# PROGRAMME NATIONAL SUR L'ENVIRONNEMENT PHYSIQUE ET BIOLOGIQUE

# Pollution des Eaux

Projet Mer

# Interactions at the sea boundaries as a handicap to modelling

by

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N 25

## Preprint

Conference given at the Fifth Liège Colloquium on Ocean Hydrodynamics Liège, April 3-6, 1973

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#### 1.- Modelling a marine system

In constructing a model of a marine system, it is first necessary to clearly demarcate the system, separate it from the "outside world" and identify the exchanges between the system and the exterior (imputs and outputs).

The marine system is part of the general geophysical system. The first delimitation is obtained by separating the ocean from the atmosphere, thus defining a major boundary: the air-sea interface. Physical and chemical boundary interactions at the air-sea interface are essential factors in the marine system's dynamics.

Similarly, the system is limited at the sea floor.

It is conceivable to divide the marine system into three subsystems, physical, chemical and biological with the necessary imputoutput links between them. Most models in the past have more or less conformed to this simplified view. One realizes now - in particular, when faced with pollution problems - the limitations of such models and the necessity of a truly interdisciplinary approach.

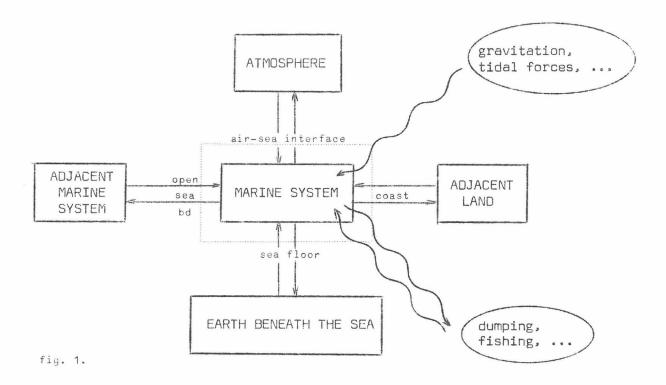
A more natural limitation is geographical. One is practically interested in modelling an estuary, a gulf, a coastal region, a channel, a sea, ..., according to one's particular design. One introduces thus coastal and open sea boundaries separating the system from land and adjacent marine regions.

The situation is summarized in figure ! [Nihoul (1973a)].

Boundary conditions on coasts and open sea frontiers concern the surface elevation, the velocity vector, the flow (either in or out) of contaminants, etc. They are usually badly known - especially on open sea boundaries - because experimental data are often not sufficient or accurate enough. The shortcomings of "lateral" data and their implications on modelling have been discussed by Nihoul (1973b).

In this paper, attention will be restricted to the sea surface and bottom.

The specification of boundary conditions at the air-sea interface and at the sea floor illustrates a typical problem of modelling.



In any simulation process, one must expect, at each step, a feed-back of necessary adjustments suggested by the evaluation and testing of preliminary results. In some cases the revising of preceding stages may go as far as the definition of the system itself. This is the case for the marine system, at least as far as the tracing of the upper and lower boundaries is concerned.

Indeed, one builds a model with the purpose of simulating a specific phenomen. Let T be its characteristic time scale. Oscillating or erratic processes with characteristic times much smaller than T will tend to cancel each other over a time of order T. Thus they will contribute to the dynamics only through non-linear terms which one may hope to take into account by some appropriate empirical closure (eddy or shear effect diffusivities, ...). This qualitative argument can be enforced mathematically by introducing a time average — in the sense of the Krylov Bogoliubov Mitropolsky method — over some intermediate time period sufficiently small that the phenomenon under investigation can "pass through untouched" but sufficiently large to eliminate the details of the rapid erratic and oscillating processes encumbering the analysis

[e.g. Nihoul (1972)]. Thus, if  $x_3 = \zeta$  and  $x_3 = -h$  are the equations of the air-sea interface and the bottom, respectively, and if **u** denotes the "mean" velocity as defined above, the boundary surfaces will satisfy:

(1) 
$$\frac{\partial \zeta}{\partial t} + u_1 \frac{\partial \zeta}{\partial x_1} + u_2 \frac{\partial \zeta}{\partial x_2} = u_3 \quad \text{at} \quad x_3 = \zeta$$

(2) 
$$\frac{\partial h}{\partial t} + u_1 \frac{\partial h}{\partial x_1} + u_2 \frac{\partial h}{\partial x_2} = -u_3 \quad \text{at} \quad x_3 = -h .$$

The "model" boundaries defined by (1) and (2) are of course far from coinciding with the real complicated boundaries. (In particular, problems with different time scales require different boundaries as, for example, "surface elevation" has a different meaning in tidal models and residual current models).

Schematically, one might say that each model has its particular conception of the sea surface and of the bottom. Interpreting in mathematical terms the interactions at the sea boundaries is a difficult task. It is complicated in modelling by the obligation to comply with the model's refinement and formulate the boundary conditions with just the right degree of sophistication.

## 2.- Boundary interactions

Specifying boundary conditions appropriate to modelling is a complex problem which "modellers" often explain, with disheartened irony, by the fact that these boundaries just do not exist.

It would be nice indeed to have, as upper and lower frontiers, respectively, a well defined free surface and a non ambiguous rigid bottom. Unfortunately, none of these boundaries may be taken as entirely free or entirely rigid and, definitely, none can be considered as well defined.

Waves at the sea surface create a first problem. They influence the instantaneous local wind and the subsequent air-sea interactions but they are themselves (their speed, form and height) function of the wind stress in the past. Waves break frequently as a result of

interaction between waves of different wave numbers. Wave breaking, which can occur at low wind speed, is more frequent and efficient under strong wind conditions. In the process, spray is produced. Depending on the relative velocities of wind and waves, the disruption of wave crests into spindrift can also produce spray. {At winds above 8 m/s, there is apparently a sudden increase in the spray load which could be attributed to this mechanism [Kraus (1967)].} Bubbles of air are entrained by the breaking waves. When bubbles burst on reaching the surface, drops are ejected into the air.

An irregular mixing of air and water results on both sides of a jagged interface which one finds difficult to position practically within a complicated zone of transition between the atmosphere and the sea.

The situation is similar at the bottom. Sea water contains suspended sediments. Whether the mean turbidity is low (open sea) or high (coastal area), there is inevitably a marked increase in the concentration of sediments as one approaches the sea floor which itself can be permeated with water to some depth. It is almost impossible to draw a line through the solid-water mixture at the bottom of the sea and the so-called bottom boundary is often nothing more than a conventional limit corresponding to some chosen concentration of the sediments. Moreover, the shear stress exerted by the sea on the bottom may recirculate deposited sediments and erode the sea floor.

Blowing sand (like flying spray) contributes to shade off the border between sea and soil.

A further difficulty in specifying boundary conditions at the sea surface occurs evidently when some alien substance is interposed between air and water. Organic surface-active films are frequently formed on the sea surface. Damping capillary waves, they produce anomalies in the reflection of light which one calls "slicks". Natural sea slicks constituted by monomolecular films of adsorbed organic molecules have been known for a long time. As a result of pollution, contaminant films or layers such as slicks of petroleum products or other chemicals are now more and more often observed.

Surface films modify air-sea interactions, in general in an assuaging way, attenuating small waves, hindering wave formation, reducing air drag, inhibiting gas exchange, ... [e.g. Garrett (1972)].

Momentum is presumably transferred from wind to waves and, for a substantial part, passed over to the water column by combing waves or other mechanisms [Stewart (1967), Krauss (1967)] . The detailed machinery of this transfer is rather complicated because the wind profile is itself affected by the state of the sea surface, the nature and the shape of the waves and, both wind and waves, are largely influenced by the stratification. The complicated interplay between air stability, sea state and wind field reflects on the resulting stress exerted by the wind. Heavy rainfall and wave breaking will increase it. {Momentum transfer is presumably enhanced by the agitation of air caused near the interface by the interpenetration of air and water layers [Tobas and Kunishi (1970)]}. Oil slicks decrease the stress. In unstable conditions, a definite augmentation of momentum transfer seems to occur when the wind speed exceeds some 7 m/s, associated with the onset of foaming [De Leonibus (1971)]. Acceleration of spray drops (and deceleration of bubbles) produce an additional shear stress which is presumably small at low wind speed but may become relatively more important at higher wind speeds [Krauss (1967)].

The situation is evidently similar for the transport of heat and material across the air-sea interface. Intricated surface mechanisms ensure the transmission of fluxes with variable efficiency. Infrared radiations, for instance, are absorbed at the interface, producing a discontinuity in the heat flux [Krauss (1967)]. Pesticides fallout from the atmosphere are often concentrated in sea slicks as a result of their preferential solubility in nonpolar organic materials [Seba and Corcoran (1969)]. According to MacIntyre (1971) surface active organic slicks can concentrate all ionic species with high ionic potential relative to Na and Cl. Aerosols produced by the evaporation of spray droplets transfer salt and other substances from the sea to the atmosphere. A substantial amount of volatile hydrocarbons and chemical species which can be concentrated in slicks can be carried into the atmosphere from the sea by this process [Garrett (1972)].

Wave generation can improve significantly the rates of evaporation and aeration. According to Hidy (1972), this must be attributed to the disturbance of the diffusional sublayer near the water boundary, which plays a key role in the material transfer across the naviface.

A similar situation prevails at the bottom. When a viscous layer can be maintained on the sea floor, the flux of material is simply due to the slow sedimentation of particles. When the layer is disrupted by turbulence, the material concentrated in the viscous layer can be recirculated. For still higher values of the turbulent shear, the flow is able to erode the bottom and blow solid sediments into the water column and its turbulent dynamics [McCave (1970), Cormault (1971)].

A few examples suffice to demonstrate the complexity of the interactions at the upper and lower sea boundaries. One understands the difficulty of specifying judicious boundary conditions appropriate to a model. The uncertainties in the boundary conditions however deeply reflect on the model's predictions and the inaccuracy of boundary data is often the most severe handicap to modelling.

## 3.- Boundary conditions

At the air-sea interface, it is generally assumed that the fluxes are proportional to the magnitude of the wind velocity at some reference height (10 m , say). If V ,  $\theta$  , q denote the velocity, the temperature and the moisture (or perhaps the concentration of some contaminant) at the height of reference and  $\theta_0$  ,  $q_0$  the corresponding surface values, one writes [Krauss (1972)] :

(i) momentum flux:

(3) 
$$\tau_s = \rho V C_{10} ||V||$$
;

(ii) heat flux (divided by the specific heat) :

(4) 
$$h_s = \rho(\theta_0 - \theta) C_{10} \|V\|;$$

(iii) moisture flux :

(5) 
$$\omega_s = \rho(q_0 - q) C_{10} \|V\|$$
,

where the "drag coefficient"  $C_{10}$  is assumed to be constant.

According to Krauss (1972), "the preceding expressions, applied to good meteorological data, probably can yield flux estimates over the open ocean with a mean expected error of less than 30 % ". Hidy (1972) anticipates a "more realistic error" as large as 50 %. This would seem to be the range in which the data from various observations scatter under comparable wind conditions.

In addition, observations from the Bomex experiment reported by Pond et al. (1971) cast doubt on the validity of (4) for the heat transfer which is significantly affected by radiation. One may also question the extension of (5) to the flux of chemical species which may dissolve or interact preferentially in organic surface films.

At the bottom, an expression similar to (3) is generally adopted for the flux of momentum although, for the convenience of two dimensional models, one often prefers to express the bottom shear in terms of the depth-averaged velocity  $\bar{\bf u}$ . Thus :

(6) 
$$\tau_b = \rho \, \overline{Du} \, \|\overline{u}\| - m \, \tau_s$$

where D is a suitable drag coefficient and where the last term in the right-hand side is a small correction introduced to account for the fact that, in case of negligible volume transport of water, there is still a stress exerted by the bottom [Groen and Groves (1966)].

The literature is very poor on the fluxes of heat, sediments, etc. If one assumes, for want of something better, that the heat flux can be derived by analogy with the momentum flux, a different formulation is required for the flux of sediments to account for the recirculation and erosion described in section 2.

If a stable viscous layer exists on the bottom, the turbulent flux vanishes there and the total flux (b, say) is reduced to the sedimentation. Thus, one would have, in that case:

$$b = -\sigma \rho_h$$

where  $\sigma$  is the sedimentation velocity and  $\rho_b$  the nearbed concentration.

Equation (7) assumes complete entrapment in the viscous sublayer. To allow for periodic disruption of the layer and ejection of sediments, a correcting factor is introduced, indicating the effective degree of sublayer instability. One writes [Owen and Odd (1970)]:

(8) 
$$b = -\sigma \rho_b \left(1 - \frac{\tau_b}{\tau_c}\right)$$

where  $\tau_{\rm c}$  is the limiting shear stress above which no deposition takes place.

In equation (8),  $\tau_b$  is of course variable and the limiting shear stress can be exceeded part of the time (say, part of the tidal cycle) still allowing for a positive budget of the sedimentation in the mean.

When the shear stress exceeds a second critical value  $\tau_e$ , the turbulence is able to erode the sea bed. The flux of eroded material seems to obey the same type of law and Cormault (1971) suggested that it can be expressed by (8) where  $\tau_c$  is replaced by  $\tau_e$  and  $\sigma\rho_b$  by some constant M depending on the nature of the sea floor.

It is not quite clear if formulas of that type incorporate the action of waves on the sediment transport in the nearshore zone and if expressions like (6) for  $\tau_b$  are accurate enough to be substituted in (8) [McCave (1971)].

The expressions (6) and (8) are really quite amazing. For one thing, they pretend to be more refined than the formulas for the fluxes at the sea surface. [One could imagine, on their model, to introduce critical values of the wind stress for spray formation or foaming and include correcting factors in the surface drag coefficients as one did in equation (8).] Then, they provide completely wrong boundary conditions from a mathematical point of view as they specify the variables at the boundary in terms of their unknown (indeed sought) values inside the system. Finally they rely on the knowledge of the nearbed concentration  $\rho_b$  (which, if it were known would provide a much simpler boundary condition, incidently).

Near the bottom boundary (one should say "inside" the bottom boundary since it is so vaguely defined) the concentration of sediments varies extremely rapidly and equation (8) is prohibitively sensitive to the exact height where the "nearbed" concentration is specified [McCave (1973)].

Perhaps — at least if equations (7) and (8) make any sense —  $\rho_b$  can be taken as the concentration at the top of the viscous layer. Then, if the water column is fairly well-mixed (as it is the case in the Southern Bight of the North Sea, for instance)  $\rho_b$  can be approximated by the depth averaged concentration, with eventually a slight adjustment of the value of  $\sigma$ . Equation (8), modified in that way, has been reasonnably successful in simulating the sedimentation in the Southern Bight of the North Sea [Nihoul (1973c)].

The boundary conditions discussed briefly in this section have been shown to be often inadequate, always unprecise as a result of the introduction of empirical coefficients  $C_{10}$ , D,  $\tau_c$ , ... which are difficult to ascertain experimentally. In fact, too much is being asked from these coefficients. They should integrate the intricated interactions at the sea boundaries with just the right degree of refinement to provide boundary conditions appropriate to the models.

In the present stage, the situation is obviously not very satisfactory and models are often much better than the boundary imputs they elaborate on.

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