos data (fig. 19).

The power spectrum after fast fourier transforms provided a somehow log linear relationship. No clearly dominant frequencies were present. The predictions provided by the dominant frequency techniques were poor as opposed to what would be generated by periodic data.

The correlation dimension stabilized well and was of the order of 2, well below the value of five expected for white noise data. The Lyapunov exponents are all positive (max. 0.26 ± 0.13).

The correlation matrix analysis produced principle components which allowed to generate an attractor in phase space topologically similar to the phase space diagram of the observed data (fig 20). The predictions from the present point were unstable beyond one single year. This is quite consistent with the two year recruitment catch composition of the present landings.

Discussion

The North American lobster stocks must react to highly variable environmental conditions, and show a wide geographic variability of biological characteristics. The environment in the

lobster habitat widely varies from the cold oceanic waters of the northern extent of Newfoundland, to the highly seasonal Gulf of St Lawrence waters (-1 to 20°C), the deep canyons off Maine and Nova Scotia (more seasonally stable temperatures around 6°C), the warmer seasonal waters of North Carolina. A strong geographic variability exists in growth, age at first maturity, life cycles in alternating periodicities for moulting, mating and spawning. The fishing regulations also widely vary over the range of distribution of lobster.

Despite this high variability, the genetic composition of the stocks appear to be remarkably homogeneous. Published and unpublished results from modern techniques such as isoelectrophoresing of a wide variety of enzymes, on mitochondrial DNA analysis have failed to identify distinct genetic populations. The exchanges between geographic sites can occur through extensive movements at advanced benthic stages as well as larval drift during several weeks to months at early stages. The distinct geographic life history patterns revealed under different life history conditions appear to be flexible adaptations rather than fixed genetic differences between individuals.

Under such conditions, it can be logically expected that North American lobster stocks as a whole react as a homeostatic system to minute local disturbances. Local disturbances may have locally drastic effects over a limited number of years, but these effects will dampen over a longer time. Through recruitment feed back the system has its own periodicity. The lag time between oscillations of the effect and population oscillations may progressively extend until the effects are dampened out. This is possibly what Drinkwater, and Sutcliffe observed about local responses of lobster stocks to fresh water outflow. The global effect of freshwater is not noticeable or may even be reversed.

Temperature may have global long term effects on the landings. Temperature changes are less locally variable than river outflow. The 0 lag effect may be a catchability effect. It has been frequently reported that lobster are more active and enter more readily traps as temperatures seasonally raise in the Gulf of St Lawrence. Most of a recruitment group is caught over two years, but it may be more advantageous in terms of yield per recruit to catch most of the recruitment group over a single year. The 6 year lag can be explained by an enhancement of growth in early stages, and subsequently a reduction of mortality.

The null to positive relationship between cod and lobster landings was unexpected, and does not verify the intuitive perception that a depletion of cod stocks may have a positive effect on lobster landings. Predation has more subtle effects than what can be detected by a linear correlation. The most effective lobster predators are not necessarily commercial species. Conan (personal observations) has reported while diving active predation of a small but very common unpalatable fish, the blue perch (*Tautoglabrus adspersus*) on moulting lobsters.

Rather than trying to detect and predict the effects of local phenomena on the lobster stocks, it may be more promising to understand their behaviour over time as the behaviour of a global system in which order is the result of chaotic processes. The system reacts to environmental and human effects according to orbits that can be better visualized in phase-space than in a simple plot of landings vs time. Although it may be unrealistic to exactly forecast the next location in phase space from a given location, historical records allow to outline the most usual possible trajectories, their divergence and unstable vs stable locations.

In unstable locations, any event as minute as it may be can have drastic effects, simple management goals such as maximum yield per recruit will not be realistic, while they might be realistic in stable locations of the phase space orbits.

Identifying a present fishery location in phase space may provide information on the stability of a present state and can be useful for risk analysis for directing investment on fishing fleet development for instance. Chaos analysis does not lead to contemplation of unmanageable fisheries, but may provide guidance for long term management approaches more sophisticated and realistic than simple deterministic intuitively attractive tools.

Chaos analysis does provide serious clues that the North American stocks behave globally as a chaotic system. Three interrelated orbits with different degrees of stability can be suspected to exist in phase space for Canadian data. The results are similar although less well marked for regional data. The set of data from USA that we processed is shorter, 2 orbits can be recognized and a third one would probably be completed should we incorporate start of the century records. Mixed Canada/USA records behave similarly to the Canadian records *per se*.

The present increase in landings over the past ten years appears to be a return to an ancient initial situation, which vanished at the onset of the disturbances caused by fishing. Based on historic records, it does not appear to be gravitating with stability round a high (landings) point of attraction. It may be expected that this location be visited only very occasionally.

Disclaimer

This document was written in order to stimulate exchange of ideas between scientists. The opinions expressed in this document are personal and do not imply any endorsement by the Department of Fisheries and Oceans Canada. The data used is either in the public domain in the form of published or about to be published information. No specific management recommendations are implied.

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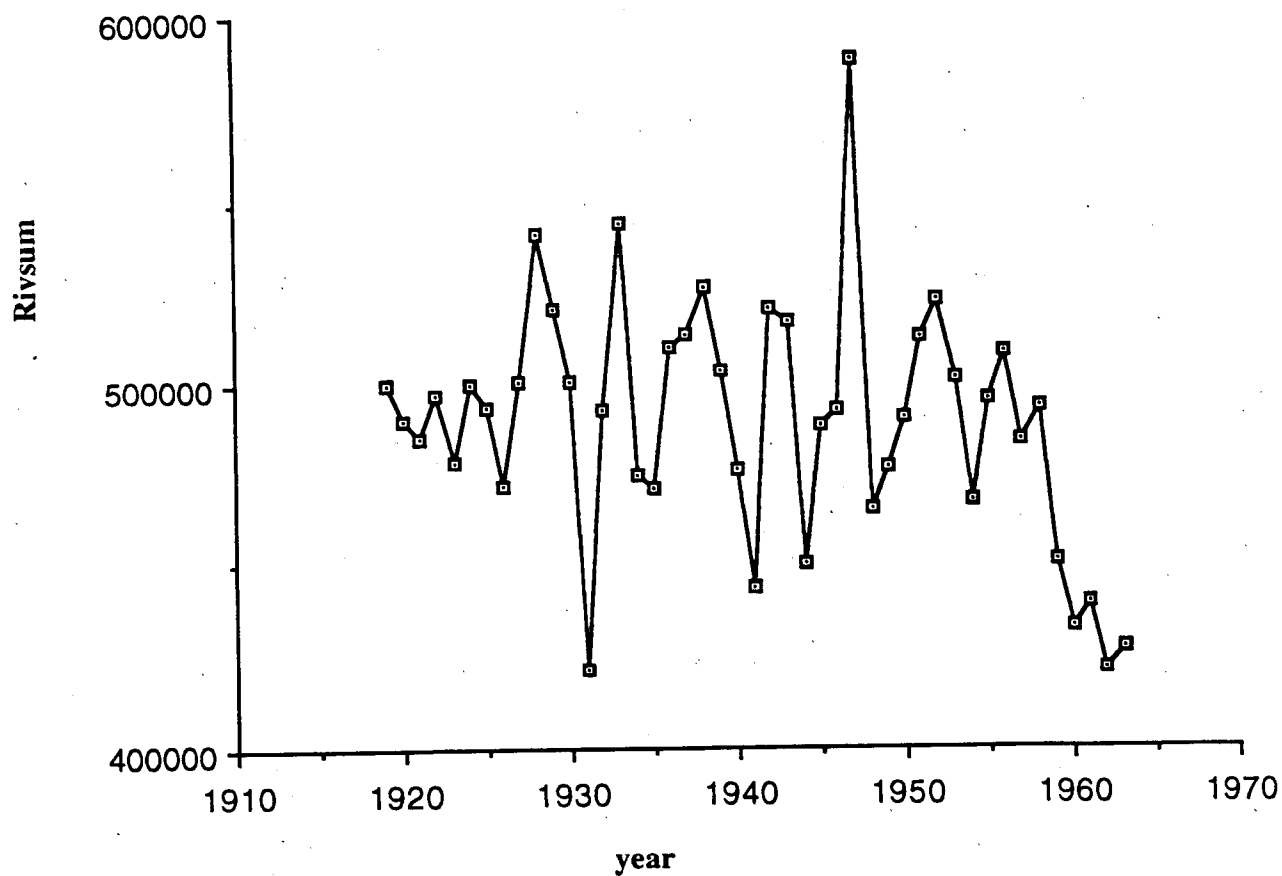


Fig. 1

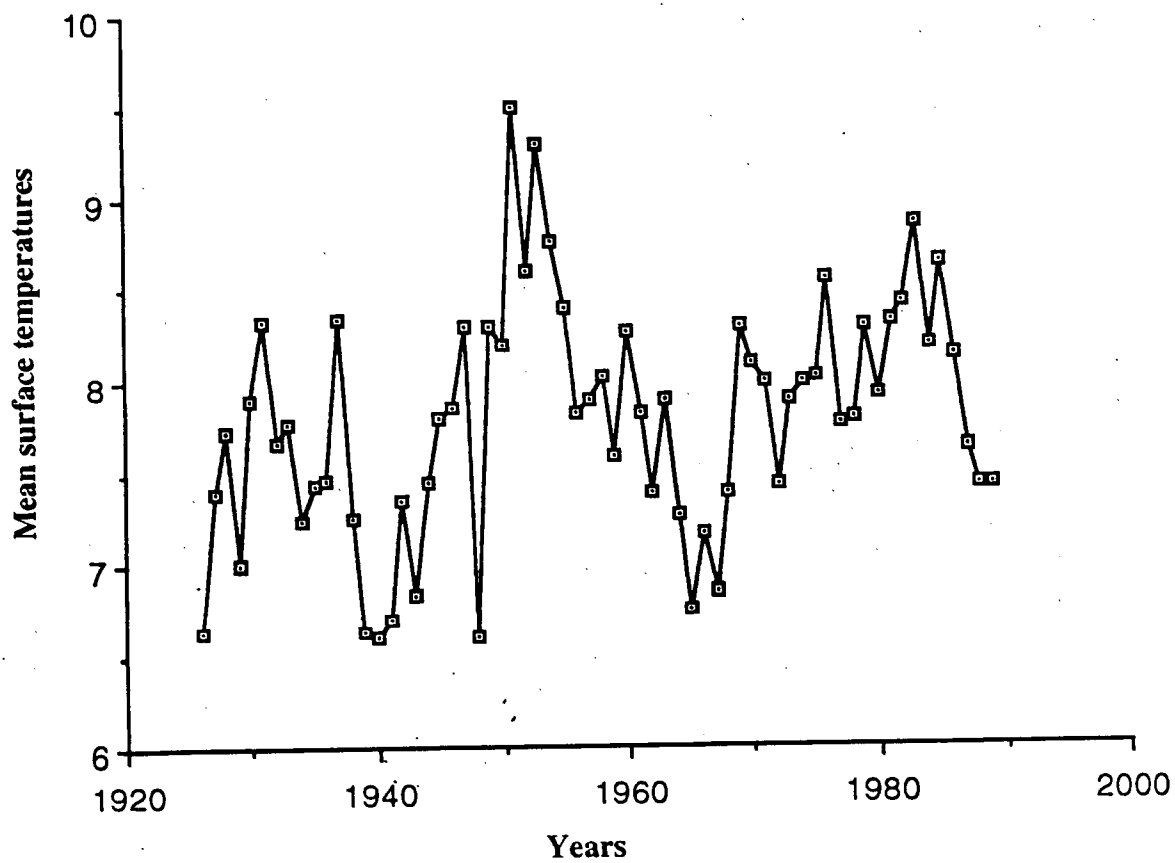


Fig. 2

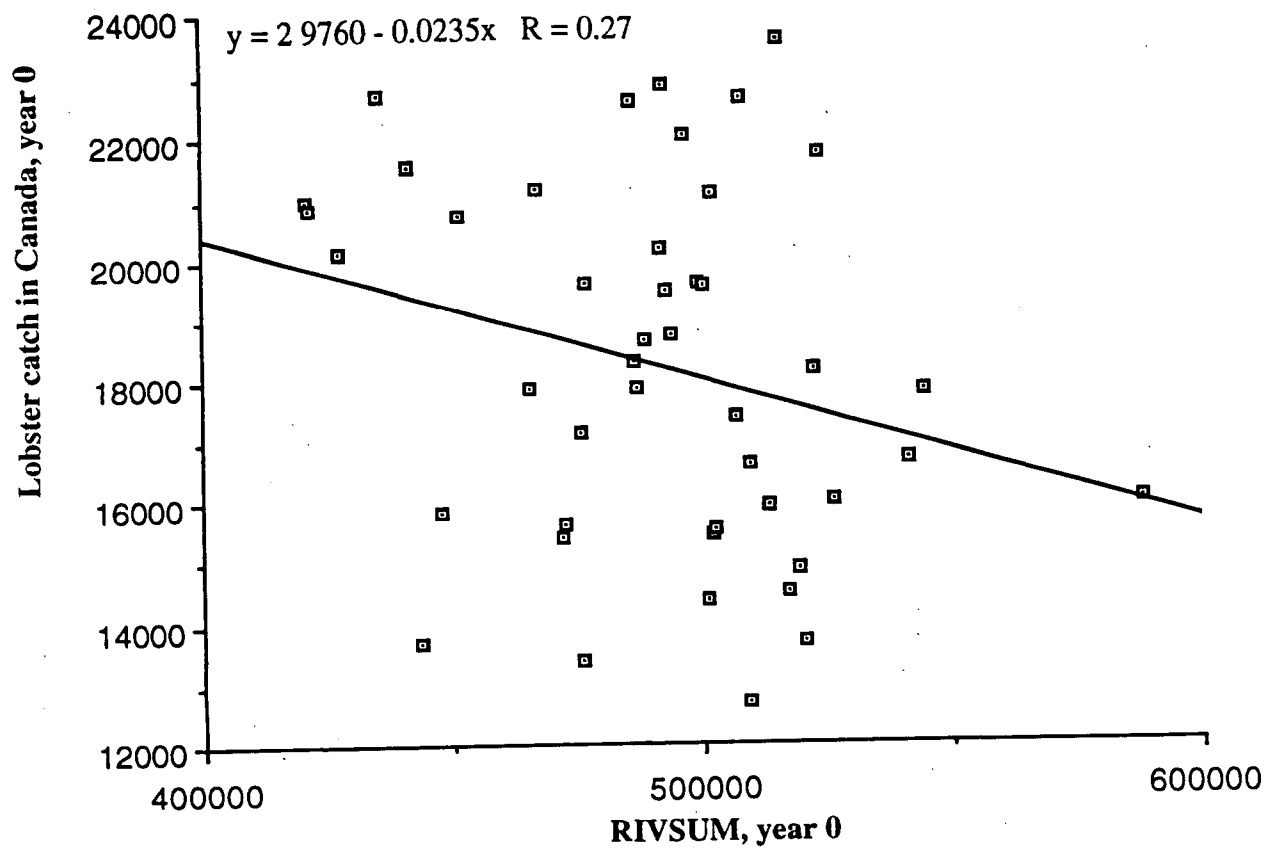


Fig. 3

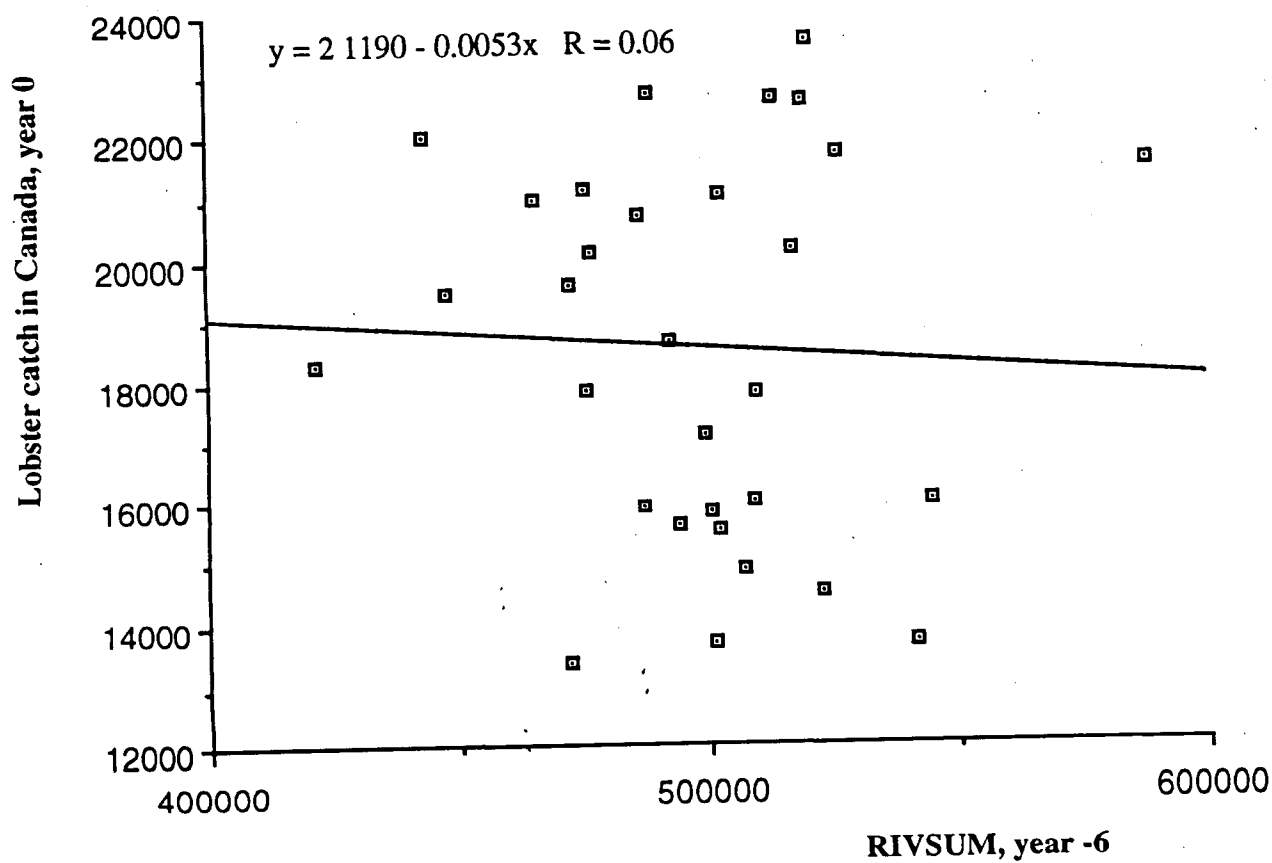


Fig. 4

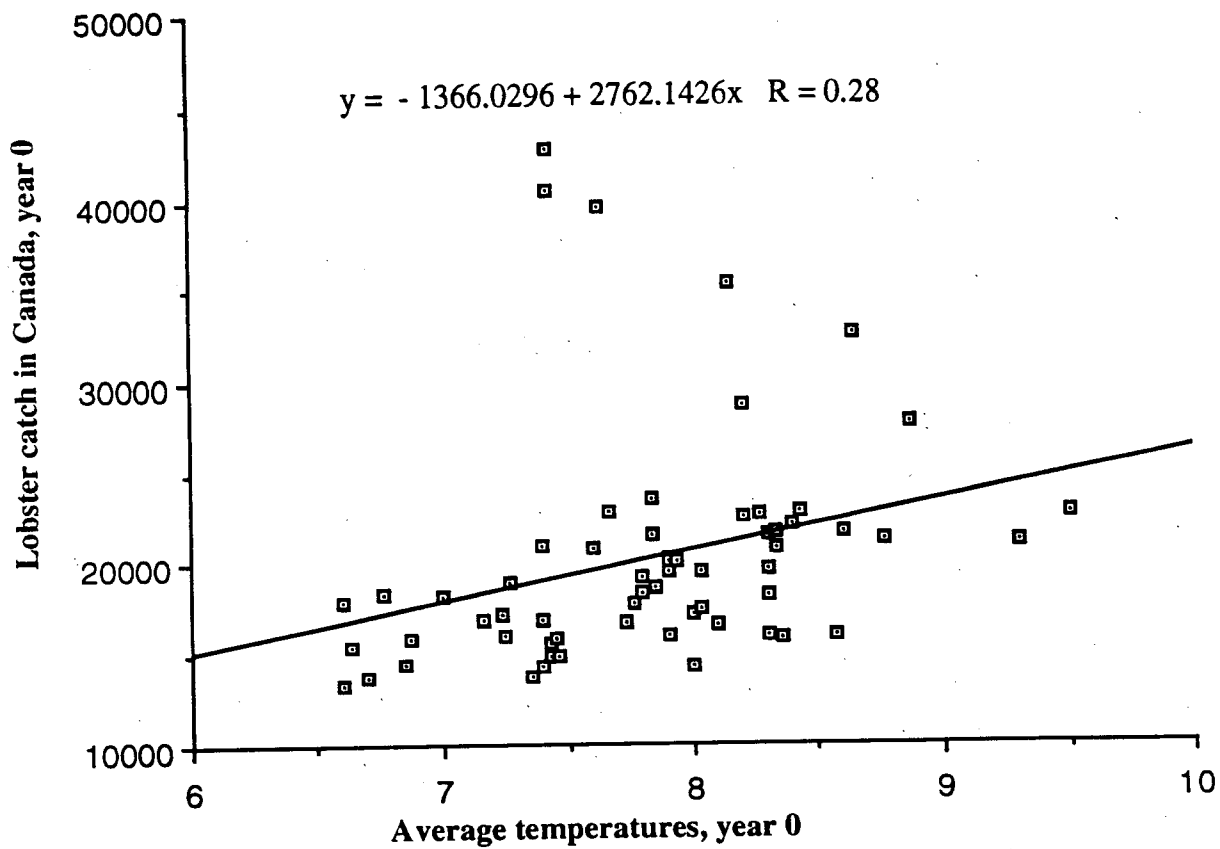


Fig. 5

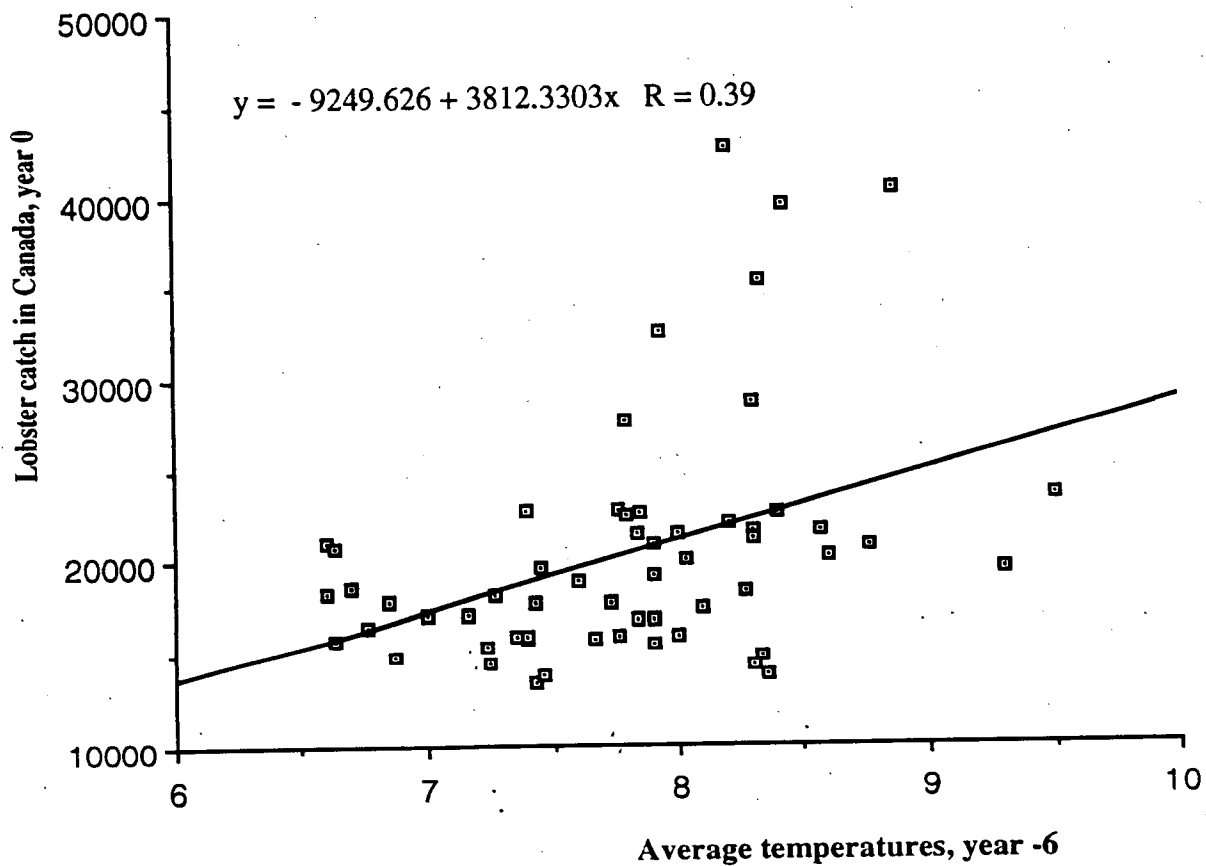


Fig. 6

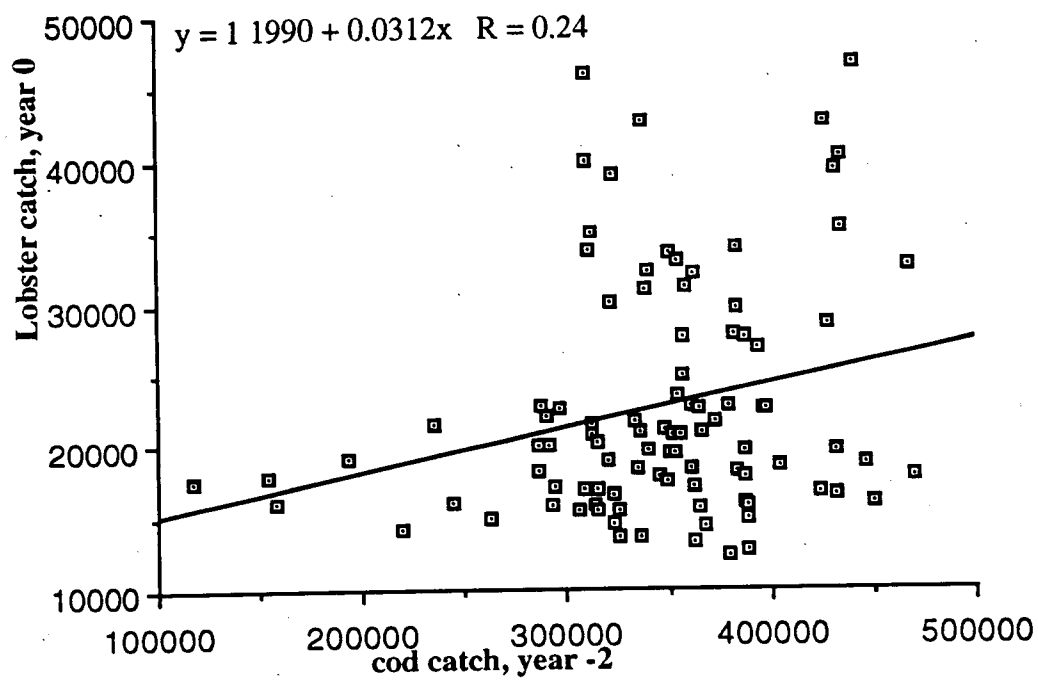


Fig. 7

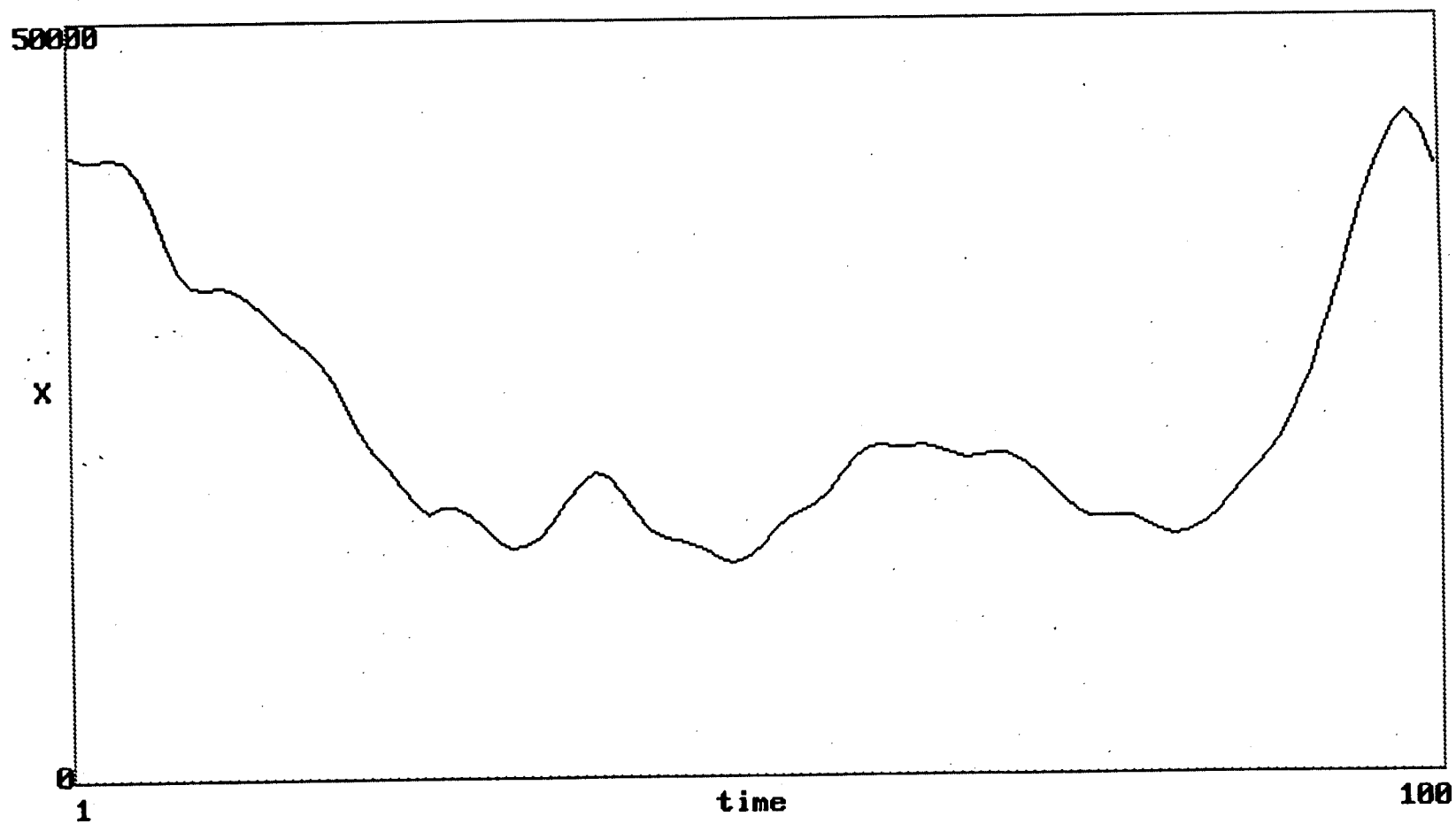


Fig. 8
Total Canadian landings of H.A.

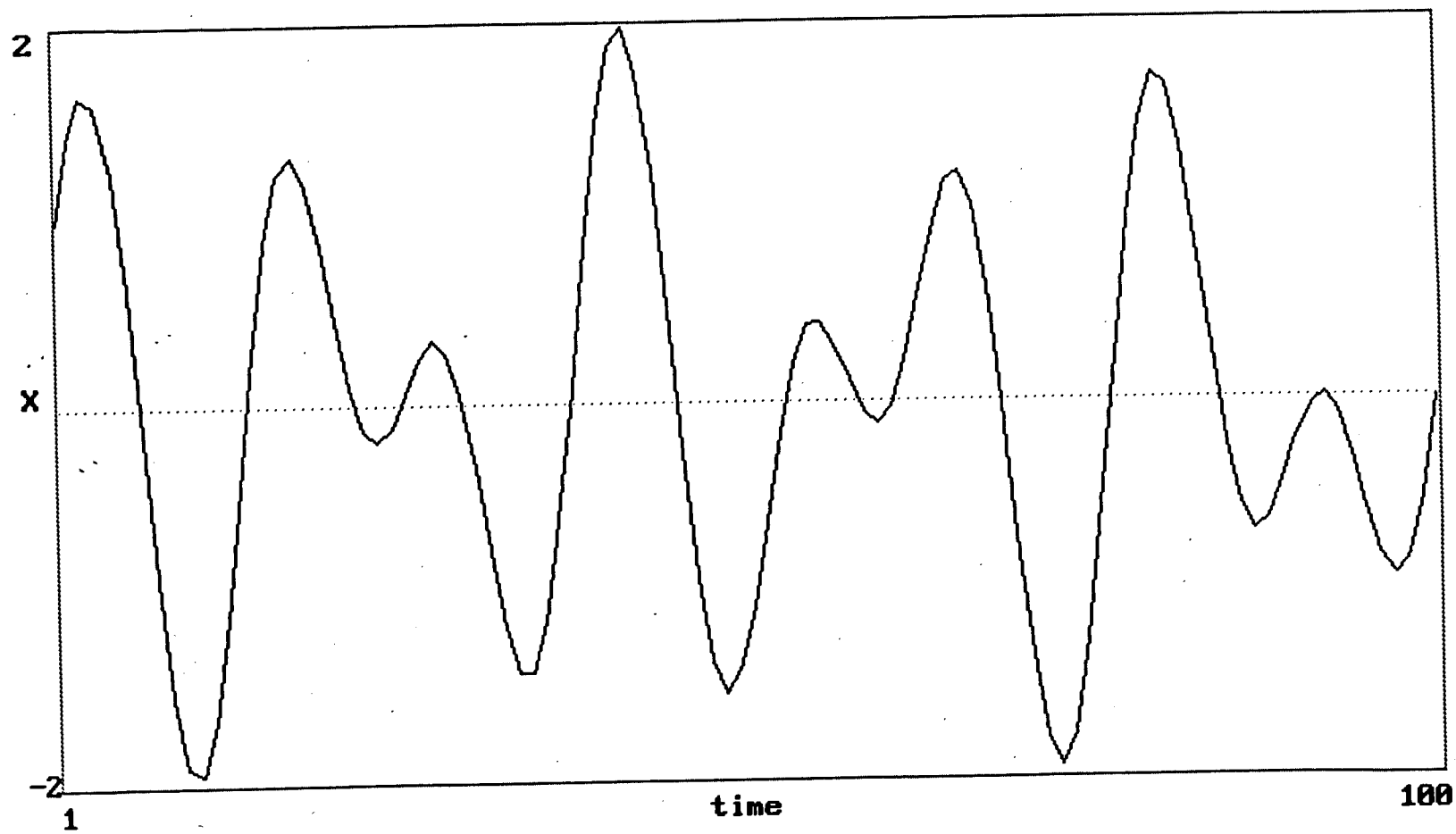


Fig. 9
 Data from $\sin\left(\frac{t}{2}\right) + \cos\left(\frac{\sqrt{5}-1}{2} \cdot \frac{t}{2}\right)$ Twosinus)

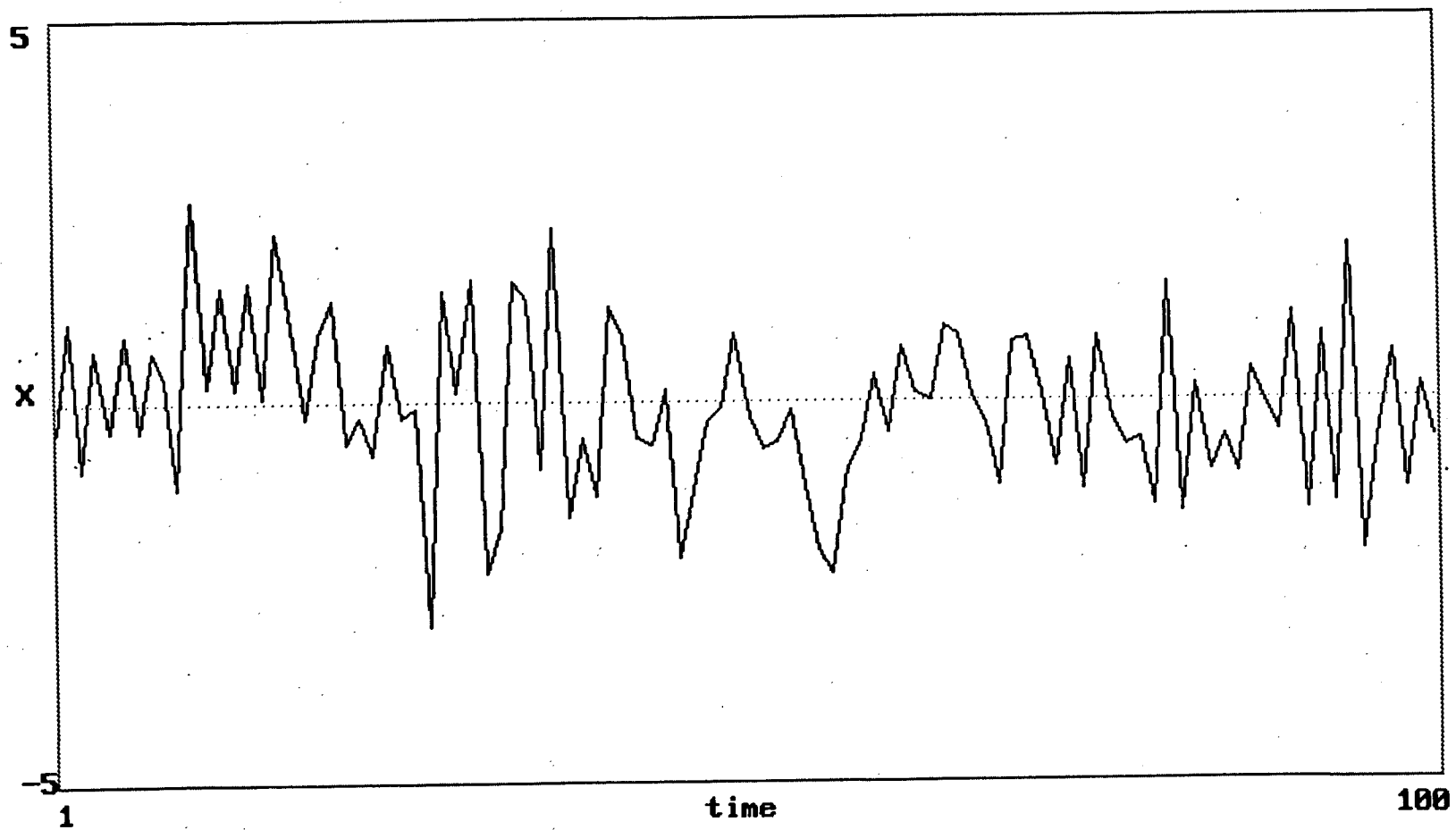


Fig. 10
Random data. White noise

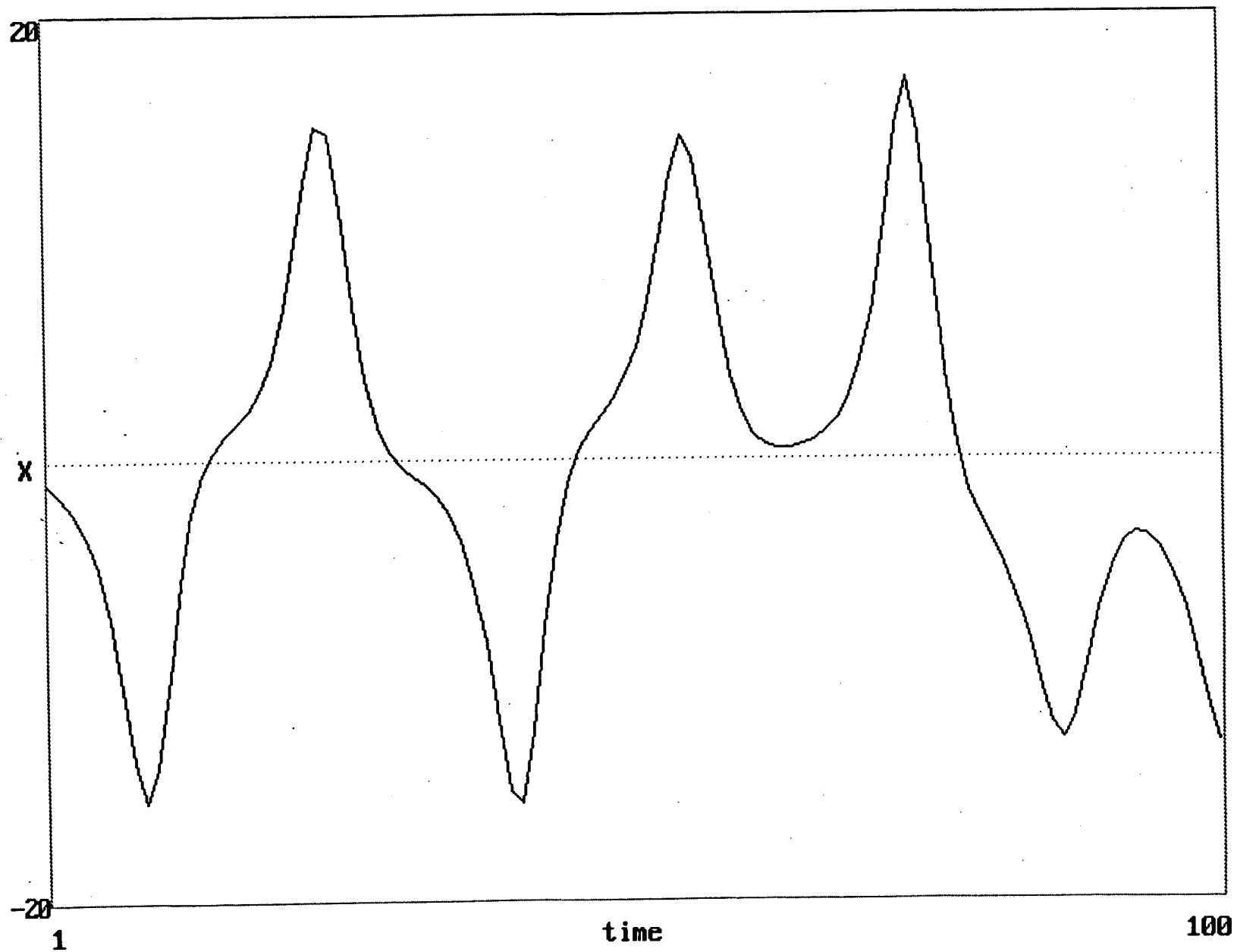


Fig. 11
Data from the Lorenz attractor

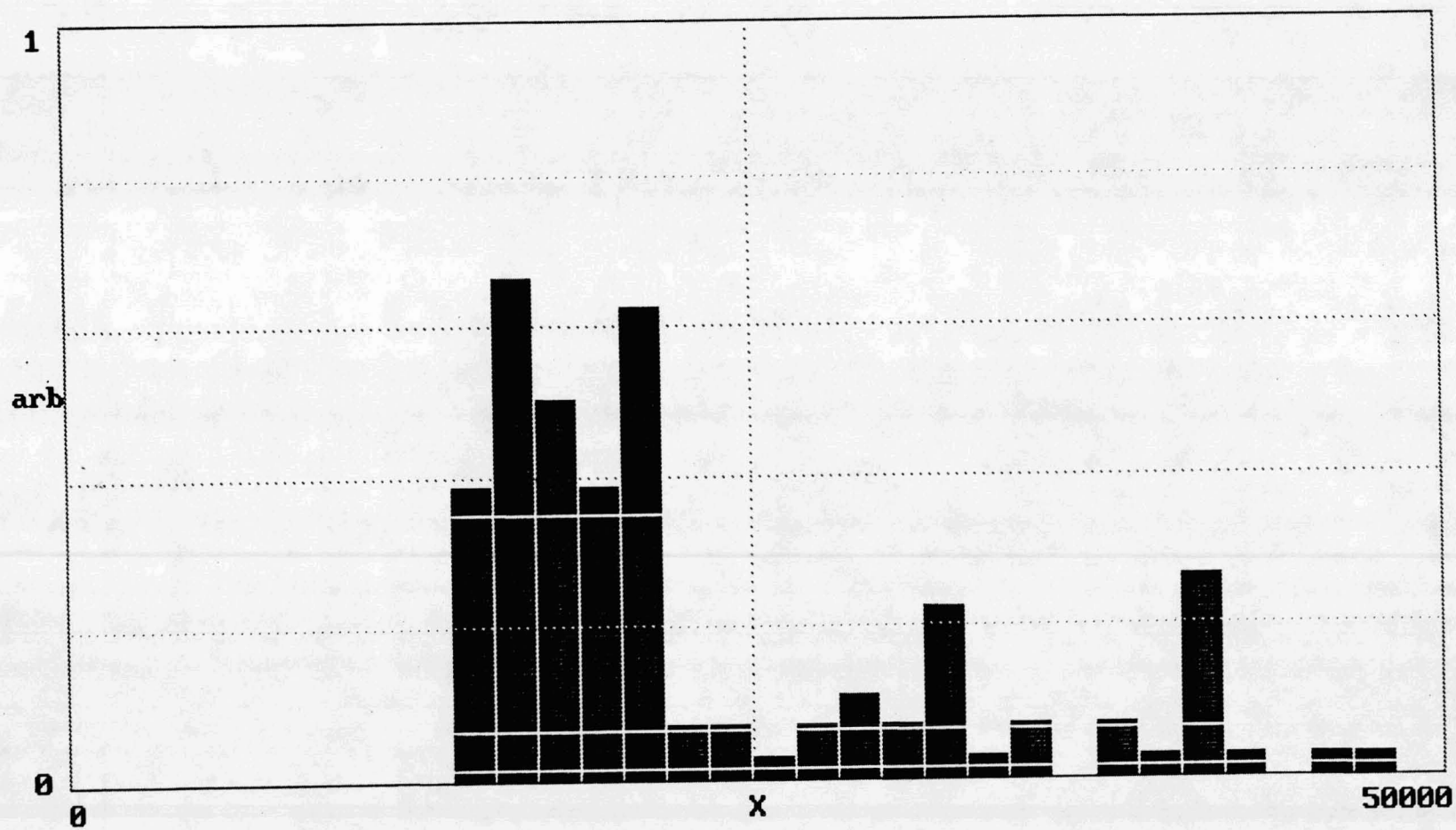


Fig. 12
Probability distribution. Canadian data of H. A. landings

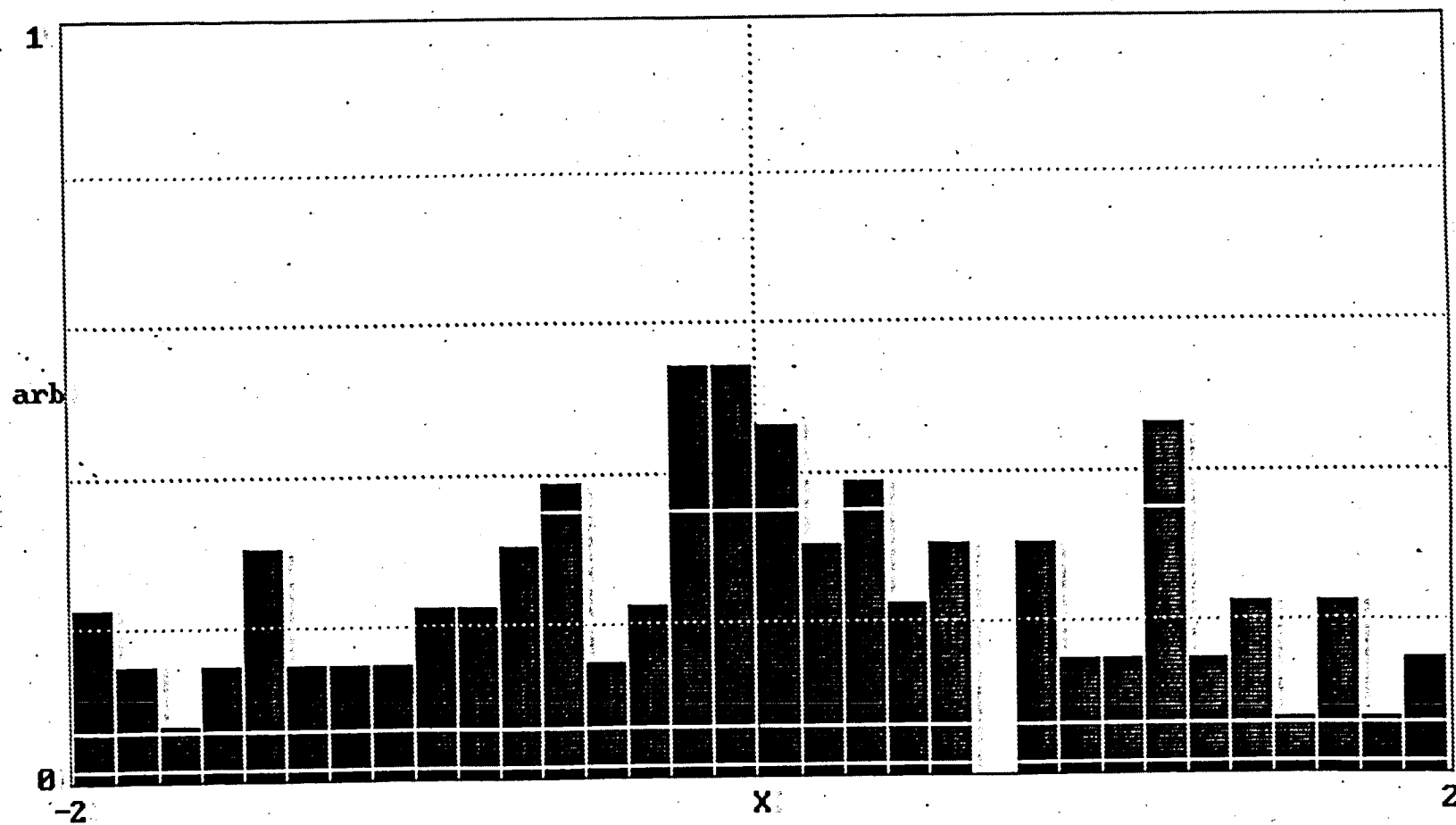


Fig. 13
Probability distribution. Twosine.

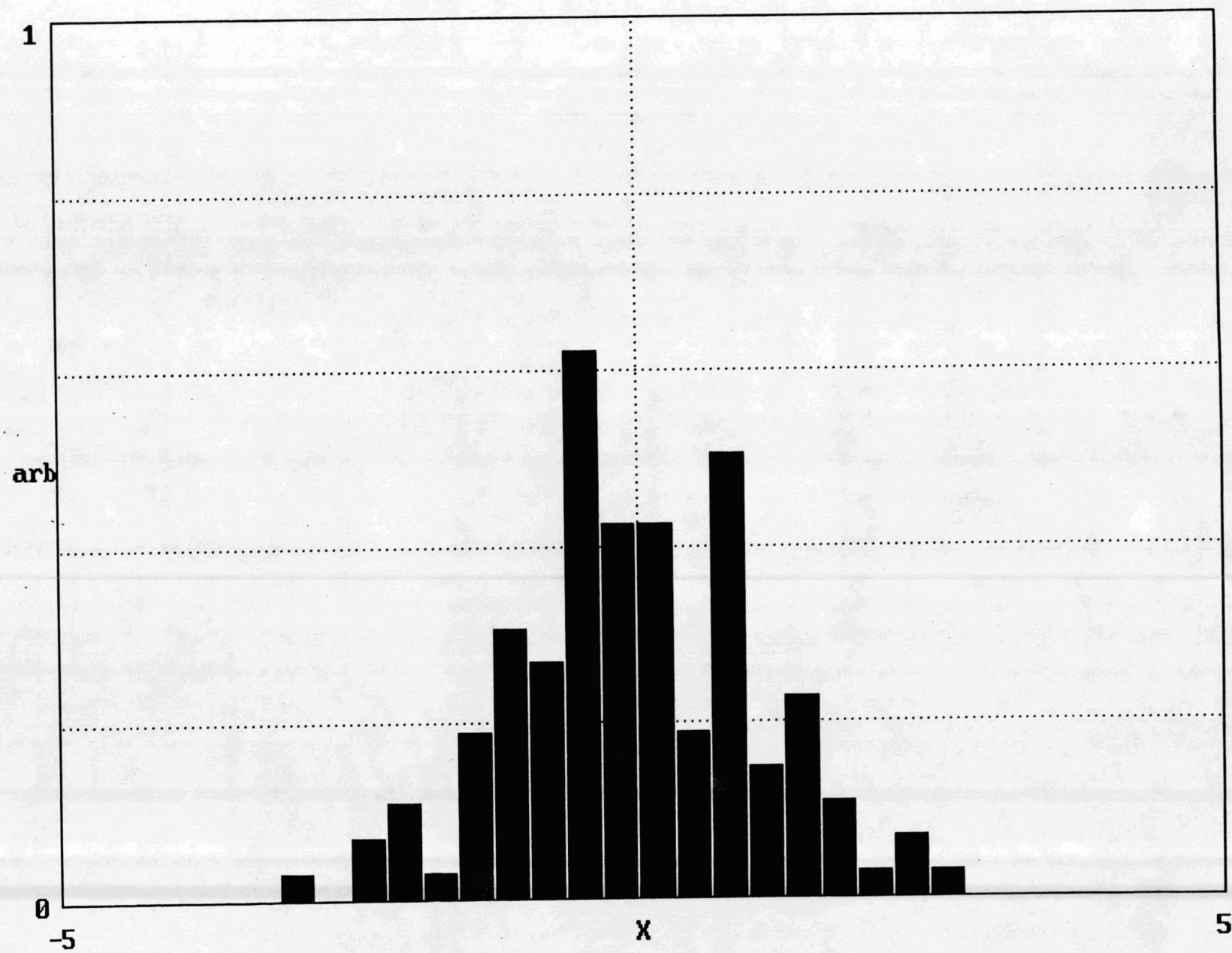


Fig. 14
Probability distribution. White noise.

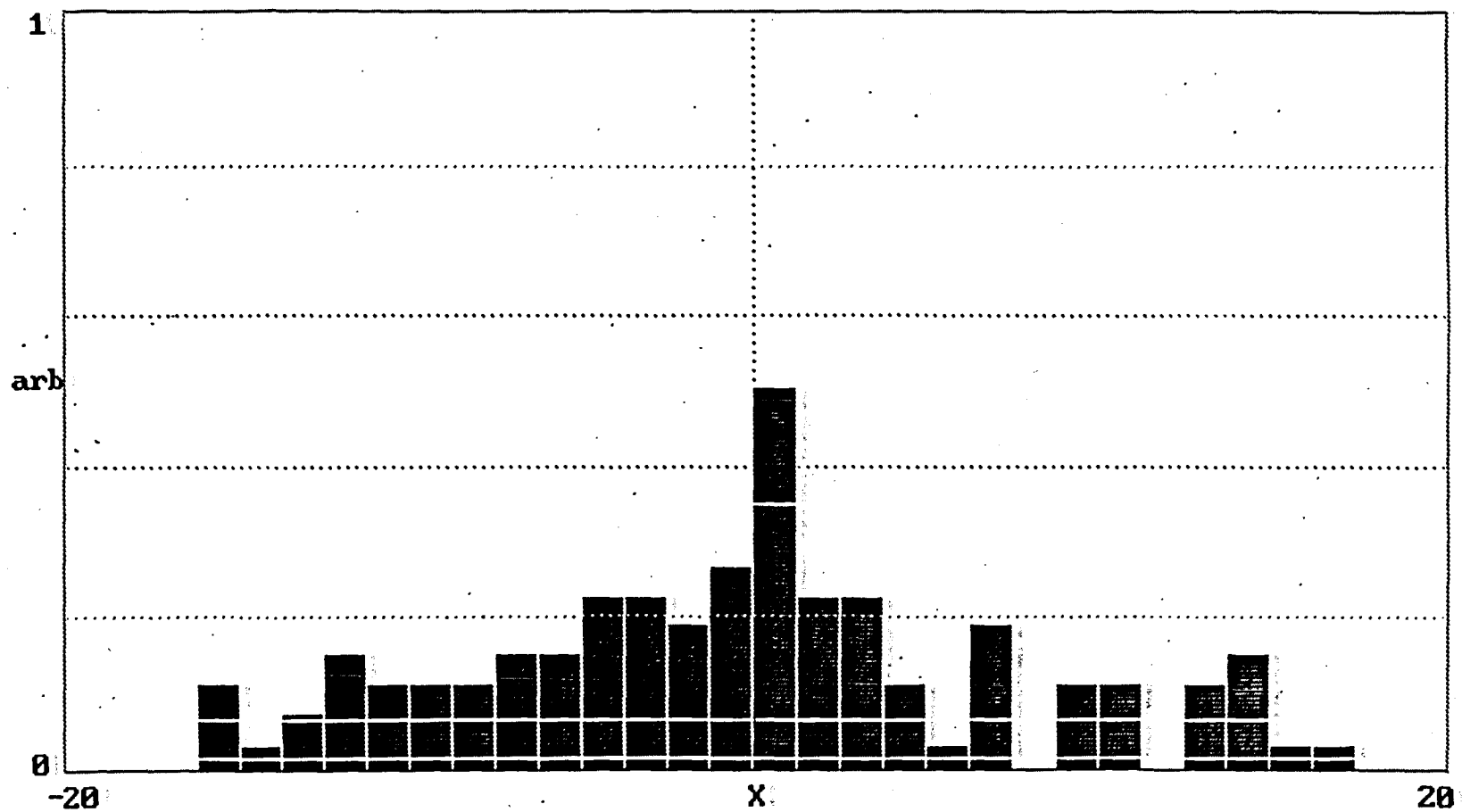


Fig. 15
Probability distribution. Lorenz attractor.

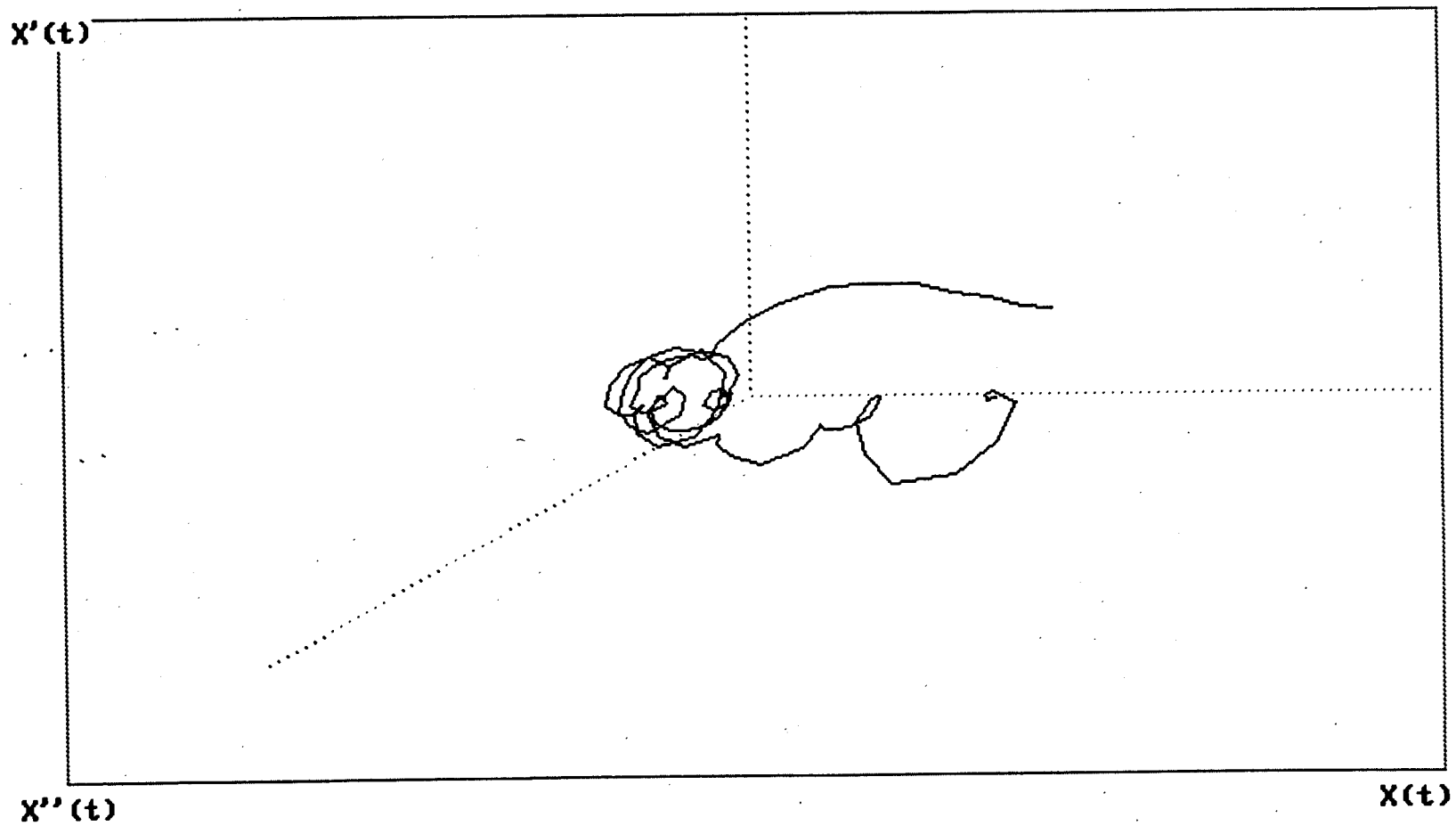


Fig. 16
Phase-space plot. Canadian landing of H.A.

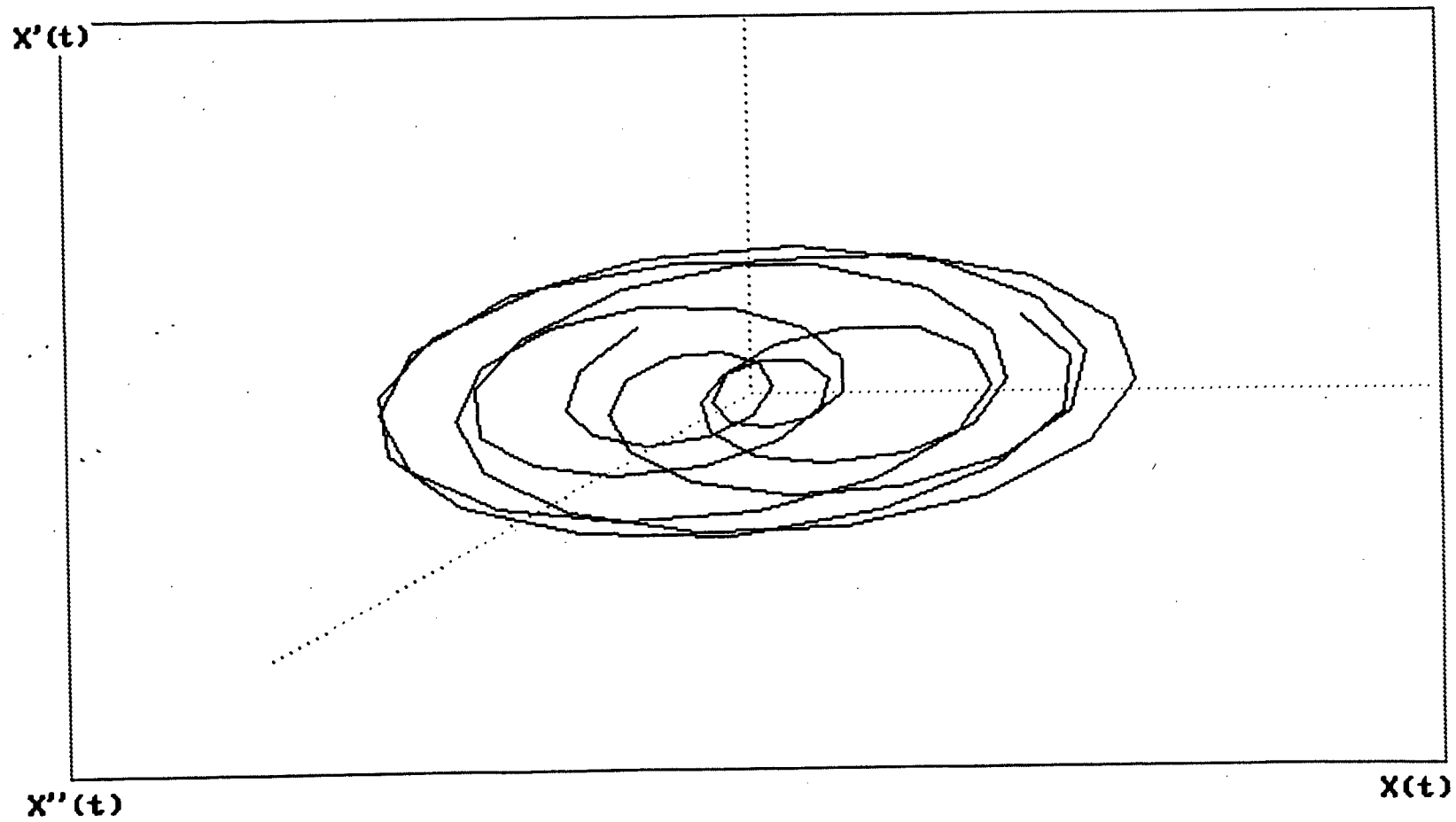


Fig. 17
Phase-space plot: Twosine.

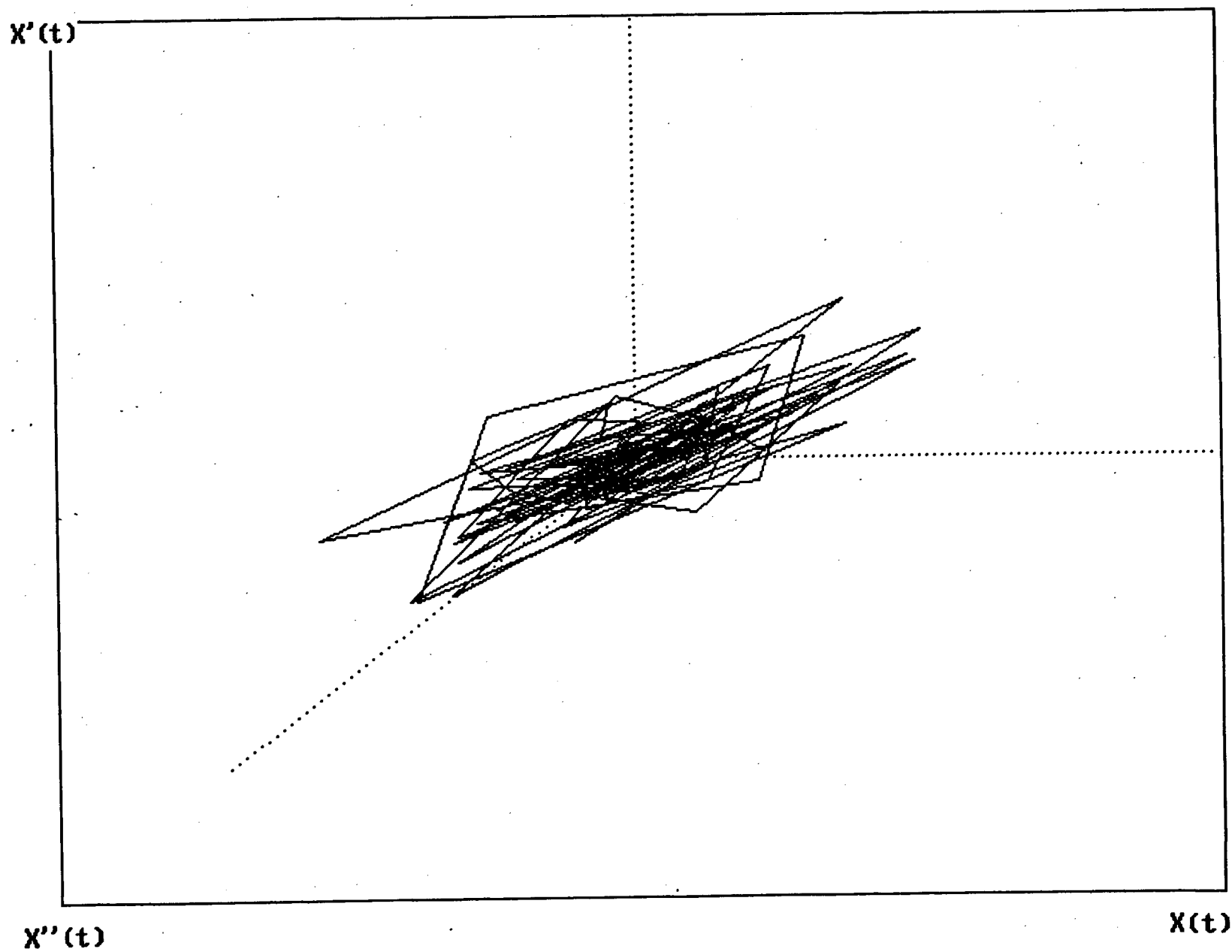


Fig. 18
Phase-space plot. White noise.

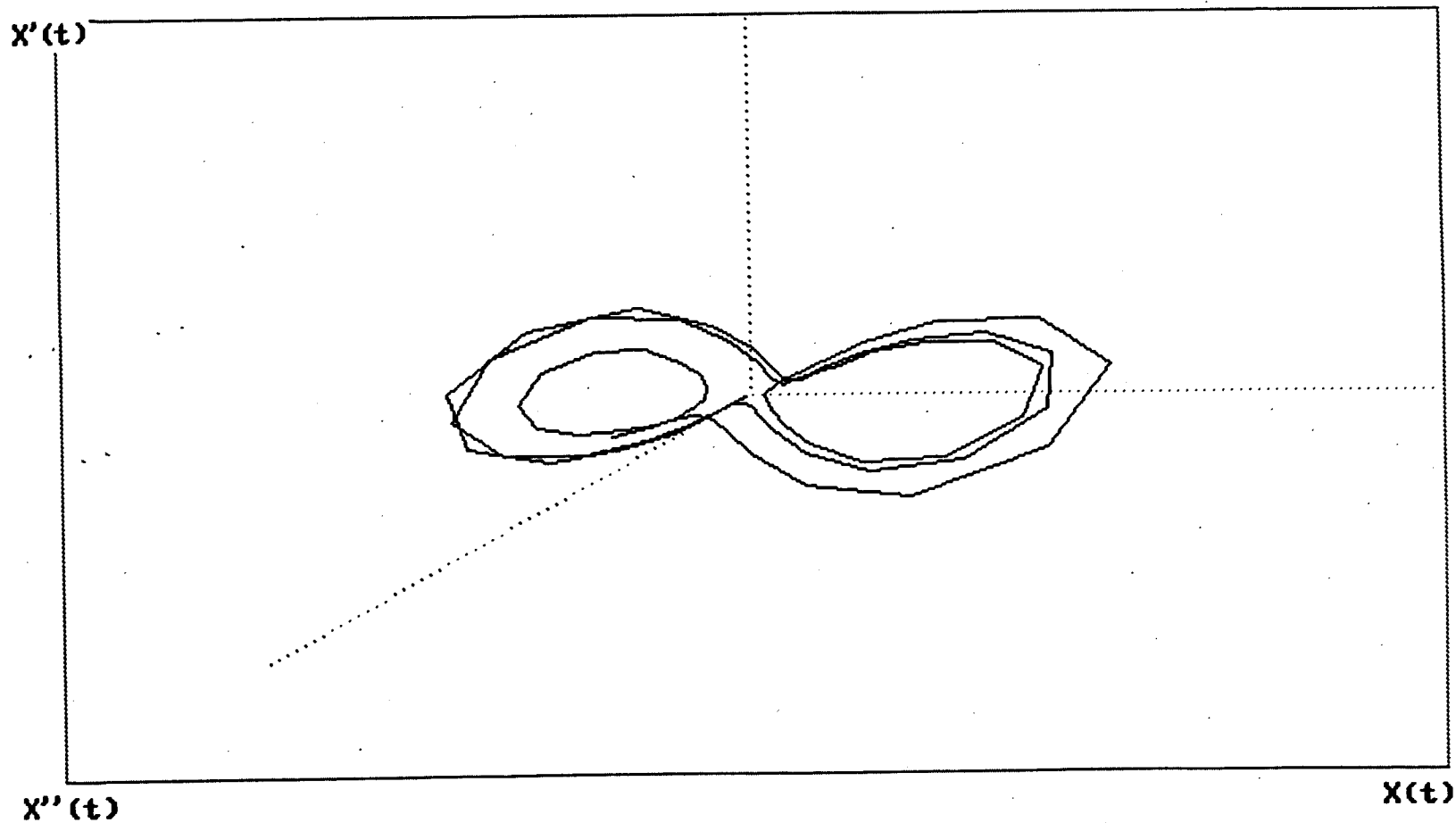


Fig. 19
Phase-space. Lorenz attractor

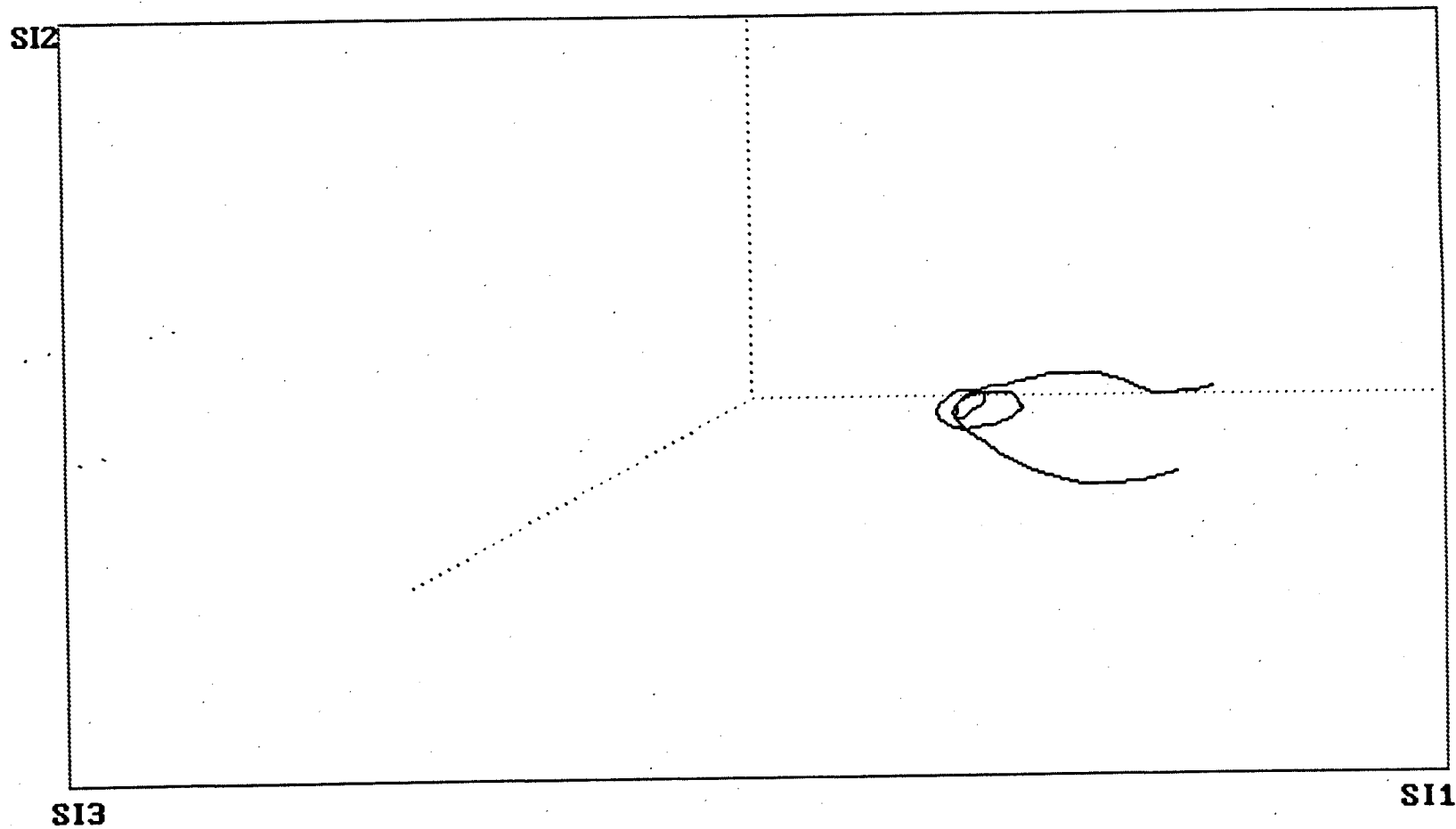


Fig. 20
Phase-space. Canadian landing of H.A.
Attractor generate from the three principal eigenfunctions.