Harmonic analysis of horizontal current data
The method and some new developments*

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Introduction

The hydrodynamics of the North Sea and of adjacent marine areas is a problem of great practical importance. A valuable modelling effort has been devoted to it in the past, and it is still the subject of many current research works (3 dimensional models, residual circulation models).
On the other hand, much information has also been gathered in situ (sea elevation and horizontal current time series, for our purpose), and they are worth being examined in detail. Field data are important because they can provide the right boundary conditions to the numerical models, and also because they can be used to estimate the quality of the model and to detect its possible shortcomings. Finally, they are very interesting by themselves because they enrich our "naturalistic" knowledge of the area.

The tide is the leading hydrodynamic process in the North Sea basin. From a practical point of view, its most interesting feature is that the motion has components with well defined and accurately known periodicities.

It was thus interesting to perform the harmonic analysis of elevation and current data, in order to extract the harmonic components out of the

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* This is JONSDAP '76 contribution n° 66.

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records. Other effects which are not purely tidal (such as storm surges), are considered as noise in the present analysis and cannot be included in this framework. It is then evident that a description in terms of tidal components alone cannot be complete.

During JONSDAP 76, a valuable mooring effort has been performed, and in Belgium, current meters (and recently tide gauges) have been moored for several years in the Southern Bight of the North Sea. The analysis of the data set has not yet been completed, but we will summarize here the method we have developed so far to analyse sea elevation as well as currents, putting much stress on the determination of the maximum number of components, and on the estimation of the quality of the results.

1.- Tidal components

The cornerstone of all works on tides is the determination of the tide generating potential and of its harmonic development. This was performed originally by Doodson (1921), who showed that the tidal potential can be written as a sum of constituents

$$\Phi = \sum_r f_r R_r \cos(V_r + u_r) \qquad (1)$$

R_r is the amplitude of the constituent,

V_r is the argument of the constituent, i.e. an angle changing steadily at the mean speed of the constituent,

R_r and V_r depend on the location considered,

f_r and u_r are nodal modulation factors, taking into account the existence of several terms in one constituent. These terms have very close frequencies and cannot be separated in one year's analysis (the difference between their frequencies is less than $2.738\ 10^{-3}$ cycle/mean solar day).

f_r and u_r are determined from Doodson's original results (Godin,1972; Loffet, 1980 a), and depend on slowly varying astronomical arguments.

The tidal components can be gathered in groups and the groups in species. The average frequency difference is :

between groups of the same species : $\sim 3.66\ 10^{-2}$ cycle/mean solar day
between species : ~ 0.966 cycle/mean solar day.

A detailed list of the constituents and of all their features can be found in Loffet (1980 a).

The tidal potential is not found as such in nature, and only some effect of the periodic forcing introduced by the tidal potential can be found in the actual tidal records :

1) the relative magnitude of the different components can vary considerably from the results of the tidal potential;

2) the non linear interactions in the dynamic balance can introduce new terms, with new frequencies unknown in the development of the tidal potential. This is particularly the case in shallow water areas as the southern North Sea.

The tidal part of the data record will still be represented by an expression in some way similar to (1), i.e.

$$h = h_o + \sum_r f_r H_r \cos(V_r + u_r - K_r) \qquad (2)$$

(h denotes the elevation or one component of the current).

V_r, f_r, u_r have the same meaning as in the development of the tidal potential.

The amplitudes (h_o, H_r) and the phase lags K_r are the harmonic constants of the tidal component considered. They depend on the location.

The expression $(V_r + u_r) - K_r$ can advantageously be replaced by its equivalent $(V_r + u_r)_{Greenwich} - g_r$, where only the last argument g_r , the Greenwich phase lag depends on the location.

We have thus

$$h = h_o + \sum_r f_r H_r \cos[(V_r + u_r)_{Gr} - g_r] \qquad (3)$$

The harmonic constants for the elevation are H_r , g_r. For the horizontal current, rather than using the harmonic constants of the east and north components (i.e. H_{rx} , H_{ry} , g_{rx} , g_{ry}), it is advantageous to express the results in terms of the tidal ellipses elements, i.e. :

- the major semi-axis length M_r
- the minor semi-axis length m_r

- the ellipse orientation θ_r (geographical coordinates)
- the phase lag ϕ_r (or time lag ϕ_r/n_r , n_r being the speed of the constituent r) between culmination of the fictitious generating celestial body at Greenwich or at the local meridian, and the occurrence of the maximum velocity.

These are defined

$$M = \left\{\frac{1}{2}[H_x^2 + H_y^2 + [H_x^4 + H_y^4 + 2H_x^2 H_y^2 \cos 2(g_x - g_y)]^{\frac{1}{2}}]\right\}^{\frac{1}{2}}$$

$$m = \left\{\frac{1}{2}[H_x^2 + H_y^2 - [H_x^4 + H_y^4 + 2H_x^2 H_y^2 \cos 2(g_x - g_y)]^{\frac{1}{2}}]\right\}^{\frac{1}{2}}$$

$$\theta = \frac{\pi}{2} - \frac{1}{2}\{\arg(H_x\cos g_x + H_y\sin g_y , H_y\cos g_y - H_x\sin g_x)$$

$$+ \arg(H_x\cos g_x - H_y\sin g_y , H_y\cos g_y + H_x\sin g_x)\}$$

$$\phi = - \frac{1}{2}\{\arg(H_x\cos g_x + H_y\sin g_y , H_y\cos g_y - H_x\sin g_x)$$

$$- \arg(H_x\cos g_x - H_y\sin g_y , H_y\cos g_y + H_x\sin g_x)\}$$

The knowledge of all harmonic constants makes it possible to derive the tidal part of the sea elevation or currents. However, in practice several limitations will occur :

- we don't know which harmonics to consider, and we will have to restrict their number;
- it is not possible to derive all those which are wanted;
- the accuracy of the results is limited.

2.- Tidal constituents considered

The main constituents arising from the tidal potential will be searched for (Doodson, 1921; Godin, 1974).

Problems appear when trying to determine which shallow water constituents have to be added :

1°) the non linear interactions in the dynamic balance are known to generate harmonics. In order to determine them, the dynamic equations can be decomposed spectrally, and the harmonics appear in different terms of the equations (Le Provost, 1974). Such developments are too far from our present purpose, and we will only consider harmonics obtained by linear combinations of the most important potential terms in species 1 and 2, i.e. :

species 1 : Q1 , O1 , P1 , J1 ,

species 2 : M2 , S2 , N2 , K2 , (ν2) , (μ2) , (L2).

Many constituents can be defined in this way, and in a first approach, we will finally restrict ourselves to the 93 components considered in the Table of Harmonic Constituents of the Tide of Monaco (Loffet, 1980 a).

2°) Many components, mainly in species 1 and 2, can have different origins (potential and/or different combinations between other components). The component with frequency 1.864547227 cycle/mean solar day, e.g. finds its origin in the potential (μ2) but also in non linear combinations (2[M2] - [S2] (= 2MS2), [N2] + [ν2] - [M2] , a.s.o.). Depending on the point of view adopted, the definition of V_r , f_r and u_r is then not unique. In very special cases, methods can be developed to discriminate between separate origins (Le Provost, 1974). This will not be applied here, and every time a multiple interpretation is possible, the expression for the term which seems the most important has been chosen. If the interpretation seems too dubious, the nodal correction is only disregarded (Loffet, 1980 a).

3.- Limitations of the analysis

Two limitations occur concerning the number of components which can be analysed in a record :

a) the data are not recorded continuously, but only at predetermined time intervals, and this could limit the analysis at high frequencies;

b) the length of the data record is finite, and then all desired tidal components cannot be separated from each other.

3.1.- SAMPLING INTERVAL

It is well known that for a phenomenon sampled with a time interval Δt , the highest frequency which can be analysed is the Nyquist frequency $f_N = \frac{1}{2\Delta t}$.

If energy is present at higher frequencies, it will be projected on (o, f_N). In tidal analysis, the highest interesting frequencies occur at species

12 or so [very seldom above, but often below (~ 8)] , so that with a Δt not exceeding one hour, the sampling interval does not introduce limitations. However, care must be taken to avoid aliasing during the recording of the data and the preparation of the time series.

3.2.- FINITE LENGTH OF THE RECORD

A signal of the form $f(t) = A\cos(2\pi nt + \phi)$ and defined for $t \in \,]-\infty, +\infty[$ has the Fourier transform $g(\lambda) = \frac{A}{2}[e^{j\phi}\delta(\lambda-n) + e^{-j\phi}\delta(\lambda+n)]$, where $\delta(\lambda)$ is Dirac's delta generalized function.

The same signal, but restricted to the finite time interval $[0,T]$, has the Fourier transform

$$\tilde{g}(\lambda) = \frac{AT}{2}\left\{e^{j\phi}e^{-j\pi(\lambda-n)T}\,\mathrm{sinc}\,[(\lambda-n)T] + e^{-j\phi}e^{-j\pi(\lambda+n)T}\,\mathrm{sinc}\,[(\lambda+n)T)]\right\}$$

where $\mathrm{sinc}(x) = \dfrac{\sin\pi x}{\pi x}$

The former spikes have now a finite width (the first zero crossing occurs at $\lambda = n \pm 1/T$), and as a consequence of this, it will be impossible to separate spikes occuring at two frequencies if they are too close.

The Rayleigh criterion considers that it becomes possible to separate two frequencies n_k and n_l if $|n_k - n_l|\, T \geqslant 1$, and that they are fully distinguishable if $|n_k - n_l|\, T \geqslant 2$.

Given a record of length T, if the Rayleigh criterion is applied, only a limited number of harmonic constants will be accessible, i.e. those for which $|n_k - n_l|\, T \geqslant 1$ holds. This will be restrictive in practice since the record lengths (for current data) seldom exceed two months.

This problem is directly related to the solution of the linear system of equations derived from the application of the least squares method to the estimation of the harmonic constants. The matrices to be inverted are

$$A_{kl} = \frac{1}{2}[S_N(n_k - n_l) + S_N(n_k + n_l)]$$

$$k,\ l = 0,\ \dots\ m;\ n_o = 0;\ \text{for the cosine terms,}$$

$$B_{kl} = \frac{1}{2}[S_N(n_k - n_l) - S_N(n_k + n_l)]$$

$$k,\ l = 1,\ \dots\ m,\ \text{for the sine terms.}$$

with $S_N(\lambda) = \dfrac{\sin(\pi N\lambda\Delta t)}{N \sin \pi\lambda\Delta t}$

If the Rayleigh criterion is satisfied, the matrices A and B have their main terms on the principal diagonal, and are thus well conditioned for inversion. The analysis of two close frequencies can lead to an ill conditioned system and computational problems.

However, even without computational problems, the consideration of two close frequencies can lead to a very poor estimation of the harmonic constants, as will be shown in the next paragraph.

The Rayleigh criterion has thus been used so far as a general guiding line when searching which components it is possible to analyse.

4.- Estimation of the accuracy of the results

The recorded sea elevations or currents are not due to the tide only, and it is customary to consider them as made of several contributions :

a) the components which are analysed;

b) the components which are not analysed, either because they cannot be separated from the ones which are analysed, or simply because they are not wanted;

c) other processes, called noise and which can be attributed to different causes :

- instrumental inaccuracies or failures, accidental errors occuring during the preparation of the time serie;
- physical processes which are not considered in the tidal analysis (storm surges, e.g.).

The presence of all these contributions has some effect on the quality of the estimation of the analysed components :

a) There is a systematic error introduced by the components which are not analysed. If the system is well conditioned, it can be shown to be of the order

$$\sim \gamma_{n.a.}\, S_N\,(n_a - n_{n.a.})$$

where $\gamma_{n.a.}$ is the amplitude of the non analysed component, n_a and $n_{n.a.}$ are the frequencies of the analysed and non analysed components, respectively.

The importance of this error increases with the relative magnitude of the non analysed component (with respect to the analysed one), and with the closeness of both frequencies

$$\left[S_N \, (n_a - n_{n.a.}) \sim 1 - \frac{[\pi(n_a - n_{n.a.})T]^2}{6} \qquad \text{for } (n_a - n_{n.a.})T \ll 1 \right]$$

A new method is currently being developed to force separation in some way, but it is still too early to account for its results.

b) The noise introduces statistical variability in the estimation of the analysed components. This problem has been examined in details, and the results of the least squares theory have been applied to harmonic analysis (Hannan,1970; Rao,1973).

The noise was first assumed to be white, and the variances-covariances matrix of the estimators has been derived (Loffet, 1980 b). However, if the assumption of a white noise is reasonable for accidental failures and instrumental inaccuracies, it is far too restrictive if additional geophysical phenomena have to be included.

The least squares theory has then been developed in the case of a second order stationary noise process, and it makes use of its spectral properties (Loffet, 1980 b). The variances-covariances matrix of the estimators has been derived, as well as its asymptotic behaviour.

Using a particular application of the central limit theorem, it has become possible to derive statistical tests of hypotheses, and to build up confidence regions for the harmonic constants (Loffet, 1980 b). So far, the method has only been developed and applied when the system is well conditioned (the Rayleigh criterion is satisfied).

A detailed description of the method and of all the results involved is too lenghty to be described here. However, it is worth noting that if the system is well conditioned, the spectra sufficiently smooth, and the frequencies different from zero, one has :

$$\mathrm{cov}(a_i \, , \, a_{i'}) \approx s(n_i)/T \, . \, \delta_{ii'}$$

$$\mathrm{cov}(b_i \, , \, b_{i'}) \approx s(n_i)/T \, . \, \delta_{ii'}$$

$\mathrm{cov}(a_i\ ,\ b_{i'}) \approx 0$

for the sea elevation, where a_i , b_i denote respectively the cosine and sine terms of the i^{th} component, $s(n_i)$ is the value of the noise spectrum at frequency n_i , and $\delta_{ii'}$ the Kronecker symbol.

One also has :

$$\mathrm{cov}(a_{ix}\ ,\ a_{i'x}) \approx s_{xx}(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(b_{ix}\ ,\ b_{i'x}) \approx s_{xx}(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(a_{ix}\ ,\ b_{i'x}) \approx 0$$

$$\mathrm{cov}(a_{iy}\ ,\ a_{i'y}) \approx s_{yy}(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(b_{iy}\ ,\ b_{i'y}) \approx s_{yy}(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(a_{iy}\ ,\ b_{i'y}) \approx 0$$

$$\mathrm{cov}(a_{ix}\ ,\ a_{i'y}) \approx s_c(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(b_{ix}\ ,\ b_{i'y}) \approx s_c(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(a_{ix}\ ,\ b_{i'y}) \approx s_q(n_i)/T\ .\ \delta_{ii'}$$

$$\mathrm{cov}(a_{iy}\ ,\ b_{i'x}) \approx -\ s_q(n_i)/T\ .\ \delta_{ii'}$$

for the horizontal current, where a_{ix} , b_{ix} , a_{iy} , b_{iy} denote the cosine and sine terms along East and North directions of the i^{th} component, $s_{xx}(n_i)$ and $s_{yy}(n_i)$ are the values at n_i of the noise spectrum of the East and North components, $s_c(n_i)$ and $s_q(n_i)$ are the values at n_i of the co- and quadrature spectra of the two-dimensional noise process.

To be applied, the method needs the knowledge of the noise spectrum. This one is not known a priori, but an estimate of it is obtained from the residuals (i.e. what remains after removing the analysed components from the original time serie). The estimate is obtained using standard FFT techniques (Jenkins and Watts, 1969; Otnes and Enochson, 1972; Bath, 1974) which will not be developed here.

If the Rayleigh criterion is not satisfied, it is still possible to derive the variances-covariances matrix of the estimators and to determine the order of magnitude of the statistical error affecting the estimation of the tidal components. This can be used to analyse more components than the criterion allows, if the level of the noise is sufficiently low so that the results are still meaningful. We have not yet used this technique, but we have concentrated our attention on the estimation of the quality of the results derived when the Rayleigh criterion is satisfied.

5.- Classification of the components considered

Applying the Rayleigh criterion, the determination of all constituents considered needs a record lasting at least one year.
The determination of one component in each group needs at least 27 days.
The species (1, 2, 3, 4, ...) are separated in one day.

The records we examined last from one week (when data were lost because of a failure in the instrument) to two months at most. Different record lengths are thus available, and the considerations above are rather crude, because they don't consider the detailed structure of the frequencies of the desired components, and because they don't take their relative importance into account. It is thus interesting to classify them in some way, taking into account the points mentioned above. This was performed as follows : every component is given a weight,and the time needed for the determination of all of them is computed, applying the Rayleigh criterion; the ones giving way to this record length are determined, and among them, the one having the smallest weight is then extracted; the process goes on with the remaining ones, and is stopped when all have been extracted.

In this way, a classification giving the components which can be analysed versus the record length is obtained. In theory, the weights are unknown a priori, and the aim of the harmonic analysis is to determine them. However, it is possible to run the classification using known results at a close location.

Table 1

This table lists the harmonic components which can be determined, versus the length of the record. 60 components; results for Oostende after data from Melchior *et al.* (1967).

Record length (days)	Components
0.5	M2,M4,2MS6
1.1	+ O1,MO3
9.6	+ MNS2
13.7	+ K1,MK3
14.8	+ S2,M6,MS4,2SM2,MSf,2SM6,SK3,SO1,S4
27.3	+ M3,M1
27.6	+ N2,2MN6,N4,Q1,2Q1
31.8	+ L2,2MS2,MSN2,MSN6,Mm,SN4,J1
182.6	+ K2,P1,MK4,2MK6,MKS2,OQ2,OP2,SO3,Mf,MSK6,MP1, SK4,KJ1,OO1
193.6	+ χ1
205.9	+ ν2,λ2,2N2,ρ1,σ1,θ1
365.2	+ Sa,Ssa,S1,ψ1,φ1,T2,R2,π1

In the present analysis, we took the sea elevation given at Oostende (Melchior *et al.*, 1967) and the mean sea elevation at three belgian coastal harbours (Hydrographische Dienst der Kust, private communication) as weights (60 components). The results are listed in table 1 and 2. They are very similar, and will be used as a guide in the Southern Bight of the North Sea.

Table 2

This table lists the harmonic components which can be determined, versus the length of the record. 60 components; mean for Nieuwpoort, Oostende and Zeebrugge. (Hydrographische Dienst der Kust, private communication.)

Record length (days)	Components
0.5	M2,M4,2MS6
1.1	+ O1,MK3
9.6	+ MNS2
13.7	+ K1,MO3,OO1
14.8	+ S2,M6,MS4,2SM2,MSf,2SM6,SK3,S4
27.3	+ M3,M1
27.6	+ N2,MN4,Q1,2MN6,2Q1,J1
31.8	+ 2MS2,L2,MSN2,Mm,MSN6,SN4
182.6	+ K2,P1,MK4,2MK6,OP2,Mf,SO3,MSK6,MKS2,KJ2,φ1, SK4,MP1,SO1
193.6	+ χ1
205.9	+ ν2,2N2,λ2,ρ1,σ1,θ1,
365.2	+ Sa,Ssa,S1,ψ1,T2,R2,π1

6.- Practical realization of the harmonic analysis

The harmonic analysis of the current data records is made of several steps we will develop here.

6.1.- PREPROCESSING OF THE DATA

The data are usually recorded with a time interval of the order of 5 to 10 minutes. As mentioned above, a time step of 0.5 to 1 hour is usually sufficient for the purpose of tidal analysis. However, in order to avoid any aliasing of high frequency noise, the original time serie is first low-pass filtered, and then decimated to the desired time interval. The filter we use is an autoregressive sine-Butterworth , modified to avoid any phase-shift at all frequencies.

6.2.- SPECTRAL ANALYSIS OF THE ORIGINAL DATA

The spectrum of the time serie is computed using standard FFT techniques, or the Blackman-Tukey method. This allows us to see at which frequencies variance can be found. In particular, it is possible to determine all species worth being considered. However, in order to achieve some statistical reliability for the estimate, it is necessary to consider relatively large bandwiths, and usually it is not possible to separate components or even groups within one species. This would be possible with longer records (one year).

6.3.- HARMONIC ANALYSIS

From 6.2. and the length of the tidal record, the components which are desired can be determined, and the harmonic analysis is performed.

6.4.- SPECTRAL ANALYSIS OF THE RESIDUALS

The residuals are computed and spectrally analysed as in 6.2.

6.5.- ESTIMATION OF THE QUALITY OF THE RESULTS

Confidence limits are determined for the harmonic constituents, and several tests can be applied. The consideration of the spectrum of the residuals makes it possible also to get some idea of the effectiveness of the analysis. The effect of the non analysed components is not considered separately. They must appear in the residual spectrum, and thus enlarge the statistical variability of the current estimate.

If needed, a new analysis can be performed, and its results compared to the previous ones.

7.- Application

As an example, the method is applied to current data from a current meter moored off the belgian coast by the "Unité de Gestion du Modèle Mathématique de la Mer du Nord".

Characteristics

latitude : 51° 27' N	;	longitude	: 2° 59' 18" E
depth : 3 m vs bottom	;	total depth	: 15.6 m
start of analysis	:	27 jan. 1977 at 21 h	
length of the record	:	61.4 days	
time interval	:	0.5 hours	

a) The time serie was obtained after filtering and decimating the original data. The - 3dB cut off frequency was 0.8 cycle/hour.

b) Figures 1 and 2 show the spectrum of the total variance of the horizontal current $S(\lambda) = s_{xx}(\lambda) + s_{yy}(\lambda)$. As we are in a shallow water area, many high order species can be noticed (2, 4, 6, 8, 10, 12, 14, even 16). However, the species higher than 8 carry a very small part of the variance and will be disregarded in the harmonic analysis.

c) 29 components have been searched for.
The results are listed in table 3.

d) Figure 2 shows the total residual spectrum.
It can be seen that most of the energy has been removed in species 2, 4, 6 and 8. As expected there is no change in the species 10 and higher. The low frequency end of the spectrum looks also very similar to its original shape. However, the "noise" level within each species is still much higher than between them. This is due to the non analysed components, and to interactions between tide and noise ("harmonic" constants is only a first approximation). The hypothesis : $M = 0$, $m = 0$ for every component has been tested at 95 % global confidence level.

Table 3

Results of harmonic analysis

Constituent	Major semi-axis length (cm/sec)	Minor semi-axis length (cm/sec)	Major semi-axis direction (°, geog. coord.)	Lag related to culmination at Gwch (°)	
mean	2.78	-	109.	-	
MSf	1.54	0.65	229.	177.	*
Q1	0.62	- 0.05	236.	132.	*
O1	1.20	- 0.27	62.	39.	
K1	1.59	- 0.29	219.	17.	
MNS2	1.31	- 0.47	46.	129.	
2MS2	3.40	2.10	60.	120.	
N2	6.94	1.73	44.	24.	
M2	48.63	16.58	47.	37.	
L2	1.93	0.66	247.	1.	
S2	18.43	5.33	47.	96.	
MSN2	1.10	0.05	209.	91.	*
2SM2	1.53	0.34	226.	142.	
MO3	0.29	0.15	296.	151.	*
MK3	0.56	- 0.05	63.	175.	*
MN4	1.30	- 0.11	233.	120.	*
M4	4.80	0.25	240.	135.	
MS4	3.42	- 0.02	58.	21.	
S4	0.74	- 0.21	92.	125.	
2MN6	1.03	0.57	286.	54.	
M6	2.93	0.97	265.	79.	
MSN6	0.96	0.30	273.	93.	
2MS6	3.91	1.55	267.	135.	
2SM6	0.78	0.32	97.	28.	
S6	0.33	0.08	234.	61.	*
M8	0.73	0.19	207.	90.	*
3MS8	1.59	0.87	206.	146.	
2(MS)8	0.98	0.24	40.	29.	
3SM8	0.20	0.04	102.	74.	*
S8	0.11	0.01	40.	23.	*

A negative minor semi-axis indicates clockwize rotation.

The components with an asterisk in table 3 are those which cannot be considered as different from zero at this level of confidence.

Some interesting conclusions can be drawn from this simple example :

a) The high level of the noise due to non tidal phenomena at low frequency ($\lambda < 1d^{-1}$) makes it difficult or impossible to derive the low frequency components of the tide (MSf in our case) with any confidence.

b) In the other frequency ranges, the noise introduces a limitation to the number of components which can effectively be extracted. It seems reasonable

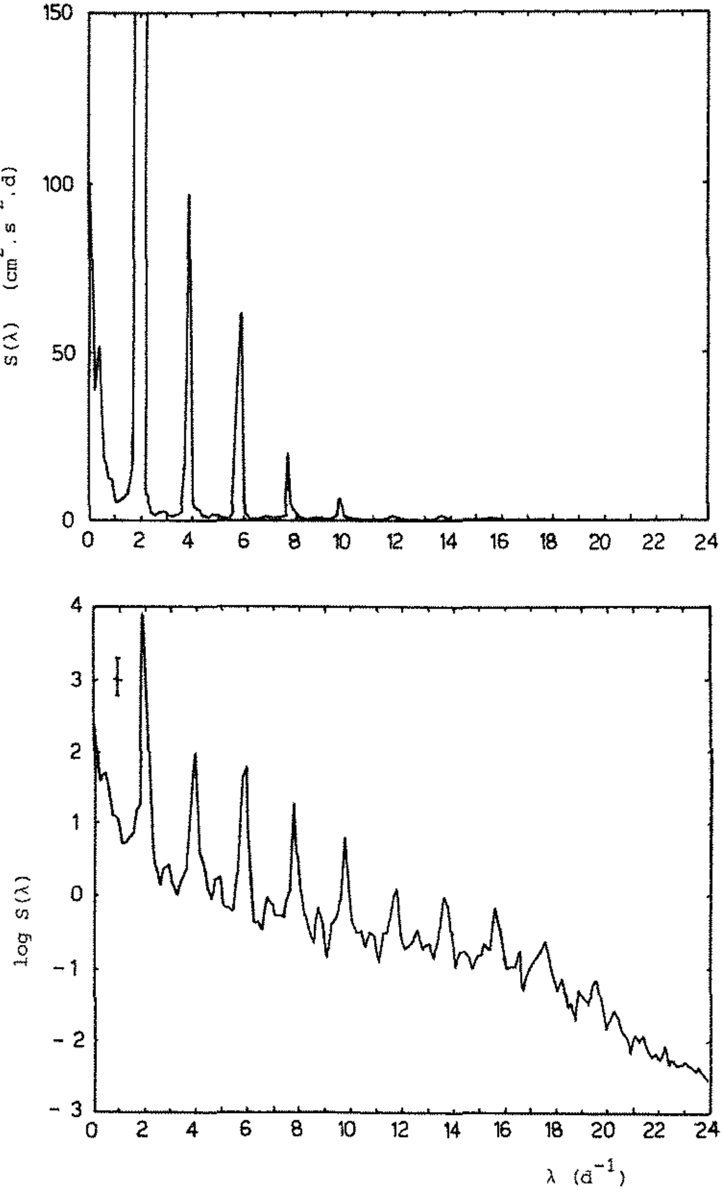

fig. 1.

Spectral representation of the variance of the horizontal current before harmonic analysis.

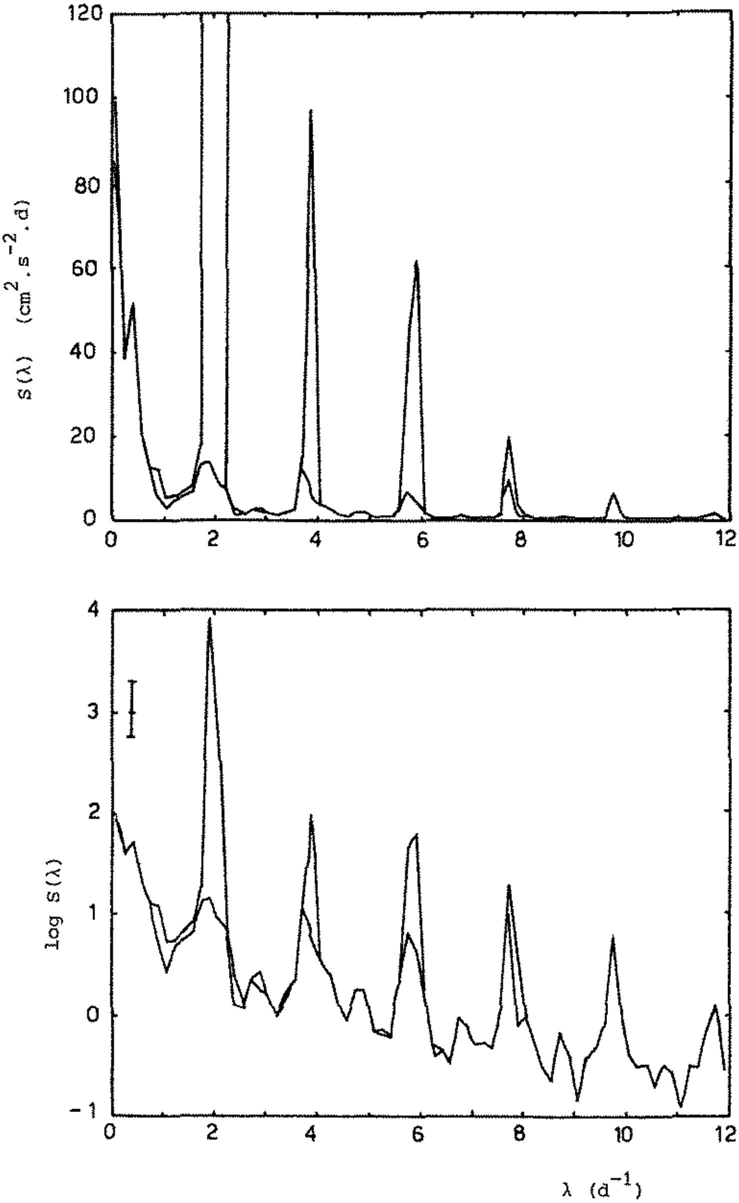

fig. 2.

Spectral representation of the variance of the horizontal current before (upper curve) and after (lower curve) harmonic analysis.

to discard those which are not significant (Q1, MSN2, MO3, MK3, MN4, S6, M8, 3SM8, S8, in our case). Other terms,even if they are significant, have large confidence regions, and it is difficult to use them directly for prediction purposes (O1, K1, ... in our case).

c) There is a need for longer records (6 months, one year or more), in order to separate all the tidal constituents and avoid some of the problems encountered with short records.

Conclusion

The method described above is applied to our current data set. In this way, we hope to derive the harmonic constituents in the area with some level of confidence.

New developments with the aim of enforcing separation but still keeping a sufficient confidence level for the estimates are under way.

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