Development of a hybrid two-phase/mixture model for concentrated sediment transport in open channel flow

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The general research strategy for the development of an innovative numerical sediment transport model applicable as engineering tool for coastal and estuarine morphodynamics is presented . It consists of two major steps: (1) the creation of a high-resolution hybrid model based on two-phase flow theory, where the model can easily be reduced to a computationally less expensive mixture theory model in the dilute part of the domain; (2) a scale analysis and the development of a strategy to upscale the high-resolution strategy to a coarse scale model applicable to large-scale 3D engineering problems.

Key Words: *two-phase flow theory, mixture theory, high-concentrated sediment transport, particle lag velocity*

1. Introduction

The mismatch between sediment transport models and experimental data lies at the basis of the research efforts of the authors. Despite all the efforts since Rouse [1], at present it is still impossible to reproduce all the details (especially near the bed) of measured sediment concentration profiles in steady uniform flume experiments for non-dilute conditions.

The basic principles and motivation for the development of a hybrid model combining advantages of mixture theory and two-phase flow theory have been presented during the previous THESIS Workshop [2]. The main goal is the development of a computationally effective 3-dimensional sediment transport model applicable to large coastal and estuarine areas (implemented in the OpenTELEMAC software), accounting for high-concentrated near-bottom sediment transport phenomena (bed load transport, sheet flow and density currents) in a less empirical way by considering all the energy dissipation mechanisms in a more physical way.

The development of the large-scale model starts from a small domain 2DV model with a high resolution vertical mesh, fine enough to capture the details of the flow in the low-Reynolds inner boundary layer above the bed where most of the transported sediment is found. This fine-scale model is used to generate numerical data to complement the very few usable experimental data on non-dilute sediment transport.

2. Basic Equations

The basic set of equations consists of the continuity equation, mass conservation of the sediment, the conservation of momentum for the mixture, a drift velocity closure and a turbulence closure model. There are different options to define these equations, depending on the choice of independent variables and the choice of averaging of the turbulent fluctuations in time. Although the ensemble-averaged form yields mathematically the simplest formulation, as proposed initially [2], eventually, preference is given to a Reynolds-averaged form expressed in terms of fluid velocity u and particle lag velocity $w = v - u$ (with v the particle velocity). The main motivation for this choice is the validation with experimental data.

2.1. Instantaneous equations

The instantaneous continuity (mass conservation) and momentum conservation for the suspension are obtained by summing up the equations for each phase, multiplied by their volumetric fraction:

$$\frac{\partial\left(u_j + w_j\phi\right)}{\partial x_j} = 0 \tag{1}$$

$$\rho\frac{\mathrm{d}u_i}{\mathrm{d}t} = -\frac{\partial p}{\partial x_i} + \rho g\delta_{iz} + \frac{\partial\sigma_{ij}}{\partial x_j} - \rho_p\phi\left(\frac{\mathrm{d}w_i}{\mathrm{d}t} + \frac{\partial\left((u_i + w_i)w_j\right)}{\partial x_j}\right) \tag{2}$$

where $\rho = (1-\phi)\rho_w + \phi\rho_s$, the suspension density, p = the pressure, and the material derivative, advected with the fluid $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x_j}(.\,u_j)$.

In addition, one needs to solve the mass balance equation to obtain the solids concentration:

$$\frac{\partial\phi}{\partial t} + \frac{\partial\left((u_j + w_j)\phi\right)}{\partial x_j} = 0 \tag{3}$$

and a closure equation for the lag velocity. The latter can be either obtained from a particle force balance, or (preferably) from elimination of the pressure between the two phase momentum equations, which after some rearrangement can be written as:

$$\rho_p\phi\left(\frac{\mathrm{d}w_i}{\mathrm{d}t} + \frac{\partial\left((u_i + w_i)w_j\right)}{\partial x_j}\right) = \left(\rho - \rho_w\right)\left(g\delta_{iz} - \frac{\mathrm{d}u_i}{\mathrm{d}t}\right) + \frac{f_i}{1-\phi} \tag{4}$$

with: f = the phase interaction force. Notice that the LHS of Eq.(4) is exactly the extra term in Eq.(2).

Equation (2) clearly shows the extra term in the RHS of the momentum transfer due to the velocity lag, which is missing in the currently applied traditional sediment transport models in engineering software. As it is proportional to the solids concentration, it is indeed of second order as long as the concentrations are low. And since the high-concentrations usually are restricted to a subgrid scale thin near-bed layer, it may seem justified to ignore it, since it is assumed to be corrected by the bedload transport closure used to compute the near-bed sediment concentration boundary condition. Notice that in this approach the velocity lag closure reduces to replacing the lag velocity by the settling velocity, usually taken as a (semi-empirical) constant value.

2.2. Time-averaged equations

The real problems start when applying time-averaging to the above equations. Since Reynolds-averaging is the most straightforward technique in data-processing of experimental data, it is preferred to apply this also to the above equations, in order to allow validation. The relative particle flux ($w\phi$) then leads to an average flux and a so-called turbulent drift flux, which traditionally is modelled as a diffusive flux, following the Boussinesq hypothesis. Notice that it makes a difference here when the lag velocity is defined relative to the instantaneous or the Reynolds-averaged fluid velocity. The latter will reduce the number of terms. Again, it is the extra lag term in the RHS of Eq.(2) that generates the problems. The other terms will produce the well-known RANS equation with appearance of the Reynolds stress terms. The same terms of Eq.(2) also allow to construct the traditional *k*-epsilon turbulence closure model, including the buoyancy term originating from the fluctuating suspension density ρ. The extra term in the RHS of Eq.(2) leads to the many extra terms in the two-phase version of the *k*-epsilon turbulence model [3], most of which are practically impossible to evaluate. Ultimately, the major problem remains the closure for the phase interaction force term [4].

3. Energy Considerations

3.1. Suspension capacity

The mismatch between sediment transport models and the experimental flume data of Cellino [5] has been a major concern to the authors for many years [6, 7]. While some published simulations give the impression that it can be simulated, one important validation step has not been dealt with: turbulence and buoyancy effects. When processing the unique data of turbulent fluctuations of the sediment concentrations and the turbulent fluxes, it turns out that the flux Richardson number (the ration of gravity to turbulent suspension force) is of the order 0.03, which is about one order of magnitude lower than the famous value of 0.25, known from thermal stratification studies [8]. Application of the new suspension capacity criterion of Toorman [6] to other data, including river data, confirmed these low values of flux Richardson numbers. The physical meaning of this observation has important implications: the suspended load transport is carried by only order 10% of the available energy, which implies that the remaining 90% is consumed by the non-dilute near-bottom transport. Subsequently, the near-bottom boundary conditions for turbulence for the suspended load transport, which are not adapted from the well-known wall-functions for clear water, yield a significant overprediction of the actual turbulence. Therefore, 3D engineering sediment transport models, which commonly use the *k*-epsilon turbulence model for the vertical turbulent entrainment of sediment particles require new near-bottom boundary condition treatment.

3.2. Low-Reynolds turbulence modelling

The data of Cellino show that non-dilute conditions prevail in the lowest 2-3 cm in his experiments, characterized by turbulence which is no longer fully-developed and therefore require a low-Reynolds modelling approach. A two-layer low-Reynolds modelling technique is under development and tested in the fine-scale 2DV model. It solves a low-Reynolds mixing-length type model in the bottom layer and a low-Reynolds *k*-epsilon model in the rest of the water column [2]. In view of future applications where considerably thick gravity currents of sand or fluid mud need to be simulated, an attempt is made to define new wall-distance free damping functions, different from what has been proposed previously in the literature (e.g. [9]), since the latter do not consider the correct asymptotic behavior towards the laminar sublayer. The basic formulation of the new damping functions is done by analysis of various DNS data for open-channel flow up to (shear velocity based) Reynolds numbers of 4000. This reveals a singularity in the “constant” σ_k in the turbulent diffusion term of the *k*-equation in the transient region, causing numerical stability problems when this transient region falls within the numerical grid where the *k*-epsilon equations are solved. This numerical problems needs to be resolved before further progress can be made.

Moreover, the latest high-*Re* DNS results as well as the Princeton Super Pipe Facility experimental data [10] both reveal significant Reynolds number dependence of the previously though standard “constants” of the *k*-epsilon model.

3.3. Bottom friction

Another important parameter in sediment transport modelling is the bottom friction which governs the relation between flow field, bed shear stress and turbulence production by shear flow. Two scaling problems appear: The first one deals with the definition of the bottom level, i.e. the level where the non-slip flow velocity is imposed. This is best illustrated by considering flow over sediment ripples in combination with the thickness of the non-dilute suspension layer. This leads to the second scaling problem, i.e. the scale of bed form length relative to the distance of the order 10^2-10^3 m between neighboring grid points in a morphodynamic model of a coastal area. It is clear that the detailed flow over these ripples cannot be resolved, and hence, neither the actual non-dilute suspension transport. Therefore, ultimately, the model results from the fine-scale 2DV model need to be upscaled in a final step before being usable in a large scale model. The subgrid method proposed by Volp [11] will be investigated

That this will be possible is encouraged by the successful implementation of a depth-integrated form of the proposed improved drag coefficient predictor into a 2DH morphodynamic model for the Belgian coast and the Scheldt estuary [4]. This new physics-based friction model is based on matching the above mentioned low-Reynolds mixing-length model with velocity profiles for clear water of smooth and rough surfaces, as well as the experimental flume data for sand transport of Cellino, therefore also accounting for the energy dissipation by transport and interparticle friction of the sediments.

4. Conclusions

This extended abstracts presents an overview of the ambition of the KU Leuven Sediment Mechanics research group to develop a sediment transport model that can properly account for the different energy dissipation mechanisms which occur in nature.

Progress is hampered in the first place by lack of reliable experimental data of separate turbulent fluctuations of fluid and particles in non-dilute conditions. Furthermore, scale effects need to be overcome by subgrid scale models which can incorporate turbulence production over topographic features and bed forms and in the wake of particles. Preliminary results indicate that the proposed multi-scale modelling approach can indeed improve sediment transport predictions.

Acknowledgments

This work is funded in part by the BELSPO BRAIN.be project INDI67, the VLAIO SBO project CREST (Climate Resilient Coast – www.crestproject.be) and a KU Leuven IRO PhD Scholarship for the second author.

References

[1] Rouse, H: Modern conceptions of the mechanics of fluid turbulence. *Trans. ASCE*, 102, 463-543, 1937.

[2] Toorman, E.A. & Q. Bi:. Hybrid two-phase/mixture modeling of sediment transport. *Extended abstract THESIS 2013* (Chatou, 2013), 4 pp., 2013.

[3] Elghobashi, S. E., Abou-Arab, T. W.: A two-equation turbulence model for two-phase flows. *Physics of Fluids*, 26(4), 931-938, 1983.

[4] Bi, Q: Numerical modelling of near-bottom sediment transport: turbulence modulation, new process models and application to the Scheldt & the Belgian coast. *Phd dissertation*, Dept. of Civil Engineering, KU Leuven, 228 pp., 2015.

[5] Cellino, M.: Experimental study of suspension flow in open-channels. *PhD dissertation*, Dept. de Génie Civil, Ecole Polytechnique Fédérale de Lausanne, 1998.

[6] Toorman, E.: Modelling of turbulence damping in sediment-laden flow. Part 2: Suspension capacity of uniform shear flows. *MAST III COSINUS Project Report* HYD/ET/00/COSINUS4, Hydraulics Laboratory, K.U. Leuven, 23 pp., 2000.

[7] Toorman, E.: Validation of macroscopic modelling of particle-laden turbulent flows. *Proc. 6th Belgian National Congress on Theoretical and Applied Mechanics*. (Gent, Belgium, May 26-27, 2003), 7 pp., 2003.

[8] Turner, J.S.: *Buoyancy Effects in Fluids*, Cambridge University Press, Cambridge, 1973.

[9] Hrenya, C.M., Bolio, E.J., Chakrabarti, D., Sinclair, J.L.: Comparison of low Reynolds number $k-\varepsilon$ turbulence models in predicting fully developed pipe flow. *Chemical Engineering Science*, 50(12), 1923-1941, 1995.

[10] Smits, A. J. McKeon, B. J., Marusic, I.: High Reynolds number wall turbulence. *Annu.Rev. Fluid Mech.*, 43, 353–375,2011.

[11] Volp, N.D., van Prooijen, B.C. Pietrzak, J.D., Stelling, G.S.: A subgrid based approach for morpho-dynamic modelling, *Advances in Water Resources*, 93, 105-117, 2015.