

## Falling Speed of Aerosol Particles

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For studying the aerosol budget in the lower and higher atmosphere, knowledge of the sedimentation rate of the aerosol particles is required. This note provides tables and graphs of the falling speed of spherical particles as function of their radius and height above ground on the basis of the best information presently available from aerosol physics.

According to Fuchs (1964), the mobility  $M$  of a spherical particle of radius  $r$  in a gas of dynamic viscosity  $\eta$  and mean free path length  $\lambda$  is given by

$$M = w/F = [1 + (\lambda/r)(A + B \exp\{-Cr/\lambda\})]/6\pi\eta r, \quad (1)$$

where  $w$  is the speed of the particle and  $F$  the external force. If no other forces act on the particle,  $F$  is the gravity less the buoyancy force, i.e.,

$$F = (\rho_p - \rho)(4/3)\pi r^3 g, \quad (2)$$

where  $\rho_p$  is the mass density of the particle,  $\rho$  the mass density of the gas, and  $g$  the acceleration of gravity.

For aerosol particles in the natural atmosphere,  $\rho/\rho_p < 10^{-3}$ , so that from (1) and (2)

$$w = (2/9)(\rho_p r^2 g / \eta) [1 + (\lambda/r)(A + B \exp\{-Cr/\lambda\})]. \quad (3)$$

$A$ ,  $B$ ,  $C$  are dimensionless empirical constants, their numerical values having been determined by several authors. According to Fuchs (1964), the values  $A = 0.864$ ,  $B = 0.29$  and  $C = 1.25$  given by Millikan (1923a, b) had to be considered as most reliable. But Flanagan and Tayler (1967) have shown that Millikan evaluated his measurements on electrically charged oil droplets on the basis of a value of the electric elementary charge which was 0.6% too high so that his calculated radii were too high by 0.2%. According to (3), the Millikan constants are therefore too low by 0.2%; however, a correction by +0.2% only affects the numerical value of  $A$ ; thus  $A = 0.866$ .

The numerical values of  $A$ ,  $B$ ,  $C$  furthermore depend on the mean free path length  $\lambda$ . Millikan used the value

$\lambda = 9.417 \times 10^{-6}$  cm (for a pressure of 760 mm Hg and a temperature of 23C). According to more recent measurements,  $\lambda = 6.53 \times 10^{-6}$  cm has to be taken for the same pressure and temperature (Fuchs, 1964; Flanagan and Tayler, 1967). Since the products  $\lambda A$ ,  $\lambda B$  and  $C(1/\lambda)$  must not be influenced by the choice of  $\lambda$ , the relations  $\lambda A = \lambda_M A_M$ ,  $\lambda B = \lambda_M B_M$ ,  $C/\lambda = C_M/\lambda_M$  hold, where the quantities indicated by the subscript  $M$  refer to the value of the mean free path length used by Millikan. With  $\lambda_M/\lambda = 1.442$ ,  $A_M = 0.866$ ,  $B_M = 0.29$ , and  $C_M$

$= 1.25$ , one finally has

$$A = 1.249, \quad B = 0.42, \quad C = 0.87. \quad (4)$$

The mass density  $\rho_p$  of the aerosol particles enters the expression for  $w$  only as a factor of proportionality. For sulphate particles, the tables compiled by Scott (1960) give the densities 1.769, 1.78, 1.834 and 1.788 gm cm<sup>-3</sup> for (NH<sub>4</sub>)<sub>2</sub>SO<sub>4</sub>, (NH<sub>4</sub>)HSO<sub>4</sub>, H<sub>2</sub>SO<sub>4</sub> and H<sub>2</sub>SO<sub>4</sub>·H<sub>2</sub>O, respectively. A value  $\rho_p = 1.8$  gm cm<sup>-3</sup> may therefore be considered as representative for sulphate particles.

TABLE 1. Falling speed (cm sec<sup>-1</sup>) of spherical particles of mass density 1.0 gm cm<sup>-3</sup> in the 1962 U. S. Standard Atmosphere for various value of height  $z$  and particle radius  $r$ . The number following the character "E" gives the power of 10 by which the foregoing decimal number should be multiplied.

$z$ (km)	$r$ ( $\mu$ m)							
	0.003	0.01	0.03	0.1	0.3	1	3	10
0	4.1150E-06	1.4283E-05	4.8096E-05	2.3182E-04	1.4008E-03	1.3188E-02	1.1264E-01	1.2280E 00
1	4.6069E-06	1.5930E-05	5.3080E-05	2.4864E-04	1.4580E-03	1.3524E-02	1.1493E-01	1.2506E 00
2	5.1727E-06	1.7825E-05	5.8811E-05	2.6796E-04	1.5224E-03	1.3890E-02	1.1738E-01	1.2745E 00
3	5.8253E-06	2.0009E-05	6.5415E-05	2.9023E-04	1.5951E-03	1.4289E-02	1.1999E-01	1.2997E 00
4	6.5809E-06	2.2537E-05	7.3058E-05	3.1600E-04	1.6778E-03	1.4725E-02	1.2279E-01	1.3264E 00
5	7.4591E-06	2.5474E-05	8.1936E-05	3.4593E-04	1.7726E-03	1.5205E-02	1.2579E-01	1.3548E 00
6	8.4837E-06	2.8901E-05	9.2287E-05	3.8083E-04	1.8818E-03	1.5736E-02	1.2903E-01	1.3848E 00
7	9.6837E-06	3.2912E-05	1.0440E-04	4.2168E-04	2.0085E-03	1.6325E-02	1.3254E-01	1.4168E 00
8	1.1096E-05	3.7630E-05	1.1864E-04	4.6970E-04	2.1565E-03	1.6985E-02	1.3635E-01	1.4509E 00
9	1.2765E-05	4.3208E-05	1.3547E-04	5.2644E-04	2.3305E-03	1.7730E-02	1.4052E-01	1.4875E 00
10	1.4747E-05	4.9831E-05	1.5544E-04	5.9377E-04	2.5363E-03	1.8575E-02	1.4510E-01	1.5269E 00
11	1.7116E-05	5.7741E-05	1.7929E-04	6.7411E-04	2.7813E-03	1.9546E-02	1.5017E-01	1.5694E 00
12	2.0005E-05	6.7369E-05	2.0814E-04	7.6918E-04	3.0490E-03	2.0299E-02	1.5235E-01	1.5769E 00
13	2.3385E-05	7.8635E-05	2.4190E-04	8.8072E-04	3.3665E-03	2.1191E-02	1.5484E-01	1.5848E 00
14	2.7337E-05	9.1809E-05	2.8140E-04	1.0115E-03	3.7427E-03	2.2259E-02	1.5776E-01	1.5942E 00
15	3.1959E-05	1.0721E-04	3.2758E-04	1.1646E-03	4.1873E-03	2.3540E-02	1.6118E-01	1.6052E 00
16	3.7362E-05	1.2522E-04	3.8159E-04	1.3440E-03	4.7117E-03	2.5077E-02	1.6521E-01	1.6183E 00
17	4.3679E-05	1.4628E-04	4.4474E-04	1.5539E-03	5.3291E-03	2.6919E-02	1.6997E-01	1.6335E 00
18	5.1065E-05	1.7090E-04	5.1858E-04	1.7995E-03	6.0547E-03	2.9122E-02	1.7559E-01	1.6515E 00
19	5.9699E-05	1.9968E-04	6.0491E-04	2.0868E-03	6.9066E-03	3.1751E-02	1.8229E-01	1.6725E 00
20	6.9792E-05	2.3332E-04	7.0582E-04	2.4227E-03	7.9057E-03	3.4878E-02	1.9027E-01	1.6972E 00
21	8.1619E-05	2.7274E-04	8.2405E-04	2.8162E-03	9.0760E-03	3.8553E-02	1.9939E-01	1.7208E 00
22	9.5383E-05	3.1862E-04	9.6165E-04	3.2743E-03	1.0441E-02	4.2890E-02	2.1034E-01	1.7489E 00
23	1.1140E-04	3.7200E-04	1.1218E-03	3.8076E-03	1.2032E-02	4.7999E-02	2.2354E-01	1.7829E 00
24	1.3000E-04	4.3399E-04	1.3077E-03	4.4268E-03	1.3882E-02	5.3987E-02	2.3936E-01	1.8237E 00
25	1.5160E-04	5.0598E-04	1.5237E-03	5.1462E-03	1.6034E-02	6.0997E-02	2.5826E-01	1.8725E 00
26	1.7666E-04	5.8953E-04	1.7743E-03	5.9812E-03	1.8533E-02	6.9179E-02	2.8074E-01	1.9311E 00
28	2.3937E-04	7.9856E-04	2.4013E-03	8.0706E-03	2.4790E-02	8.9785E-02	3.3872E-01	2.0855E 00
30	3.2344E-04	1.0788E-03	3.2420E-03	1.0872E-02	3.3186E-02	1.1757E-01	4.1873E-01	2.3077E 00
32	4.3586E-04	1.4535E-03	4.3661E-03	1.4619E-02	4.4418E-02	1.5484E-01	5.2775E-01	2.6247E 00
34	5.8605E-04	1.9541E-03	5.8679E-03	1.9623E-02	5.9418E-02	2.0463E-01	6.7402E-01	3.0564E 00
36	7.8270E-04	2.6096E-03	7.8342E-03	2.6176E-02	7.9065E-02	2.6993E-01	8.6733E-01	3.6503E 00
38	1.0383E-03	3.4617E-03	1.0390E-02	3.4695E-02	1.0461E-01	3.5491E-01	1.1201E 00	4.4497E 00
40	1.3688E-03	4.5633E-03	1.3695E-02	4.5710E-02	1.3764E-01	4.6488E-01	1.4482E 00	5.5071E 00
42	1.7933E-03	5.9782E-03	1.7940E-02	5.9857E-02	1.8008E-01	6.0619E-01	1.8706E 00	6.8847E 00
44	2.3359E-03	7.7868E-03	2.3365E-02	7.7942E-02	2.3432E-01	7.8688E-01	2.4114E 00	8.6620E 00
46	3.0255E-03	1.0085E-02	3.0261E-02	1.0093E-01	3.0327E-01	1.0166E 00	3.0993E 00	1.0934E 01
48	3.8933E-03	1.2978E-02	3.8939E-02	1.2985E-01	3.9004E-01	1.3058E 00	3.9660E 00	1.3808E 01
50	4.9889E-03	1.6630E-02	4.9896E-02	1.6638E-01	4.9961E-01	1.6710E 00	5.0615E 00	1.7453E 01
52	6.3919E-03	2.1307E-02	6.3925E-02	2.1314E-01	6.3990E-01	2.1386E 00	6.4642E 00	2.2124E 01
54	8.1778E-03	2.7260E-02	8.1784E-02	2.7267E-01	8.1850E-01	2.7340E 00	8.2506E 00	2.8080E 01
56	1.0492E-02	3.4973E-02	1.0492E-01	3.4981E-01	4.0999E 00	3.5054E 00	1.0565E 01	3.5799E 01
58	1.3510E-02	4.5035E-02	1.3511E-01	4.5043E-01	1.3518E 00	4.5117E 00	1.3585E 01	4.5868E 01
60	1.7463E-02	5.8210E-02	1.7463E-01	5.8217E-01	1.7470E 00	5.8293E 00	1.7538E 01	5.9050E 01
62	2.2645E-02	7.5486E-02	2.2646E-01	7.5493E-01	2.2653E 00	7.5569E 00	2.2722E 01	7.6336E 01
64	2.9500E-02	9.8334E-02	2.9501E-01	9.8342E-01	2.9508E 00	9.8420E 00	2.9578E 01	9.9205E 01
66	3.8766E-02	1.2922E-01	3.8766E-01	1.2923E 00	3.8774E 00	1.2931E 01	3.8846E 01	1.3011E 02
68	5.1373E-02	1.7124E-01	5.1374E-01	1.7125E 00	5.1381E 00	1.7133E 01	5.1455E 01	1.7216E 02
70	6.8892E-02	2.2964E-01	6.8893E-01	2.2965E 00	6.8901E 00	2.2974E 01	6.8977E 01	2.3058E 02
72	9.3289E-02	3.1096E-01	9.3290E-01	3.1097E 00	9.3298E 00	3.1106E 01	9.3376E 01	3.1193E 02
74	1.2782E-01	4.2605E-01	1.2782E 00	4.2606E 00	1.2782E 01	4.2615E 01	1.2791E 02	4.2705E 02
76	1.7731E-01	5.9103E-01	1.7731E 00	5.9104E 00	1.7732E 01	5.9114E 01	1.7740E 02	5.9207E 02
78	2.4925E-01	8.3082E-01	2.4925E 00	8.3083E 00	2.4926E 01	8.3093E 01	2.4934E 02	8.3189E 02
80	3.5574E-01	1.1858E 00	3.5574E 00	1.1858E 01	3.5575E 01	1.1859E 02	3.5584E 02	1.1869E 03

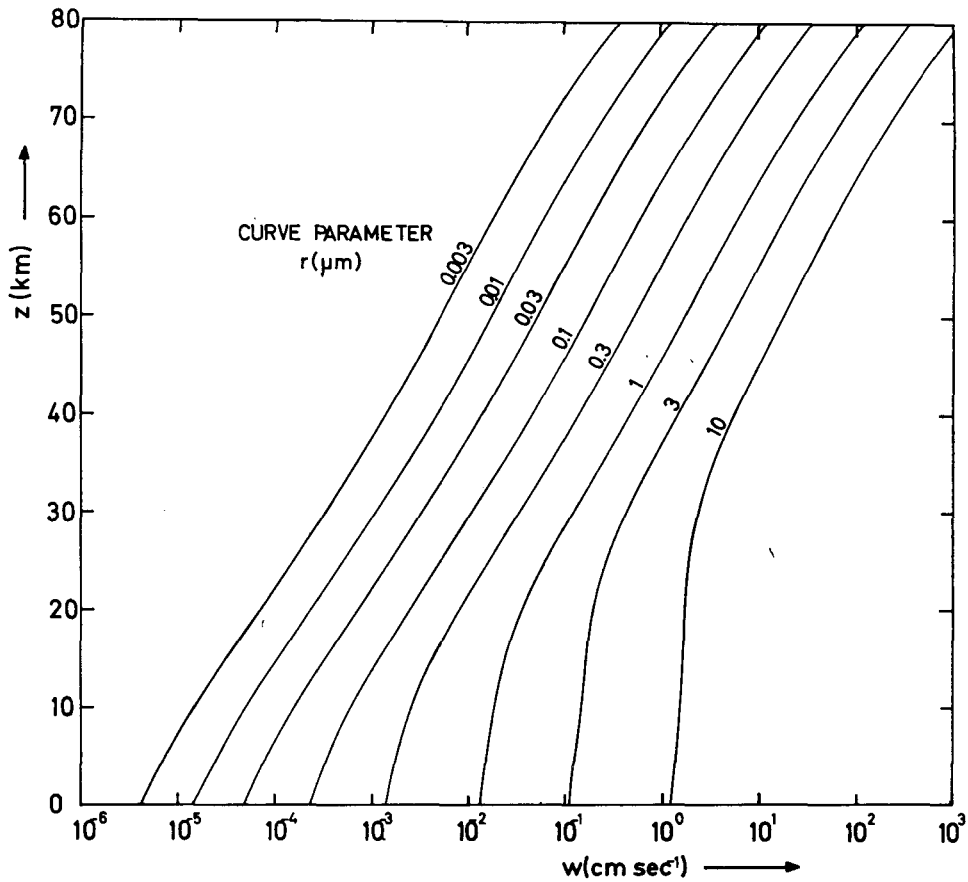


FIG. 1. Falling speed  $w$  of spherical particles of mass density  $1.0 \text{ gm cm}^{-3}$  in the 1962 U. S. Standard Atmosphere as a function of height  $z$  above mean sea level, for several particle radii  $r$ .

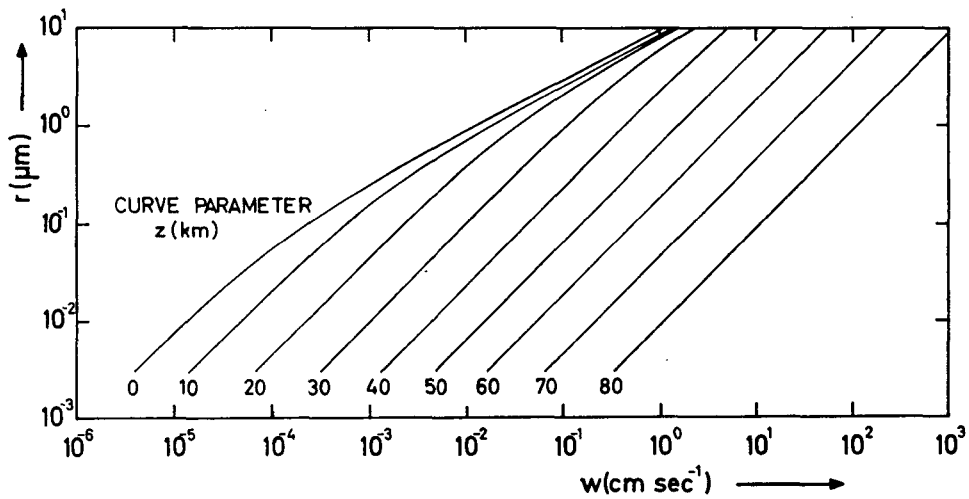


FIG. 2. Falling speed  $w$  of spherical particles of mass density  $1.0 \text{ gm cm}^{-3}$  in the 1962 U. S. Standard Atmosphere as a function of particle radius  $r$ , for several heights  $z$  above mean sea level.

However, the present computations of  $w$  are based on unit particle density so that the falling speed of particles of density  $\rho_p \neq 1.0 \text{ gm cm}^{-3}$  can easily be obtained by merely multiplying the tabulated values of  $w$  by  $\rho_p [\text{gm cm}^{-3}]$ .

The acceleration of gravity  $g$  depends slightly on the height  $z$  above mean sea level and on the geographical latitude  $\phi$ . The dynamic viscosity  $\eta$  is a pure function of temperature  $T$ . The mean free path length  $\lambda$  is inversely proportional to the mass density  $\rho [\text{gm cm}^{-3}]$  of the air or the number density  $N [\text{cm}^{-3}]$  of its molecules. Since in the atmosphere  $T$  and  $\rho$  (or  $N$ ) depend on the height  $z$ , the quantities  $\eta$  and  $\lambda$  can also be expressed as functions of  $z$ . For a mean vertical temperature and density profile of the atmosphere at  $\phi = 45^\circ$  latitude,  $\eta(z)$  and  $\lambda(z)$  are tabulated in the 1962 U. S. Standard Atmosphere (see Cole *et al.*, 1965). The function  $g(z)$  is found in the 1959 ARDC Model Atmosphere (Minzner *et al.*, 1959).

Using these tabulated values of  $g(z)$ ,  $\eta(z)$  and  $\lambda(z)$ , the particle mass density  $\rho_p = 1.0 \text{ gm cm}^{-3}$ , and the constants  $A$ ,  $B$ ,  $C$  according to (4), Eq. (3) was numerically evaluated for  $0 \leq z \leq 26 \text{ km}$  in steps of 1 km, for  $26 \text{ km} \leq z \leq 80 \text{ km}$  in steps of 2 km, and for  $r = 0.003, 0.01, 0.03, 0.1, 0.3, 1, 3$  and  $10 \mu\text{m}$ , as shown in Table 1. Graphical representations of  $w(r, z)$  are given in Fig. 1 as  $w$  vs  $z$ , with  $r$  as a parameter, and in Fig. 2 as  $w$  vs  $r$ , with  $z$  as a parameter.

Eqs. (1) and (3) are essentially based on Stokes' friction law modified by the Cunningham-Millikan correction. Their validity is therefore confined to the range of validity of Stokes' law which is restricted to small Reynolds numbers,  $\text{Re} = 2r\rho w/\eta$ . According to Fuchs (1964), the Stokes formula gives a deviation of  $-1.5\%$  from measured values for  $\text{Re} = 0.1$ . From the calculated values of  $w$  listed in Table 1, the corresponding Reynolds numbers were computed for the most unfavorable case of largest  $r$  ( $10 \mu\text{m}$ ). The results are shown in Table 2, where use was made of the relationship

$$\rho = \alpha/\lambda, \quad (5)$$

with  $\alpha = 8.125 \times 10^{-9} \text{ gm cm}^{-2}$ . Table 2 demonstrates that within the whole atmosphere up to 80 km height,

TABLE 2. Reynolds numbers of spherical particles of mass density  $1.0 \text{ gm cm}^{-3}$  and radius  $10 \mu\text{m}$  in the 1962 U.S. Standard Atmosphere as function of height  $z$  above mean sea level.

$z$ (km)	Re	$z$ (km)	Re
0	$1.68 \times 10^{-2}$	50	$2.10 \times 10^{-4}$
10	$8.66 \times 10^{-3}$	60	$2.22 \times 10^{-4}$
20	$2.12 \times 10^{-3}$	70	$2.81 \times 10^{-4}$
30	$5.76 \times 10^{-4}$	80	$3.90 \times 10^{-4}$
40	$2.75 \times 10^{-4}$		

the Reynolds number remains  $< 0.1$  as long as the particle radius does not exceed  $10 \mu\text{m}$ . Remembering the proportionality  $\text{Re} \sim w \sim \rho_p$ , we learn from Table 2 that the above statement holds for particles of any mass density  $\rho_p$  up to  $6 \text{ gm cm}^{-3}$ . Thus, the use of (3) for computing the falling speed of aerosol particles in the atmosphere is justified.

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