PROGRAMME NATIONAL SUR L'ENVIRONNEMENT PHYSIQUE ET BIOLOGIQUE

Pollution des Eaux

Projet Mer

COEFFICIENTS OF SHEAR EFFECT DISPERSION IN THE SOUTHERN BIGHT

by

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Passive dispersion models are described by Nihoul in *Modelling of Marine Systems*, Elsevier Publ. Amsterdam 1974, Chapter 3.

The present report complements the material presented in the book by providing a table of the dispersion coefficients in the Southern Bight and a simple method to find rapidly the essential features of the dispersion patche following a localized release. The mathematical introduction preceding the table closely follows the ordering and notations of the book and some parts are reproduced from it without modification. All references must be made to the book.

1. Introduction

One of the main objectives of marine models is the prediction of the distribution in space and time of temperature, nutrients, pollutants,...

As shown by Nihoul (1974), the evolution of a marine constituent results from the advection by the currents, the migration (in particular the sedimentation), the dispersion by molecular diffusion, turbulence and in general all small scale motions which contribute to the agitation of the sea and the chemical, biochemical or ecological interactions.

The cumulative effect of these processes on the space and time variations of the concentrations r_a of a given constituent α is expressed in mathematical form by a partial differential equation.

Advection and dispersion are governed by the sea dynamics. There is thus a coupling between the state variable r_a and the mechanical variables describing the system's hydrodynamics. In general also, the interactions introduce coupling between r_a and the concentrations of other constituents.

If the hydrodynamics is known experimentally or given by a preliminary model, one can calculate the velocity field and the dispersion coefficients and substitute them in the dispersion equation. If, furthermore, the interactions with other constituents have only little influence on the evolution and, in the first instance, can be neglected, this equation can be solved independently of the other evolution equations of the model.

A non-interacting constituent which can be described in this simple way is called a *passive constituent* and the model which is subsequently reduced to a single evolution equation with given mechanical coefficients of advection and dispersion is generally called a *passive dispersion model.*

It is obviously important in marine modelling to master the techniques of passive dispersion models.

In addition to adequately describing the evolution of some - effectively passive - constituents, such models always provide valuable estimates of the distribution of non passive constituents by, at least, appraising their possible transport by the sea motions.

Moreover, in many cases, the interaction processes are not yet entirely understood and cannot be formulated with sufficient accuracy to be used in a reliable simulation model.

It is then preferable to base one's predictions on the estimates of a passive dispersion model until new experimental and theoretical data are available and the interaction terms can be determined with the required precision.

In any case, if the interactions play a significant role, they can be taken into account in a passive dispersion model if one assumes that their net result is a production or destruction of constituent α which can be expressed as a function of r_a alone.

Indeed the condition for the dispersion equation to be independent of the other evolution equations is that the interaction term I_a is zero or a function of r_a only. This is the case, for instance, with a radioactive substance which decays in time with a rate proportional to its concentration.

On the model of radioactive decay, the resulting effect of all interactions is often approximated by a linear production or destruction term of the form

(1) $$I_a = K\, r_a \,.$$

With this approximation, passive dispersion models can be used to describe the general features of the space and time variations of even non-passive constituents.

This is again an example of the general policy which consists in extracting, from the exhaustive multi-purpose mathematical model one tries to elaborate, the elements of less ambitious but more practical sub-models to accelerate simulation, prediction and diagnosis.

2. The equation of passive dispersion

If the interactions of the constituent α with other constituents as well its disappearance by sedimentation can be globalized in a destruction ($\beta > 0$) or production ($\beta < 0$) term $\beta\, r_\alpha$, the dispersion equation can be written (Nihoul 1974) :

$$(2) \qquad \frac{\partial r_a}{\partial t} + \nabla . r_a \underset{\sim}{u} = Q_a - \beta\, r_\alpha + \nabla_h . (\kappa\, \nabla_h r_a) + \frac{\partial}{\partial x_3} \left(\lambda \frac{\partial r_a}{\partial x_3}\right) .$$

where Q_α is the rate of production (destruction) of the constituent by external sources (sinks) and where κ and λ are the horizontal and vertical eddy diffusivities respectively.

Restricting attention to mean vertical concentration, one defines

$$(3) \qquad \bar{c}\,(x_1\,,\,x_2\,,\,t) = H^{-1}\, e^{\beta t} \int_{-h}^{\zeta} r_\alpha\, dx_3$$

where h is the depth, ζ the surface elevation and

(4) $$H = h + \zeta$$

Integrating eq.(2) from $-h$ to ζ, using (3), neglecting small order terms (and in particular turbulent dispersion as compared to shear effect dispersion) and assuming that the release takes place at some initial time with otherwise no continuous external source, one obtains (Nihoul, 1974) :

(5) $$\frac{\partial \bar{c}}{\partial t} + \bar{\underset{\sim}{u}}_h \cdot \nabla \bar{c} = H^{-1} \nabla \cdot \left(\gamma_1 \frac{H^2}{\bar{u}_h} \bar{\underset{\sim}{u}}_h (\bar{\underset{\sim}{u}}_h \cdot \nabla \bar{c}) \right)$$

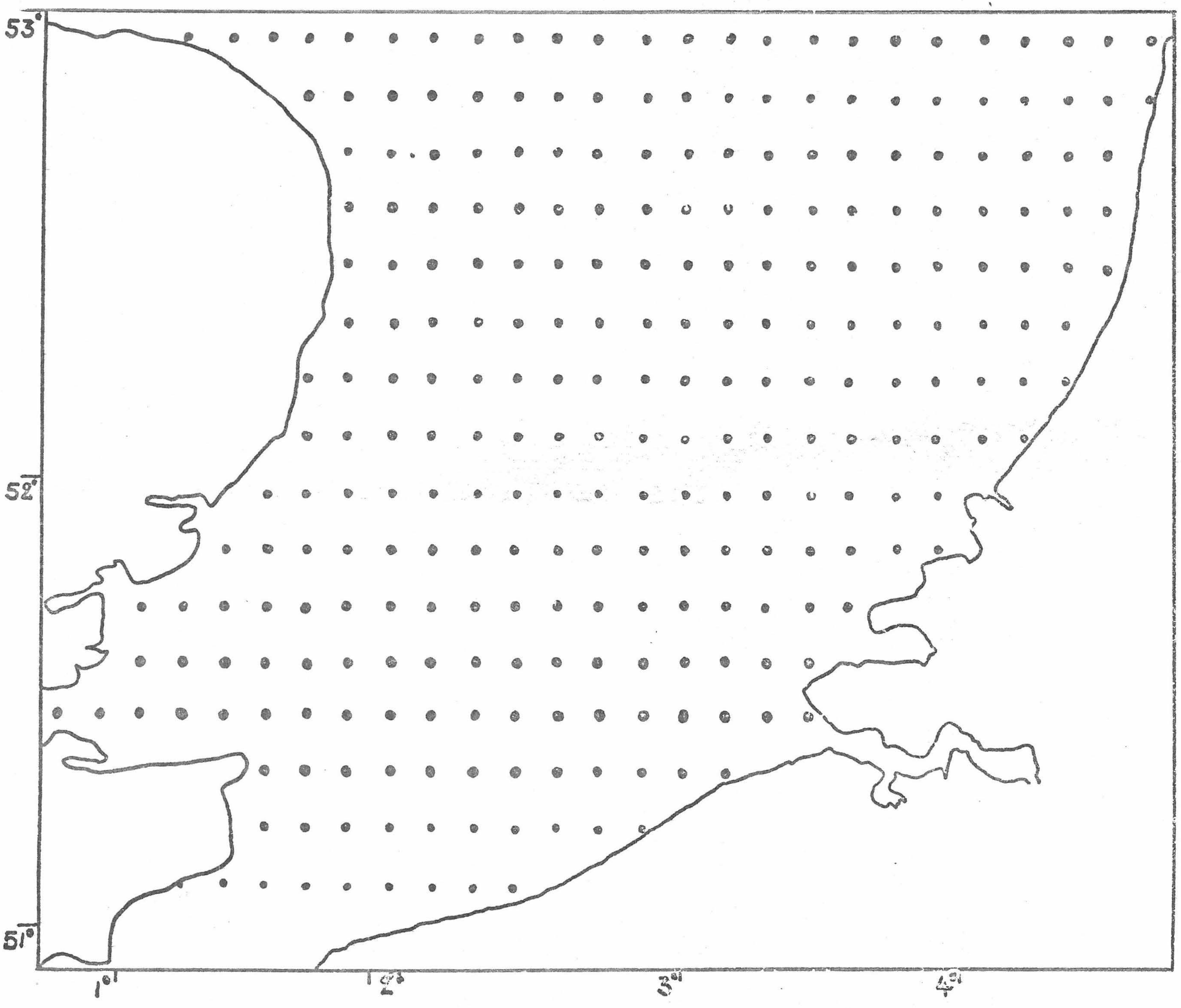
where $\bar{\underset{\sim}{u}}_h$ is the depth - average horizontal velocity and γ a shear effect coefficient of order 1.

In many cases, one is only interested in a prediction of the pollutant's concentration every one or two tidal periods. It is then advantageous to average first eq.(5) over a tidal period.

Changing to a frame of reference which moves with the advection velocity $\bar{\underset{\sim}{u}}_h$ and neglecting small order terms, one obtains thus, noting c_m the tidal average of $\bar{c}$

(6) $$\frac{\partial c_m}{\partial t} = \kappa_{GA} \frac{\partial^2 c_m}{\partial x_1^2} + \kappa_{PA} \frac{\partial^2 c_m}{\partial x_2^2}$$

where the x_1 - axis is taken along the great axis of the best ellipse which fits the tidal velocity vector diagram, the x_2 - axis is perpendicular to it , κ_{GA} and κ_{PA} are dispersion coefficients in the corresponding directions.



53°
52°
51°
1°
2°
3°
4°

κ_{GA} and κ_{PA} have been calculated for a grid of points covering the Southern Bight (figure 1) and the numerical values are given in the annexed table.

3. Iso-concentration curves in axes moving with the center of the patche

Eq.(6) shows that the simplest good estimates of the curves of equal concentration following a point release, are in axes moving with the local advection velocity of the patch, ellipses given by

$$(7) \qquad \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$$

with

$$(8) \qquad a^2 = 4 \, \kappa_{GA} \, t \, \ln \frac{c_o}{c_m}$$

$$(9) \qquad b^2 = 4 \, \kappa_{PA} \, t \, \ln \frac{c_o}{c_m}$$

where c_o is the concentration at the center of the patch and is given by

$$(10) \qquad c_o = \frac{R \, H^{-1}}{4\pi \, \kappa_{GA}^{1/2} \, \kappa_{PA}^{1/2} \, t}$$

where R is the amount of initial release and H is the depth.

The depth-mean concentration of the constituent α at the center of the patch is obtained by multiplying (10) by $e^{-\beta t}$.

The annexed table gives, for each point-located by its longitude and latitude in degrees and minutes - the dispersion coefficients κ_{GA} and κ_{PA} in m^2/sec and the direction of the great axis of the iso-concentration ellipses referred to the geographical North in degress and $\frac{1}{100}$ of degrees.

The table can be used to get a first estimation of the extension with time of a patch of pollutant following an initial release.

At a given time t (t should be an integer number of tidal periods expressed in seconds), one first estimates the position of the center of the patch and the concentration at the center of the patch.

The iso-concentration ellipses corresponding to 10^{-1} , 10^{-2} , 10^{-3}... of the central concentration are then obtained by substituting in (8) and (9) the corresponding values of t , κ_{GA} and κ_{PA} and taking $\ln \frac{c_o}{c_m} = \ln 10$, $2\ln 10$, $3\ln 10$ etc..

Reference

Nihoul J.C.J. (1974), Modelling of Marine Systems,
Elsevier Publ. Amsterdam.

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
1°22'	52°39'	8.56	1.94	21°60
1°31'	"	7.53	4.50	39°44
1°40'	"	13.74	4.18	35°15
1°49'	"	15.01	4.82	35°19
1°58'	"	15.58	3.81	28°69
2°07'	"	8.80	3.54	50°18
2°16'	"	7.24	1.74	45°48
2°25'	"	4.24	2.17	35°99
2°34'	"	8.32	0.58	12°52
2°43'	"	1.94	0.22	74°41
2°52'	"	1.09	0.12	38°06
1°22'	52°31'	6.45	1.21	20°35
1°31'	"	6.15	1.95	40°83
1°40'	"	10.06	1.93	40°61
1°49'	"	15.46	2.11	33°34
1°58'	"	15.64	1.93	35°60
2°07'	"	11.24	1.46	49°15
2°16'	"	10.02	0.92	48°09
2°25'	"	8.13	1.20	49°28
2°34'	"	8.69	0.58	44°52
2°43'	"	6.22	0.07	69°69
2°52'	"	4.90	0.37	57°65
3°01'	"	2.94	0.12	72°05
3°10'	"	1.70	0.10	78°30
3°19'	"	1.03	0.12	36°16
0°55'	52°23'	0.29	0.11	87°01
1°04'	"	0.73	0.25	58°94
1°13'	"	4.82	0.45	12°97
1°22'	"	4.71	1.00	47°65
1°31'	"	6.77	1.35	61°16
1°40'	"	8.48	2.98	40°07
1°49'	"	14.23	1.41	24°54
1°58'	"	13.12	1.55	31°83

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
2°07'	52°23	12.41	0.67	42°88
2°16'	"	11.06	0.60	42°34
2°25'	"	9.22	0.60	42°60
2°34'	"	8.27	0.11	43°10
2°43'	"	6.52	0.02	51°98
2°52'	"	6.25	0.03	50°45
3°01'	"	5.07	0.04	59°48
3°10'	"	3.72	0.39	39°47
3°19'	"	1.89	0.33	30°42
3°28'	"	0.83	0.13	35°01
0°55'	52°15	0.37	0.13	59°57
1°04'	"	1.46	0.11	49°84
1°13'	"	4.22	0.17	44°65
1°22'	"	4.41	0.26	56°07
1°31'	"	4.70	0.72	36°27
1°40'	"	7.77	0.99	31°45
1°49'	"	15.06	0.47	22°66
1°58'	"	10.51	1.66	36°09
2°07'	"	13.28	0.24	32°12
2°16'	"	11.06	0.52	36°30
2°25'	"	10.48	0.19	38°11
2°34'	"	8.98	0.09	41°32
2°43'	"	7.59	0.03	43°79
2°52'	"	6.21	0.42	40°76
3°01'	"	4.94	0.22	46°85
3°10'	"	6.12	0.48	31°99
3°19'	"	3.77	0.44	36°88
3°28'	"	2.44	0.13	15°47
0°55'	52°07'	0.42	0.02	78°01
1°04'	"	1.55	0.02	73°50
1°13'	"	3.06	0.02	76°12
1°22'	"	5.70	0.06	45°46
1°31'	"	5.72	0.40	52°08
1°40'	"	7.06	1.03	41°49

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
1°49'	52°07'	14.62	0.28	24°35
1°58'	"	9.64	1.34	44°93
2°07'	"	14.73	0.09	32°74
2°16'	"	12.45	0.32	35°07
2°25'	"	10.33	0.44	36°36
2°34'	"	9.90	0.37	38°47
2°43'	"	8.59	0.24	42°65
2°52'	"	8.17	0.40	42°16
3°01'	"	6.53	0.21	41°54
3°10'	"	6.07	0.25	38°55
3°19'	"	6.46	0.10	27°86
3°28'	"	5.05	0.13	38°20
3°37'	"	2.21	0.05	36°96
1°22'	51°59	2.58	0.00	39°54
1°31'	"	4.78	0.31	52°45
1°40'	"	6.92	0.46	33°61
1°49'	"	12.40	0.45	21°13
1°58'	"	8.92	0.06	38°46
2°07'	"	12.69	0.31	28°91
2°16'	"	13.85	0.25	29°52
2°25'	"	11.67	0.12	29°14
2°34'	"	9.84	0.03	29°29
2°43'	"	8.29	0.03	34°67
2°52'	"	7.98	0.17	33°23
3°01'	"	7.77	0.03	34°68
3°10'	"	6.14	0.04	42°89
3°19'	"	6.67	0.26	35°92
3°28'	"	6.34	0.19	36°25
3°37'	"	4.96	0.08	44°17
3°46'	"	2.67	0.16	58°56
3°55'	"	0.91	0.08	40°00
1°31'	51°51'	1.70	0.00	39°38
1°40'	"	6.19	0.02	29°91

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
1°49'	51°51'	10.87	0.07	23°96
1°58'	"	9.24	0.62	25°63
2°07'	"	9.97	0.56	23°75
2°16'	"	13.45	0.26	19°94
2°25'	"	12.24	0.16	23°39
2°34'	"	10.24	0.18	27°68
2°43'	"	8.28	0.11	32°57
2°52'	"	8.64	0.08	28°74
3°01'	"	7.30	0.10	29°86
3°10'	"	6.09	0.11	35°17
3°19'	"	6.19	0.20	33°81
3°28'	"	6.45	0.28	33°45
3°37'	"	5.10	0.13	44°76
3°46'	"	4.60	0.13	40°47
3°55'	"	3.38	0.03	21°88
1°40'	51°43'	3.81	0.03	5°28
1°49'	"	8.09	0.02	11°66
1°58'	"	9.86	0.02	17°37
2°07'	"	9.29	0.06	19°12
2°16'	"	12.29	0.21	18°30
2°25'	"	11.79	0.38	22°74
2°34'	"	10.54	0.44	25°15
2°43'	"	8.33	0.25	31°29
2°52'	"	8.89	0.25	29°00
3°01'	"	7.75	0.51	38°85
3°10'	"	6.60	0.23	36°80
3°19'	"	6.37	0.12	33°95
3°28'	"	6.24	0.16	32°81
3°37'	"	5.06	0.06	40°41
3°46'	"	4.99	0.04	43°33
3°55'	"	5.84	0.07	44°46
4°04'	"	3.58	0.10	65°84
4°13'	"	1.66	0.06	38°93
1°40'	51°35'	2.10	0.00	36°14
1°49'	"	6.87	0.01	21°51

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
1°58'	51°35'	9.22	0.00	17°81
2°07'	"	9.88	0.05	15°61
2°16'	"	10.49	0.22	12°28
2°25'	"	10.85	0.11	15°12
2°34'	"	9.95	0.16	20°82
2°43'	"	8.60	0.29	23°05
2°52'	"	8.11	0.13	22°12
3°01'	"	7.25	0.14	25°86
3°10'	"	6.46	0.17	30°94
3°19'	"	6.33	0.11	32°40
3°28'	"	6.09	0.12	36°09
3°37'	"	5.42	0.07	41°34
3°46'	"	4.61	0.06	44°40
3°55'	"	4.56	0.09	40°84
4°04'	"	4.15	0.09	42°19
4°13'	"	3.64	0.04	42°90
4°22'	"	1.86	0.07	37°63
1°49'	51°27'	5.06	0.01	10°39
1°58'	"	8.82	0.00	10°50
2°07'	"	9.77	0.08	9°35
2°16'	"	10.06	0.30	10°92
2°25'	"	9.65	0.47	12°59
2°34'	"	9.29	0.58	13°38
2°43'	"	8.56	0.59	12°85
2°52'	"	7.31	0.69	16°67
3°01'	"	6.32	0.72	23°88
3°10'	"	5.97	0.65	26°68
3°19'	"	5.47	0.66	32°27
3°28'	"	5.56	0.47	34°93
3°37'	"	5.24	0.20	35°43
3°46'	"	4.40	0.17	37°41
3°55'	"	4.16	0.09	36°06
4°04'	"	3.87	0.09	35°80
4°13'	"	3.52	0.15	33°35

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
4°22'	51°27'	3.46	0.25	23°92
4°31'	"	0.94	0.09	41°71
1°49'	51°19'	3.65	0.08	358°19
1°58'	"	8.58	0.14	359°45
2°07'	"	8.65	0.29	2°44
2°16'	"	9.27	0.55	5°94
2°25'	"	9.26	0.90	6°68
2°34'	"	9.11	1.13	6°50
2°43'	"	7.86	1.37	10°79
2°52'	"	6.68	1.34	17°04
3°01'	"	5.51	1.29	24°48
3°10'	"	5.92	0.89	25°07
3°19'	"	4.60	0.82	31°52
3°28'	"	5.18	0.43	34°59
3°37'	"	5.46	0.26	35°91
3°46'	"	4.87	0.19	36°82
3°55'	"	4.25	0.15	36°10
4°04'	"	4.10	0.12	30°34
4°13'	"	3.78	0.10	23°74
4°22'	"	3.81	0.09	18°27
4°31'	"	2.24	0.02	14°38
1°49'	51°11'	4.98	0.02	348°86
1°58'	"	8.08	0.46	347°60
2°07	"	7.59	0.95	350°29
2°16	"	8.29	0.95	354°59
2°25'	"	9.25	1.08	356°66
2°34'	"	8.62	1.47	358°95
2°43'	"	6.82	1.89	3°87
2°52'	"	6.06	2.04	11°60
3°01'	"	4.85	1.96	19°19
3°10'	"	5.25	1.47	25°48
3°19'	"	4.19	1.43	40°89
3°28'	"	4.79	0.93	32°90

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
3°37'	51°11'	4.63	0.71	33°20
3°46'	"	5.21	0.25	32°41
3°55'	"	4.37	0.19	34°14
4°04'	"	4.67	0.08	27°50
4°13'	"	4.36	0.08	24°19
4°22'	"	4.42	0.07	15°61
4°31'	"	3.48	0.02	6°75
1°40'	51°03'	1.66	0.05	320°94
1°49'	"	6.72	0.03	338°69
1°58'	"	9.19	0.32	337°83
2°07'	"	6.96	0.40	329°31
2°16'	"	7.45	1.17	341°20
2°25'	"	9.67	1.73	345°93
2°34'	"	7.67	2.64	343°30
2°43'	"	6.00	3.19	349°30
2°52'	"	4.89	3.23	6°44
3°01'	"	4.62	2.91	32°51
3°10'	"	5.01	2.01	42°22
3°19'	"	4.15	1.97	44°54
3°28'	"	5.04	1.11	34°35
3°37'	"	4.44	0.85	36°50
3°46'	"	5.27	0.22	32°28
3°55'	"	4.73	0.13	36°25
4°04'	"	5.03	0.17	33°16
4°13'	"	5.47	0.09	24°42
4°22'	"	5.15	0.04	18°80
4°31'	"	4.47	0.13	17°62
4°40'	"	0.96	0.08	39°96
1°31'	50°55'	1.16	0.03	133°45
1°40'	"	3.36	0.61	324°63
1°49'	"	6.90	0.31	337°02
1°58'	"	6.91	0.22	348°63
2°07'	"	7.37	0.94	345°97

LONG	LAT	K_{GA}	K_{PA}	DIRECTION
2°16'	50°55'	9.05	1.52	328°97
2°25'	"	10.88	2.24	329°02
2°34'	"	8.04	3.26	316°88
2°43'	"	5.85	4.27	114°91
2°52'	"	5.06	3.74	74°13
3°01'	"	4.61	3.15	66°08
3°10'	"	5.17	2.38	51°10
3°19'	"	4.99	2.13	43°47
3°28'	"	5.28	1.26	33°20
3°37'	"	4.56	0.67	34°89
3°46'	"	4.88	0.20	36°11
3°55'	"	4.85	0.23	42°42
4°04'	"	5.17	0.71	34°74
4°13'	"	6.39	0.65	24°38
4°22'	"	5.87	0.74	20°38
4°31'	"	5.48	0.32	12°78
4°40'	"	1.83	0.08	18°99