Cartometric Analyses: Methodological Issues and Their Significance for the History of Cartography and Historical Studies in General

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1 THE OUTLINE

The cartometric approach itself is a set of methods and tools used for processing quantitative historical data that are either integral components of old maps or charts or exist in some other related data formats. Yet, its proper utilisation requires possessing an additional body of knowledge and skills regarding both sciences and humanities, such as mathematics, geodesy, the history of science, the history of navigation, etc. Otherwise, there is a likelihood that cartometric analyses will be conducted in a methodologically inaccurate manner, that conclusions derived from those computations will be erroneous and scientifically invalid, or both.

This volume permits going into all of the subsidiary disciplines and techniques. Consequently, its content is confined to providing essential

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information related to the existing cartometric analyses, some important methodological issues, and how they may contribute to a better understanding of old maps and charts, the development of the history of cartography as a discipline, and broader insights into human history. The chapter is crafted to be accessible to a broad array of readers, regardless of whether they are coming from history of cartography background or not. In order to make the concept and techniques of cartometric analyses more approachable to newly interested readers, certain elements are deliberately mentioned multiple times across different paragraphs and explained in a more simplistic manner. Figures derived from the author's previous cartometric studies supplement the text by vividly and intuitively illustrating its key elements. Another volume-induced limitation relates to production periods and coverage of old maps and charts. The cartometric methods described in this chapter pertain to nautical charts and geographical maps created during the late medieval and early modern periods, before the conduction of geodetic and hydrographic surveys, and the examples provided cover the Mediterranean area and the Adriatic Sea, depending on the topic of certain passages. This is because the impact of choosing a certain map projection as a frame of reference is better understood in the example of the entire Mediterranean, whereas some approaches for processing spherical coordinates are better demonstrated by using the smaller area as an example. Since the author's research focuses primarily on the geometry of portolan charts, the majority of its examples and illustrations naturally belong to that specific sub-niche within the history of cartography.

The chapter does not contain one section solely dedicated to previous research. It was intentionally written in such a way that certain elements of the existing cartometric studies conducted by the author and other scholars are dispersed across the respective (sub-)topics. These are partially explained and addressed via cross-references, enabling the reader to search the missing information externally. The final section emphasises the applicability of cartometric analysis results, which, when coupled with other historical documentation, may operate as forensic tools that strengthen understanding of both the history of cartography and human history overall. Since cartometric analyses are not applicable to every study within the history of cartography because not all old maps were made with a realistic display of the (known) world in mind, the first logical step is to provide the reader with a brief outline of cartographers' approach to planimetric accuracy in mapmaking across certain important historical periods.

2 A Brief History of Planimetric Accuracy in Mapmaking

Between classical antiquity and the advent of modern geodetic and hydrographic surveys that took place in the eighteenth and nineteenth centuries, the attention to planimetric accuracy of old maps and charts went through numerous changes. The mathematical approach to mapmaking that included estimating the size of the Earth, defining coordinates of locations (their latitudes and longitudes) within the spherical geometry, and the development of map projections was thoroughly practised during classical antiquity.[1], [2] In contrast, the *mappae mundi* that were made during the early Middle Ages in Western Europe to depict the Christian understanding of the known world were more elementary in design (Woodward 1987), whereas in the ninth and tenth centuries, the Islamic cartography adopted a more scientific approach, inspired by classical antiquity sources that were, at the time, freshly discovered in the former Byzantine provinces (Tibbets 1998).

The most enigmatic and, perhaps, one of the most important periods in the history of cartography occurred in the late thirteenth century, when an entirely new branch of cartography, nautical cartography, was established, as evidenced by the sudden appearance of portolan charts in the Mediterranean. It is due to the extremely realistic appearance of their coastlines and a complete historical lack of their evolutionary path because the oldest known samples have already been made to a fully developed stage (Campbell 1987; Edson 2007; Nicolai 2014, 2024; Marelić 2024a, b).[3]

The rediscovery of Claudius Ptolemy's book *Geographike Hyphegesis* (commonly known as *Geography*, originally written in the second century

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[1] The most renowned calculations of the Earth's great circle in classical antiquity were performed by Eratosthenes in the third century BCE and by Posidonius of Rhodes in the second and first centuries BCE (Russo 2004).

[2] Marinus of Tyre from the first century CE is the presumed inventor of the equidistant cylindrical projection, whereas in the second century CE Claudius Ptolemy described how equidistant conic and pseudoconic projections should be constructed (Keuning 1955; Snyder 1993).

[3] The earliest-known portolan charts include the anonymous *Pisane* (see Figs. 9.1 and 9.2), *Cortona*, *Avignon*, and *Lucca* charts made in the late thirteenth or early fourteenth century, and those made by Pietro Vesconte (see Fig. 9.3) between 1311 and 1327 (Nordenskiöld 1897; Campbell 1987; Billion 2011; Mille 2023). The anonymous *Carta Riccardiana* (c. 1300–1325) was most likely drawn by Vesconte as well (Marelić 2024b).

CE) with a list of around 8000 locations, 6300 of which were designated by their latitudes and longitudes, had the greatest impact on European cartography in the early Renaissance (Berggren and Jones 2000; Gautier-Dalche 2007). The use of spherical coordinates to determine point locations—the so-called geometrisation of space, according to Richard L. Kagan and Benjamin Schmidt (2007: 663)—and the implementation of graticules on maps that were deliberately made in the map projection was a novel approach for Western European cartographers at the time, who initially followed those principles to create their geographical maps. During the early modern period, prior to the conduction of extensive geodetic and hydrographic surveys, the nautical charts that were produced in parallel with geographical maps and geometry-wise noticeably differed from them (Marelić 2023a, b) had also become supplemented with those cartographic elements, although their authors appear to have been more conservative in terms of their implementation, adopting them more cautiously at a somewhat slower rate.

3 THE ESSENTIALS OF CARTOMETRIC ANALYSIS

Simpler types of old maps, such as *mappae mundi* and their variant, the T-O maps, can only be analysed descriptively because their creators did not prioritise the metrically accurate representation of spatial relationships, whereas some more intricate works, such as the so-called *Tabula Peutingeriana*,[4] allow only a limited quantitative analysis. This is because the distance annotations accompanying its complex road network are the only exact quantitative historical data that seems to have been observed in the field, unlike the overall image of the area and the road network itself, which are longitudinally extremely compressed to fit within its 6.7 × 0.3-meter format, designed to be rolled up and carried around as a scroll when not in use.

On the other hand, the extremely accurate display of coastlines on portolan charts has prompted a scholarly approach to them from the late nineteenth century. Matteo Fiorini, for example, assumed that they were made in the equidistant azimuthal projection because the colour-coded lines of their wind roses radiate from a central point (Fiorini 1881), but

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[4] The anonymous map portraying a graphic representation of a Roman *itinerarium* (travel guide), dated to the early thirteenth century, whose original was likely made in the fourth or fifth century CE (Dilke 1985).

Hermann Wagner's first-ever cartometric analysis in 1896 explicitly refuted this assumption. Wagner coined and proposed the term *cartometric method*, examined portolan charts created by Pietro Vesconte (1318) and Gratiosus Benincasa (1480), and concluded that their metrics are very similar to a normal cylindrical projection and that they are most likely mosaic images composed of sub-parts that were graphically stitched together (Wagner 1896/1969). Wagner's methods were analogue and rudimentary in comparison to modern standards, but his revolutionary conduct exposed a completely new and previously unknown realm of methodological approaches to be taken within the history of cartography niche. Despite advancements in computer sciences and engineering since Wagner's study that lifted cartometric analyses to a whole new technological level, and some scholars pointing out that cartometric methods are beneficial to avoid generating a "heavy load of theory … on a visual impression" (Skelton 1965: 6), the number of cartometric studies of old maps is still much smaller compared to those that emerged from the traditional descriptive approach. Some notable and representative cartometric studies will be mentioned and described in the following paragraphs, based on their respective sub-topics.

Within the context of the history of cartography, cartometric methods are essential tools for extracting geometric and other quantitative data from historical sources that are often invisible to the naked eye and comparing them directly to other similar datasets (whether historical or modern), and are usually conducted with the application of GIS and statistical software. Although the input data in such analyses are primarily cartographic, as they are typically encapsulated within frameworks of old maps and charts, some other relevant quantitative input data formats, such as coordinates, distances, or bearings contained in textual sources, may be imported in GIS software and cartometrically analysed as well. The basic concept is to place historical input data—whether in the form of digitised reproductions (images) of old maps and charts or spreadsheet data containing other significant metrics—in direct geometric relationship with their reference modern equivalents, and the entire process consists of multiple consecutive steps. The initial step involves acquiring and creating the necessary reference data (treated as error-free) and properly identifying the required historical data (referred to as the *identical data*) to pair with them. In the case of old maps and charts, those are identical points typically utilised for the subsequent step: their georeferencing.

Georeferencing (Fig. 9.1) is a process conducted in GIS software within which a sufficiently redundant number of identical points identified on the old map or chart are paired with their reference counterparts on a modern map in a selected map projection and geometrically transformed. The geometric transformation enables the digital image of the old map and its identical points to be shifted, scaled, and rotated to obtain their geometric best fit to the reference map projection by achieving the *least-squares estimation* (LSE) of their axial *residuals* (dX, dY in kilometres). These are the displacements of identical points that occur along its X and Y axes when the geometry of an old map differs from the geometry of a reference map (Fig. 9.2). In addition, the residuals need to be de-projected, that is, translated into their spherical or ellipsoidal angular displacements ($d\lambda$, $d\varphi$ in degrees). The following step is to convert them into distances along the longitudinal and latitudinal arcs between the reference–identical point pairs (*dLON*, *dLAT* in kilometres). The final step is the computation of axial de-projected *root mean square errors* of residuals (RMSE *dLON*,

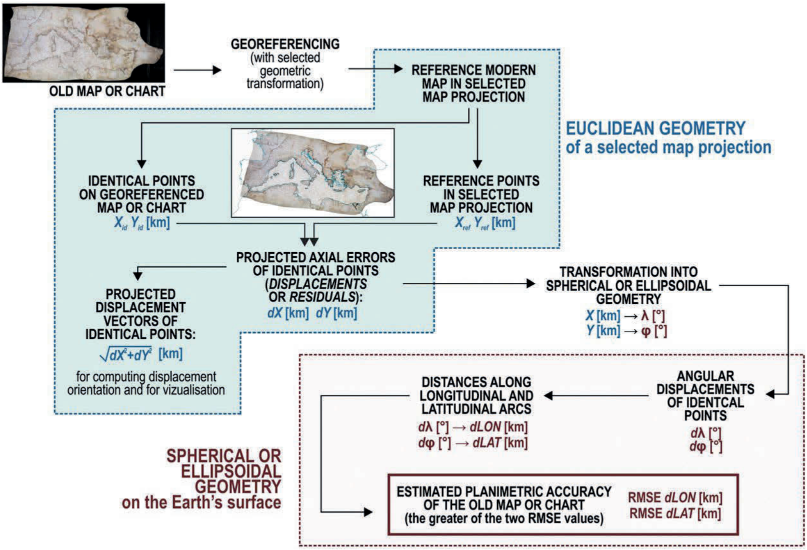

Fig. 9.1 The simplified schematic of the georeferencing process and the accuracy estimation of old maps and charts by cartometric means (Portolan chart source: *Bibliothèque nationale de France*, GE B-1118 (RES))

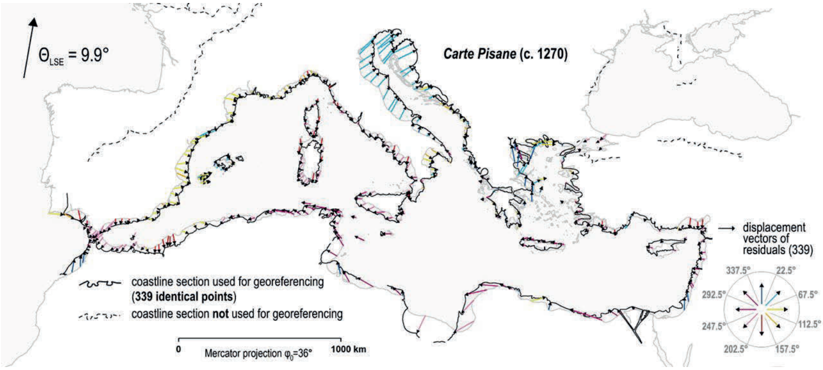

Fig. 9.2 The magnitude and orientation of displacement vectors of residuals of the anonymous *Carte Pisane* (c. 1270, the oldest known portolan chart) georeferenced to a modern map in the Mercator projection across the Mediterranean area. Basemap shapefile source: marineregions.org (Claus et al. 2017)

RMSE *dLAT* in kilometres) (Fig. 9.1). The greater of the two values (whether RMSE *dLON* or RMSE *dLAT*) is used to indicate the estimated planimetric accuracy of the entire map (Jenny and Hurni 2011; Nicolai 2014; Penzkofer 2016). Since these, in essence, represent the mapping errors, the accuracy of the map is understood to be greater if the RMSE values are smaller and vice versa. Also, when a single map is georeferenced to a modern map iteratively, each time using a different map projection (see Fig. 9.3), smaller RMSE values indicate a greater likelihood of the map being originally made in such a projection.

4 Some Important Methodological Issues

Although the georeferencing of the old map or chart seems relatively straightforward on the conceptual level, there are several important parameters that need to be considered during the procedure, since improper configuration might negatively affect both the outcome and the interpretation of the results. The first two issues are the parameters of the coordinate reference system and the geometric transformation to be used because they strongly affect the results in the early stages of the research. The third issue includes transforming projected values back to spherical or ellipsoidal geometry in order to eliminate the distortions induced by the systemic properties of the analytical framework itself.

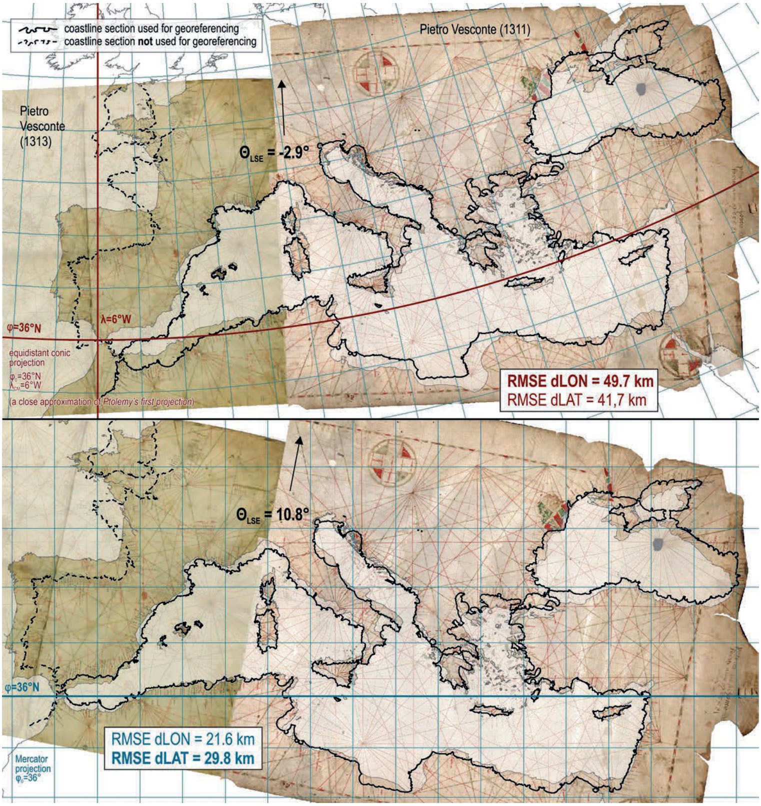

Fig. 9.3 The manually assembled composite of two Pietro Vesconte's portolan charts georeferenced to the modern map in the close approximation of the *Ptolemy I* projection centred near Gibraltar (the upper part), and to the modern map in the Mercator projection (the lower part), with their vectorised coastlines and de-projected accuracy values. The greater of the two axial values in bold indicate the estimated planimetric accuracy of the entire georeferenced unit (Portolan chart sources: *Archivio di Stato di Firenze*, CN 01; *Bibliothèque nationale de France*, CPL GE DD-687 (RES)). Basemap shapefile source: marineregions.org (Claus et al. 2017)

Those issues are important because the georeferencing process operates by comparing the image of the identical dataset to the image of the reference dataset within the same Euclidean environment, in accordance with which the geometry and visual appearances of both the reference and identical images are defined. However, the coordinates on the Earth's surface, as well as bearings and distances between those points observed during surveys, are established according to either spherical or ellipsoidal geometry. This means that the geometry of maps and charts that were deliberately made in a map projection is determined with its geometric parameters, that certain geometric features from the real world (whether distances, shapes, or areas) become distorted by definition, and that their distribution and magnitude vary across the map field as well.

4.1 Coordinate Reference System (CRS)

The first important methodological issue is that—unlike the real world—the geometry of all the data within the GIS framework is typically Euclidean and determined by the user-defined projected coordinate reference system (CRS). In other words, the structure of the whole dataset, including its visual appearance and its geometric properties, is defined in accordance with the selected map projection and geodetic datum. This means that the coordinates of points on the Earth's surface (λ, φ in degrees) are, by definition, transformed into Euclidean coordinates (X, Υ in kilometres) depending on the selected projected CRS, and that the geometry of all the identical datasets imported into that framework will likewise be expressed in accordance with the Euclidean parameters of the selected CRS. In many types of GIS software, the parameters of the CRS can be altered any time, meaning that selecting a new CRS or modifying the existing one will geometrically alter the entire reference dataset, and that a completely new set of Euclidean coordinates will be assigned to it.[5]

The user has to be well aware of the geometric properties of the CRS framework in advance because wrongly set parameters can easily lead to incorrect results, especially if the research area stretches across large territorial extents, making it more prone to projection-induced distortions. For example, if one is interested in computing certain area-values by using GIS software, the selected projected CRS should be based on some equal-area projection,[6] whereas in cases of studying bearings or distances along the rhumb lines (loxodromes), one should opt for a CRS that is based on

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[5] The upper part of Fig. 9.3 shows the modified map projection within the CRS: the equidistant conic projection was modified to become centred at the longitude near Gibraltar ($\lambda = 6°$W), and $\varphi = 36°$N was set as its single standard (true-to-scale) parallel.

[6] Some typical examples of equal-area map projections are the Mollweide pseudocylindrical projection, the Albers conic equal-area projection, and the Lambert azimuthal equal-area projection (Snyder and Voxland 1989).

the Mercator projection.[7] Otherwise, the computed geometric parameters become erroneous, and conclusions derived from them are incorrect and invalid. This is especially important for the users of *MapAnalyst*, which is a relatively simple and user-friendly software for elementary cartometric analyses of old maps (Jenny 2006, 2010; Jenny et al. 2007; Jenny and Hurni 2011). The software automatically links identical points on the old maps or charts to *OpenStreetMap*'s reference map in the Mercator projection. Those points are, then, de-projected and projected again, but this time in the transverse cylindrical equal-area projection centred at the mean longitude of all control points. This choice of a reference map is suitable only for studying smaller areas because the distortions of distances and bearings become more and more extreme on the peripheral areas as the longitudinal extent of the research area increases (Nicolai 2018).

In cases where the old maps or charts are the result of some systematic field survey and their original geodetic parameters are well known, the CRS of the reference dataset should be set in the same way. If those parameters are unknown, and if the map projection of the old map or chart cannot be estimated plausibly enough with the naked eye (as is the case with portolan charts), the cartometric analysis necessarily becomes an iterative process in which the same old map or chart is georeferenced multiple times to the same reference dataset, but each time with a different CRS assigned to it (Fig. 9.3). This means that every iteration (georeferencing of the same map or chart to the same reference dataset using a different CRS in each session) yields different estimated planimetric accuracy values. Smaller RMSE values at the end indicate greater geometric similarities between the old map or chart and a certain map projection and vice versa, meaning that this iterative process can be used to determine the most plausible map projection of an old map or chart.

4.2 Two-Dimensional Geometric Transformations

There are several two-dimensional geometric transformations to be selected for georeferencing, ranging from simpler to more complex, each affecting the geometry and the image of the georeferenced map in its own way. In

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[7] This is because, unlike the stereographic or Lambert conformal conic projection, the Mercator projection does not display the convergence of the meridians and therefore retains the same direction of north across the entire map field, and plots loxodromes as straight lines. If distances along the rhumb lines on the Earth's surface are required for the cartometric analysis, the prerequisite step is to de-project their (Euclidean) values defined within the Mercator CRS.

general, the more complex the transformation, the more adjustments the original map image undergoes during the georeferencing process.

For example, an eight-parameter projective transformation, which is based on a more complex computation and creates a perspective two-dimensional projection of a two-dimensional image (resembling its visual appearance when observed from a different angle), was used by Scott A. Loomer in the initial stage of his cartometric study of portolan charts (Loomer 1987). He applied it to the (invisible) circles of the wind roses on portolan charts—used as proxies—to convert them into perfect circles in order to rectify the perspective viewshed of cameras used to photograph them and to reduce the eventual distortions of their carrier material, the parchment, that might have occurred over centuries.

After Loomer made those corrections to prepare portolan charts for the analysis, he applied a four-parameter Helmert similarity transformation to cartometrically examine 26 portolan charts in comparison to nine map projections. Helmert transformation uniformly scales and rotates both axes of the unit (Modenov and Parkhomenko 1965), preserving the geometry of the georeferenced image of an old map or chart in relative terms. It was also used by Tome Marelić in his cartometric studies of old nautical charts and geographical maps of the Adriatic Sea (Marelić 2022, 2023a, b) and portolan charts showing the Mediterranean and Black Sea (Marelić 2024a, b). A tangible example can be seen in Fig. 9.3 that shows how the manually assembled composite of Pietro Vesconte's two portolan charts, made in 1311 and 1313 (the former of which is the oldest known portolan chart whose author and year of production are known explicitly),[8] was iteratively georeferenced to the same reference dataset twice, by using different CRS each time. Although the geometry of the reference dataset significantly differs in each iteration, leading to different scales and rotations (θ_{LSE}) of the georeferenced images, their geometry remained relatively unaltered in both cases because the same four-parameter similarity transformation was applied each time.

In his doctoral thesis, Roel Nicolai analysed portolan charts using a six-parameter affine transformation (Nicolai 2014) by which each axis becomes scaled and rotated independently. The same transformation was used by Evangelos Livieratos and Chrysoula Boutoura in their cartometric

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[8] On its eastern margin it reads: *Petrus Vesconte de Janua fecit ista carta ann[o] dni MCCCXI*, meaning *Pietro Vesconte of Genoa made this chart in 1311* (Nordenskiöld 1897: 57).

study of the *Carte Pisane* (Livieratos and Boutoura 2018). In his recent study, Nicolai used a five-parameter affine transformation by which each axis is scaled independently, whereas both of them are uniformly rotated (Nicolai 2024). Affine transformations allow more degrees of freedom in comparison to a Helmert transformation, meaning that they deform the image of georeferenced units if there are significant geometric differences between them and the reference map, and yield greater estimated accuracy values at the end.[9] They are exceptionally useful when the geodetic parameters of old maps and charts are already known and are intended to be georeferenced to a reference dataset (CRS) of the same geometric properties.

4.3 The Necessity of De-Projecting the Datasets

Since georeferencing is performed within the constraints of the Euclidean geometry, the accuracy values derived from it are not directly applicable to the real world because each map projection (defined within the selected CRS) distorts certain parameters of spherical or ellipsoidal geometry of the Earth in its own way across the map field. For example, a portion cut from a sphere along two meridians and stretching between poles is a digon, a polygon made of two straight lines on the spherical surface that meet at two points (vertices) at a certain angle, and which cannot exist in the Euclidean geometry because its most basic polygon is a triangle. Moreover, each of the three angles in the equilateral triangle in the Euclidean geometry has 60°, whereas in the spherical geometry the equilateral triangle has internal angles of 90°.

Because accurately made maps and charts are, in essence, the projected imagery of observations that were initially made on the Earth's surface, the aftermath of the cartometric analysis should be directly applicable to

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[9] The number of degrees of freedom of different two-dimensional transformations affects the computation of their RMSE of residuals. In the case of a four-parameter Helmert transformation, the formulae are: $RMSE\ dLON = \sqrt{\frac{\sum_{i=1}^{n}\left(dLON_i\right)^2}{2n-4}}$ and $RMSE\ dLAT = \sqrt{\frac{\sum_{i=1}^{n}\left(dLAT_i\right)^2}{2n-4}}$, whereas in a six-parameter affine transformation those are: $RMSE\ dLON = \sqrt{\frac{\sum_{i=1}^{n}\left(dLON_i\right)^2}{2n-6}}$ and $RMSE\ dLAT = \sqrt{\frac{\sum_{i=1}^{n}\left(dLAT_i\right)^2}{2n-6}}$.

the Earth's surface as well. This means that after the georeferencing is done, the results need to be returned into spherical or ellipsoidal values using the inverse process in order to emulate how the planimetric accuracy of maps and charts translates into displacements of identical points in the real world. Therefore, their Euclidean residuals (dX, dY in kilometres) need to be de-projected by all means, and the intermediate step for doing so is converting them into angular differences between identical-reference point pairs ($d\lambda$, $d\varphi$ in degrees).[10] The final step is to recompute those angular differences into axial distances along the longitudinal and latitudinal arcs on the Earth's surface ($dLON$, $dLAT$ in kilometres; see Fig. 9.3).[11] In 2023, Marelić published a study that illustrates the necessity of such a procedure, even in the case of a relatively small research areas. He cartometrically examined 11 early modern manuscript and printed nautical charts of the Adriatic Sea basin in comparison to the Mercator projection and the equidistant cylindrical projection, both set with φ_0=36° as their standard, true-to-scale parallel (Marelić 2023a). The research reveals that nine examined charts—the majority of which do not contain graticules, and some of which are manuscript portolan charts—showed better fit to the Mercator projection, which is methodology-wise important for two main reasons. First, the true-to-scale parallel of both reference maps is located south of the Adriatic Sea (with latitudinal extent between φ = 39.8°N and φ = 45.7°N, and mid-latitude of around φ = 42.7°N), meaning that both projections induce distance distortions across the entire research area. Second, the same true-to-scale parallel of φ_0 = 36° for both reference maps dictates that the equidistant cylindrical projection retains the same true-to-scale renderings of latitudes (distances along its images of meridians), whereas the Mercator projection progressively distorts distances towards the (north) pole. This means that all the cartometrically generated data will, in most cases, 'favour' the equidistant cylindrical projection unless they are de-projected.

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[10] The majority of GIS software allow direct translation of projected (Euclidean) coordinates into spherical or ellipsoidal coordinates (depending on the CRS), and vice versa.

[11] If the CRS contains a WGS84 ellipsoid, the following formulae can be used: $|dLON|\ [km] = |d\lambda|\ [^\circ] \times (\pi \times 6378.137 \times \cos(mid\ LAT))/(180 \surd (1 - 0.006 \times sin\ {}^\wedge\ 2(midLAT)))$ and $|dLAT|\ [km] = |d\Phi|\ [^\circ] \times (111.132 - 0.559 \times \cos(2 \times midLAT) + 0.001 \times \cos(4 \times midLAT)$. The $midLAT\,[rad]$ represents the mid-latitude between the reference point and its corresponding (georeferenced) identical point.

5 Cartometric Analyses of Historical Coordinates

The aforementioned characteristics of the georeferencing process are purely geometric in essence, which means that either no coordinates were assigned to their identical points by the cartographers who created them, or that they were intentionally ignored during the analysis. It is, for example, the only method for cartometrically analysing portolan charts. However, many later-made nautical charts, and especially the geographical maps, have been deliberately made in the map projection and contain graticules of longitudes and latitudes. In such circumstances, the identical points are (historical) coordinates that were assigned to them by their authors for their own reasons. Furthermore, some historical records, such as Ptolemy's well-known *Geography* (being the sole such survivor of classical antiquity), contain coordinates in textual format. Cartometric analyses can also be applied to historical records containing coordinates (Fig. 9.4), albeit the methodology somewhat changes in comparison to the purely geometric approach.

The first important difference is that historical coordinates, whether an integral part of old maps or textual in nature, are not imported into GIS software as an image but as a point dataset. This means that the initial step is to (manually) extract them one by one and record them on properly designed spreadsheets within which every point receives its proprietary historical longitude and latitude. Those spreadsheets can then be imported into GIS software set with the geographic CRS (made only of Earth's approximation as a sphere or some reference ellipsoid). In such a case, spreadsheet values become plotted as point datasets whose every point is positioned (georeferenced) individually according to its previously designated coordinates, which represents the second major difference in comparison to the purely geometric approach. This is because, unlike georeferencing an entire image of a map or chart at once, whose end result is determined by the parameters of the selected transformation, each point is imported and georeferenced within the GIS framework on its own, and no two-dimensional geometric transformation is tethered to the process itself. In later stages, those point datasets are usually projected by assigning some map projection to the CRS and reprojected by changing the map projection if needed. By doing so, the spatial layout of points (the geometry of the entire point dataset) changes in accordance with the selected map projection.

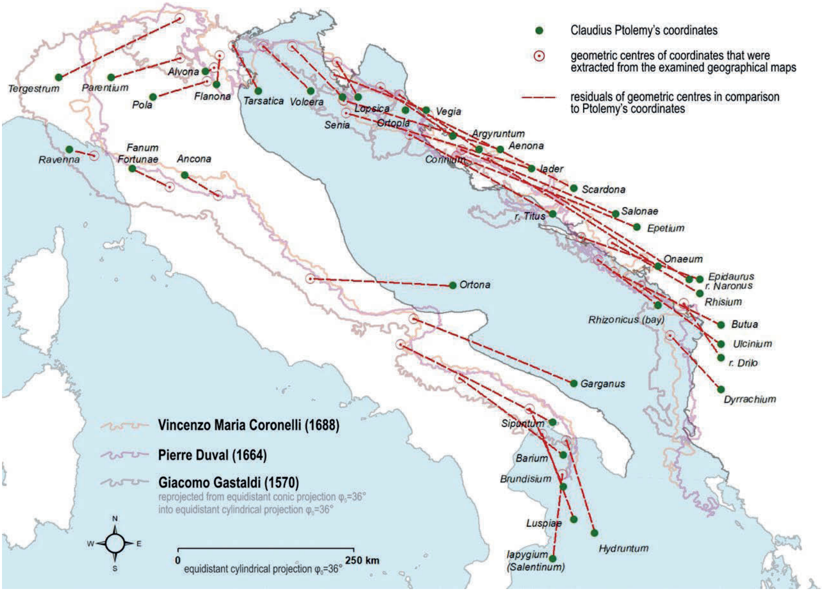

Fig. 9.4 The displacement vectors of residuals of the geometric centres of coordinates on three geographical maps (georeferenced by using the coordinates extracted from them) in comparison to Claudius Ptolemy's coordinates of locations on the Adriatic Sea coast. All datasets were longitudinally shifted by $\Delta\lambda_0 = -25.2°$. Ptolemy's coordinates source: Stevenson 1991. Basemap shapefile source: marineregions.org (Claus et al. 2017)

It is usually not possible to calculate the accuracy of historical coordinates in their current form because their longitudes have been expressed in accordance with another (historical) prime meridian. For example, Claudius Ptolemy's prime meridian passed through the so-called *Fortunate Islands* or the *Islands of the Blessed*, whose location is thought to be in the Canary archipelago. At the time, it was considered the westernmost limit of the known world (*oikoumene*), and Ptolemy chose it so that all the longitudes pointed in the same direction, ranging from 0° to 180°E (Berggren and Jones 2000). This means that historical longitudes should be translated into modern standard (recomputed with respect to the Greenwich Prime Meridian) as accurately as possible. If not, the entire historical point dataset is significantly shifted to the east or west, and the longitudinal accuracy cannot be calculated reliably. Because these point datasets represent coordinates believed to be true by the cartographers who made old maps and charts, they can be treated as pseudo-reference points and used to georeference the image of an old map or chart to them (Figs. 9.4 and 9.5).

Cartometric analyses of Ptolemy's coordinates were conducted by Livieratos and his associates (Livieratos et al. 2007, 2008), Christian Marx (2011, 2012, 2016), Irina Tupikova (2014), Marelić (2023b), and Jan Martínek and Aleš Létal (2023). In addition, Livieratos cartometrically compared the geometric properties of two *Ptolemaic maps* of Crete made in the fourteenth and seventeenth centuries[12] with Ptolemy's coordinates for the same area and concluded that there are significant geometric similarities between the historical datasets that were made to different formats (Livieratos 2006). Marelić's study (Marelić 2023b) shows that the coordinates of printed geographical maps of the Adriatic Sea and the surrounding areas made in the sixteenth and seventeenth centuries are largely similar to Ptolemy's coordinates dataset, retaining its renowned longitudinal exaggeration (Fig. 9.4). In contrast, the coordinates on the contemporaneously made nautical charts followed a completely different lineage, with longitudes that were seemingly deduced from the graphical appearance of coastlines on portolan charts (see Fig. 9.5).

6 Cartometric Analyses as the Forensics of Human History

The most obvious importance of cartometric analyses for the history of cartography as a discipline is that their application explicitly exposes metrics that are encapsulated within the historical cartographic sources but are not deducible with the naked eye, and from the phenomenological standpoint, modern cartometric analyses literally pose as the computer-aided forensics of the historical data. There is a plethora of their applications, many of which were already mentioned, and their utility covers both scientific and everyday purposes. One praiseworthy example of georeferencing old maps for everyday use is the *Arcanum project*, whose end result is a web portal that contains numerous old topographical, cadastral, and

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[12] *Ptolemaic maps* are late medieval and early modern attempts to reconstruct the original cartographic content of Claudius Ptolemy's *Geographike Hyphegesis* which has not been preserved (Gautier Dalché 2007).

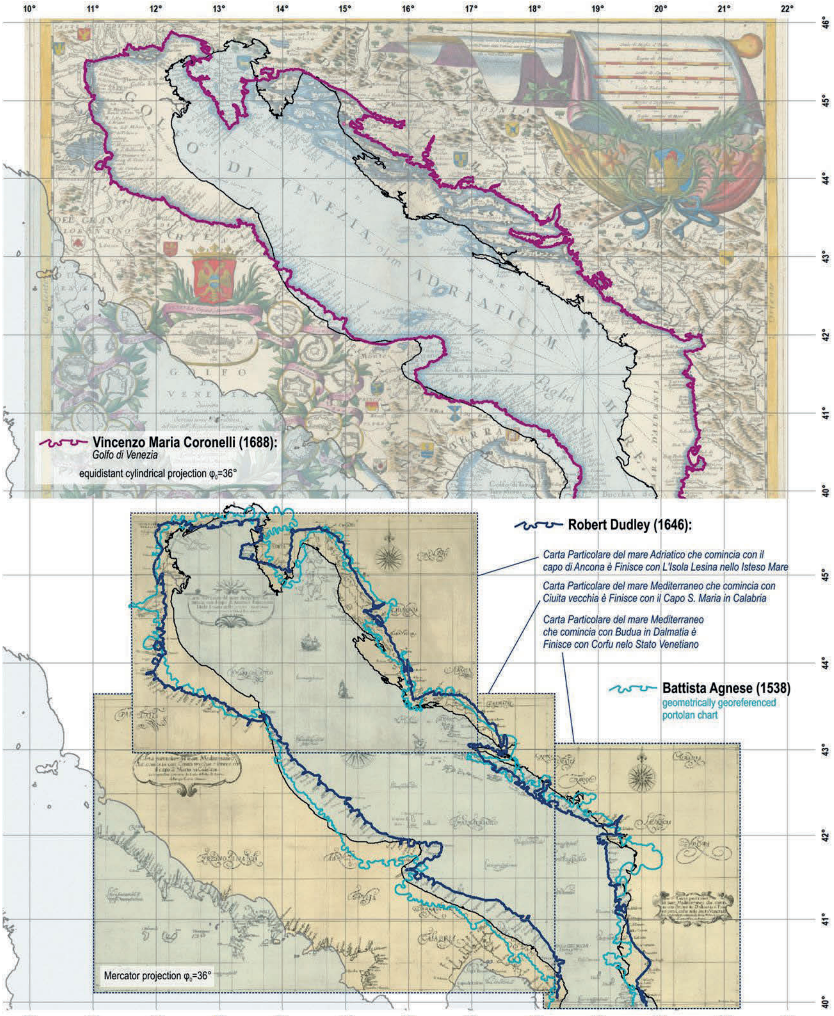

Fig. 9.5 Vincenzo Maria Coronelli's map from 1688 georeferenced to a modern map in the equidistant cylindrical projection $\varphi_0 = 36°$ (the upper part), and the composite of Robert Dudley's three nautical charts from his 1646 book *Dell'arcano del mare* georeferenced to a modern map in the Mercator projection $\varphi_0 = 36°$ (the lower part). Both units were georeferenced by using the coordinates extracted from them and with the application of proprietary longitudinal shift ($\Delta\lambda_0$) (Old map and nautical chart sources: *Nacionalna sveucilišna knjižnica u Zagrebu, zbirka karata i atlasa*, S-JZ-XVII-56; *National Library of Finland, Nordenskiöld collection* NBN:fi-fe201002051338). Basemap shapefile source: marineregions.org (Claus et al. 2017)

city maps that were scanned in high resolution and georeferenced within a web-GIS server.[13] The platform is accessible through desktop and mobile interfaces and allows the user to observe the historical data in their topologically correct order and to zoom in on desired sections for a more detailed view.

From a scholarly standpoint, cartometric analyses have proven to be the indispensable tools for the study of portolan charts. Wagner's 1896 paper not only introduced a novel method in the history of cartography, but also sparked one of the most important discoveries in human history by revealing their cylindrical map projection properties and the possibility of their sub-piece composition, and posting the hypothesis that the origin of their spatial data may be older than previously thought (Wagner 1896/1969). Regrettably, his peers, largely unfamiliar with his cartometric approach, failed to acknowledge Wagner's incredible feat (Winter 1948, 1956), and a significant segment of historians of cartography continues to express scepticism towards such concepts even today. Between Wagner's work and Loomer's extensive study that exposed their geometric similarity to the Mercator projection but did not investigate their possible origin in greater detail (Loomer 1987), there were only a few cartometric studies that are either simplistic or include only a small sample of charts (Clos-Arceduc 1956; Tobler 1966; Duken 1988).

The two most extensive and detailed ongoing cartometric studies of portolan charts, which have thus far appeared in several publications, are undertaken by Nicolai (2016, 2024) and Marelić (2024a, 2024b). The results of their analyses—interpreted in unison with known historical sources—confirm Wagner's assumptions and strongly argue that the spatial data used to establish the shapes of their coastline contours were likely acquired at some earlier period and that portolan charts are, most likely, late medieval and early modern copies of some older maps or charts. For example, the clockwise rotations of their georeferenced images in comparison to the Mercator projection, which occur due to anticlockwise-tilted renderings of their coastlines, do not agree with the regional distribution of the magnetic declination across that area at the time of their creation according to the CALS3k.4 paleomagnetic model.[14] This means that the cartometric evidence explicitly refutes the possibility that

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[13] Georeferenced historical maps on the *Arcanum* webpage can be accessed via the following link: https://maps.arcanum.com.

[14] For more information about the CALS3k.4 model, see Korte and Constable (2011).

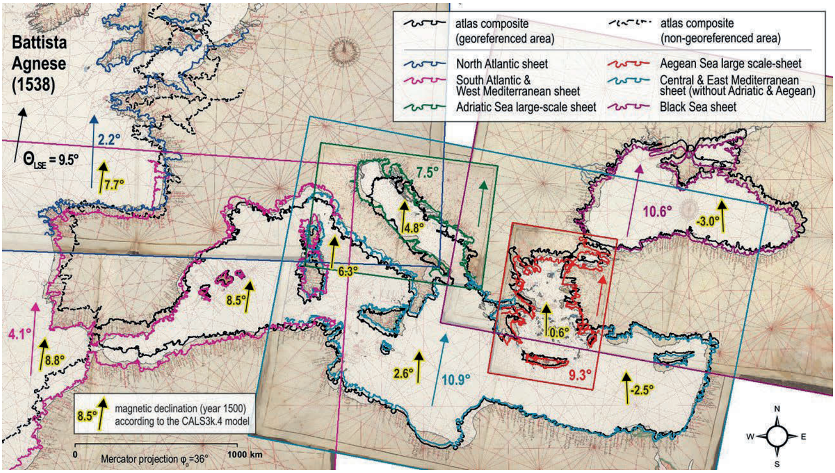

Fig. 9.6 Individually georeferenced small- and large-scale sheets from Battista Agnese's portolan atlas (1538), overlay of their vectorised coastlines with the coastlines of their coherent composite, and the magnetic declination for the year 1500 according to the CALS3k.4 paleomagnetic model (Portolan atlas source: *University of Pennsylvania, Rare Book & Manuscript Library*, LJS 28). Basemap shapefile source: marineregions.org (Claus et al. 2017). CALS3k.4 data source: GEOMAGIA50.v3.2, accessed: March 18, 2023

shipborne use of the magnetic compass led to the creation of portolan charts based on contemporaneously acquired bearing observations (Fig. 9.6). Furthermore, Marelić's recent cartometric study of portolan atlases (Marelić 2024c, 2025a) shows that individual georeferencing of their sheets produces 'broken portolan chart images' because the anticlockwise tilts and map scales of their coastline renderings differ from those of their manually assembled coherent composites that were georeferenced in parallel (compare black and colour-coded vectorised tracings in Fig. 9.6) and that later-made atlases increasingly diverged from the contemporary magnetic declination values. These findings explicitly raise suspicion about the cartographers' in-depth knowledge of their own products and additionally decrease the likelihood of their genuine late medieval origin. Other significant pieces of cartometric evidence supporting the pre-medieval origin hypothesis include the fact that large portions of coastlines on the early, supposedly less-sophisticated *Pisane*, *Cortona*, *Avignon*, and *Lucca* charts are very similar to each other and to Pietro Vesconte's charts, which are thought to be more mature and sophisticated (Marelić 2024a, 2025b); that portolan charts and atlases have not become more accurate over time (Marelić 2024b, 2025a); and that they exhibit twice as great planimetric accuracy after being split into their cartometrically determined sub-pieces that are georeferenced individually (see colour-coded vectorised tracings in Fig. 9.7). The last discovery was independently made by Marelić (2024a, b; 2025a, b) and Nicolai (2024), albeit through distinct methodological approaches. The most logical explanation for such phenomena is that late medieval cartographers were unable to properly assemble their copies of some earlier-made maps or charts (that they had somehow acquired in advance) and that they have stitched them together graphically with differences in scale and tilt.

Alongside cartometric analyses, various historical arguments challenge their medieval origin, including prolonged political upheavals in the region

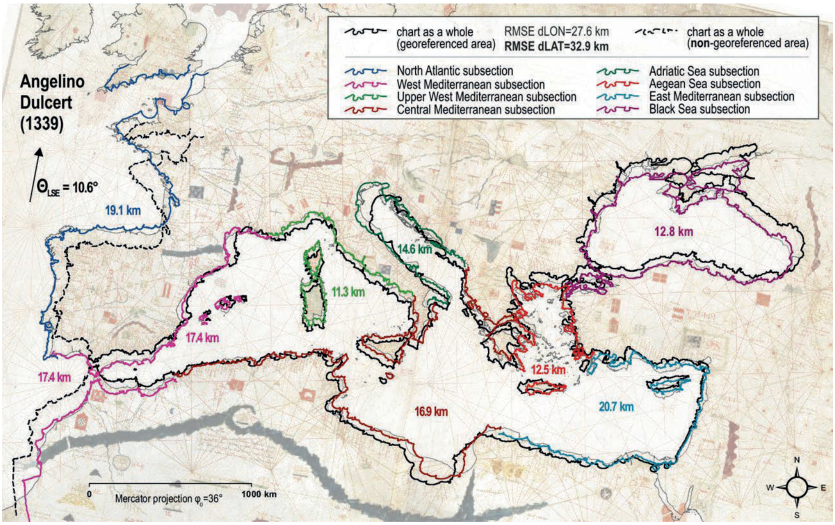

Fig. 9.7 Angelino Dulcert's portolan chart from 1339 georeferenced across the Mediterranean and Black Sea (black line) and its eight cartometrically determined subsections (colour-coded lines) with an average planimetric accuracy twice as high (Portolan chart source: *Bibliothèque nationale de France*, GE B-696 (RES)). Basemap shapefile source: marineregions.org (Claus et al. 2017)

(Abulafia 2011) that obstructed the practicality of extensive interstate campaigns for the hypothetical shipborne acquisition of substantial bearing and distance data, the pervasive lack of understanding of geodetic principles and map projection mechanics (as the earliest West European engagement with such concepts occurred through the rediscovery of Ptolemy's *Geography* during the early Renaissance), and the inadequacy of navigational instruments and sailboat manoeuvrability. Also, the earliest-known portolan charts were created decades before the invention and widespread adoption of the mariner's compass in the mid-fourteenth century, whereas the estimation of sailed distances emerged in the sixteenth and seventeenth centuries (Pujades 2007; Nicolai 2014). In contrast to the total lack of historical documentation validating the medieval origin hypothesis, three historical sources distinctly attribute their origins to classical antiquity, each created approximately a century apart. The earliest one is Qutb al-Din al-Shirazi's description (1282) of a map akin to a portolan chart, which he claims to be originally made by the sages of Greece and ancient geometers.[15] Two other historical records are the inscription on the anonymous *Rex Tholomeus* portolan chart (c. 1360),[16] and the passage from Benedetto Cotrugli's *De Navigatione Liber* (1464),[17] which unilaterally assert that portolan charts originated from Ptolemy's maps (Marelić 2024b). Marelić cartometrically established that the anticlockwise tilt of the Central Mediterranean on portolan charts corresponds to the tilt of the same area in the modern approximation of *Ptolemy's first projection* (equidistant conic projection) centred on the longitude of Gibraltar (see the upper part of Fig. 9.3). Also, the tilts of their subsections from west to east increase similarly to the tilts of the meridians on that map, in contrast to the magnetic declination of that period, which declined towards the east (Marelić 2024b). Moreover, Marelić's most recent cartometric study examines the geometry of al-Shirazi's data and some early-made portolan charts in conjunction with well-known historical sources and points at certain solid

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[15] The additional information about Qutb al-Din al-Shirazi's records and their English transcript can be found in Savadi and Campbell (2023). For its detailed cartometric analysis that points at the classical antiquity origins of their spatial data, see Marelić (2025b).

[16] For more information regarding the *Rex Tholomeus* portolan chart, see Ruderman et al. (2023) and Marelić (2024b).

[17] Benedetto Cotrugli's *De Navigatione Liber* is the property of Yale University, Beinecke rare book and manuscript library, Call No.: MS 557 (https://collections.library.yale.edu/catalog/2005406). The passage appears on pages 60v (image ID: 1029839) and 61r (image ID: 1029840). Its transcript and English translation can be found in Marelić (2024b).

connections between the portolan charts and Claudius Ptolemy's geographical opus that were previously unimagined, including the first-ever objectively proposed reason for the length of the *portolan mile* (*miglio*) of approximately 1.25 km. Ptolemy's inaccurate dimensions of the Earth's circumference, which are 1.4 times smaller than the actual circumference, led to an incorrect division of the Mediterranean into about 60° of longitude instead of 42°. As a result, a one-degree arc along the parallel $\varphi=36°$ mistakenly measures about 63 km, corresponding to 50 portolan miles. The lengths of the squares in the grids common to the *Pisane* and *Avignon* charts, as well as within al-Shirazi's matrix of 40×30 squares with a schematic appearance of the coastline remarkably similar to that on the *Pisane* and *Cortona* charts, amounting to 100 portolan miles, correspond to the length of a two-degree arc along the parallel $\varphi=36°$ according to Ptolemy's inaccurate perception of the Earth's circumference (Marelić 2025b). These results, bolstered by historical evidence, cumulatively suggest that the typical geometry of portolan charts may be the result of a process in which late medieval cartographers created a graphical composite of copies of regional maps in cylindrical projection (made based on Eratosthenes's dimensions of the Earth) that they gradually tilted more and more from west to east in order to emulate the appearance of the Mediterranean on a map in conic projection rotated so that the meridian passing through the 'beginning of the Mediterranean' (its farthest west at the area of Gibraltar) is plotted vertically and provides a novel and more credible explanation of the geometry of portolan charts, favouring the hypothesis of classical antiquity origins. Therefore, they are not only critically significant for the advancement of the history of cartography as a discipline but also greatly enhance our understanding of human history. While their implications are predominantly debated within academic circles, their cultural relevance transcends this context and warrants greater dissemination to the general public.

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