

**Kruisbestuiving van numerische aerodynamica
naar numerische hydrodynamica
H. Deconinck**

**Colloquium
Numerieke oppervlaktewater modellering
Antwerpen, 23-24 oktober 2003**



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Samenvatting

Introductie

Godunov eindige volume methode

Residie distributie methode

Tijdsafhankelijke schema's

Hybriede rooster generatie

Oplossingsafhankelijke roosters



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Doel

Enkele actuele onderwerpen van onderzoek op het VKI belichten i.v.m. methodes voor het oplossen van **hyperbolische behoudswetten**

Link leggen met simulatie van oppervlaktestromingen

(andere toepassingen van hyperbolische behoudswetten: MHD, gas-vloeistofstromingen)



Introductie (1/3)

Kenmerken van aerodynamische stromingen

- **Gedomineerd door convectie: dunne grenslagen, hoge Reynoldsgetallen**
- **Complexe geometrie: motoren, inlaten, landingsconfiguratie**
- **Hoge nauwkeurigheid vereist bvb voor berekening van de drift (.0001 op de drift oefficient)**



Noodzaak voor nauwkeurige schema's op niet gestructureerde roosters
Hybride niet gestructureerde roosters (met semi-gestructureerde grenslagen)
Oplossings-afhankelijke roosters



Introductie (2/3)

Bijkomende kenmerken van transsonen en supersonen stromingen

- Sterke samendrukbaarheids effecten:
Schokgolven
Shear layers
- Zeer dikwijls tijdsafhankelijk : schok interacties ...

➔ **Noodzaak voor “monotone shock-capturing” schema’s, nauwkeurige tijdsintegratie**
Technologie van upwind schema’s : Godunov schema’s



Introductie (3/3)

Oppervlaktewater modellering: gelijklopende behoeften

- Gedomineerd door convectie, golfpropagatie fenomenen, hyperbolische systemen
- Schokgolven
- Complexe geometrie: estuaria ...
- Bewegende randen (dry out, getijden)

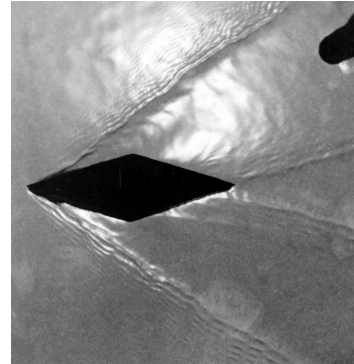
➔ **Monotone shock capturing schema’s, hoge precisie**
Tijdsnauwkeurigheid
Niet-gestructureerde bewegende roosters



Similitude tussen aerodynamica en oppervlaktestroming



Supersone stroming van lucht
(S1 windtunnel van het VKI)



Supercritische dunne laag stroming
(Watertafel van het VKI)



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De ondiep-water vergelijkingen

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{0},$$

$$\mathbf{U} = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + h\frac{a^2}{2} \\ huv \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + h\frac{a^2}{2} \end{pmatrix}$$

h : waterhoogte
u, v: snelheid
a: golfsnelheid van
storingen

$$a = \sqrt{gh}$$

$$g = 9.81 \text{ m s}^{-2}$$

Froude getal: $Fr = \frac{V}{a} = \frac{\sqrt{u^2 + v^2}}{a}$

Energie behoud langs stroomlijn: $H = a^2 + \frac{u^2 + v^2}{2} = C^t$



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De Euler vergelijkingen voor gasdynamica: zijn formeel identiek

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{0},$$

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + \rho \frac{a^2}{2} \\ \rho uv \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + \rho \frac{a^2}{2} \end{pmatrix}$$

rho: densiteit
u, v: snelheid
a: geluidssnelheid

$$a = \sqrt{\frac{\gamma p}{\rho}}, \quad \gamma = 2$$

Mach getal: $Ma = \frac{V}{a} = \frac{\sqrt{u^2 + v^2}}{a}$

Bernouilli

Dichtheid speelt de rol van waterhoogte h

$$H = a^2 + \frac{u^2 + v^2}{2} = C^t$$



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Algemene theorie: Hyperbolische behoudswetten

Conservatieve vorm

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{0}, \quad \frac{d}{dt} \int_V U dV + \oint_S \vec{F} \cdot \vec{n} dS = 0$$

Quasi-lineaire vorm

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A} \frac{\partial \mathbf{U}}{\partial x} + \mathbf{B} \frac{\partial \mathbf{U}}{\partial y} = \mathbf{0}, \quad \mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} \text{ and } \mathbf{B} = \frac{\partial \mathbf{G}}{\partial \mathbf{U}}$$

Voor ondiep water vgl:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ a^2 - u^2 & 2u & 0 \\ -uv & v & u \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ -uv & v & u \\ a^2 - v^2 & 0 & 2v \end{bmatrix}$$

Matrix $C_n = A n_x + B n_y = R_n \Lambda_n L_n$ heeft als eigenwaarden de golfsnelheden in de richting \mathbf{n} : $\mathbf{u} \cdot \mathbf{n}$, $\mathbf{u} \cdot \mathbf{n} + a$ en $\mathbf{u} \cdot \mathbf{n} - a$



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Krachtigste numerische methodes: gebaseerd op karakteristieke decompositie:

Discretisaties die rekening houden met de
golfvoortplantingseigenschappen:

- ✓ de golf voortplantingsrichtingen : stroomopwaartse beïnvloeding, Invloedsgebieden (bvb Mach kegel)
- ✓ niet-lineaire effecten: shock capturing:
 - ✓ Monotoon: voldoen aan een maximum principe
 - ✓ Capturing in 2-3 cellen voor een stationnaire schok



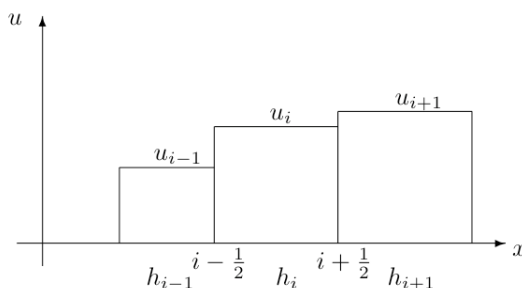
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Godunov - Eindige volume methode

In 1D, scalair: $\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = q$ $\frac{\partial u}{\partial t} + a(u) \frac{\partial u}{\partial x} = q$

Integraal vorm: $\frac{d}{dt} \int_a^b u \, dx + f(b) - f(a) = \int_a^b q \, dx$

Toepassing op een set "eindige volumes" met cell gemiddelden u_i



$$h_i \frac{du_i}{dt} + (f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}) = h_i q_i$$

$f_{i+\frac{1}{2}}$ = numerieke flux:
bekomen door oplossing
van een Riemann probleem
(Godunov)



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Riemann solver

De numerieke flux: bekomen door oplossing van een Riemann probleem voor elke interface:

Scalaire vgl:

$$f_{i+\frac{1}{2}} = \frac{f_i + f_{i+1}}{2} - \frac{\text{sgn}(a_{i+\frac{1}{2}})}{2} (f_{i+1} - f_i)$$

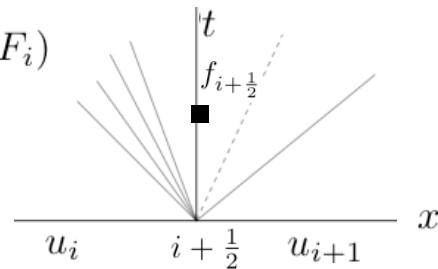
}

f_{i+1}
 f_i

Stelsel: eigenvector decompositie $A = R\Lambda L$

$$F_{i+\frac{1}{2}} = \frac{F_i + F_{i+1}}{2} - \frac{I_{i+\frac{1}{2}}}{2} (F_{i+1} - F_i)$$

$$I = R \frac{\Lambda}{|\Lambda|} L$$



Eindige Volume methode in 2D, 3D

$$\frac{d}{dt} \int_V U dV + \oint_S \vec{F} \cdot \vec{n} dS = 0$$

De numerieke flux:

$$H = \vec{F} \cdot \vec{n}$$

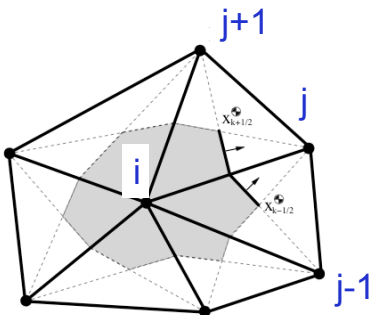
bekomen door oplossing van een Riemann probleem in de richting normaal op de cell rand tussen cell i en j:

$$H_{i,j} = \frac{H_i + H_j}{2} - \frac{I_{i,j}}{2} (H_j - H_i)$$

$$I = R_n \frac{\Lambda_n}{|\Lambda_n|} L_n$$

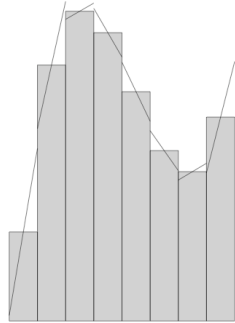
$$C_n = A n_x + B n_y = R_n \Lambda_n L_n$$

= 1-dimensionale fysica

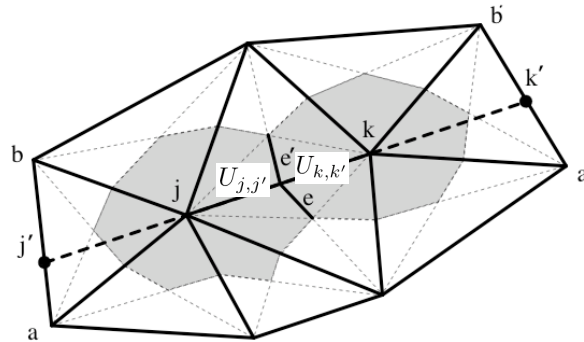


Eindige Volume methode: Hogere orde schema's via MUSCL reconstructie (1)

1. Lineaire (of kwadratische) reconstructie



Lineaire reconstructie in 1D



Lineaire reconstructie voor edge e, e' :

$$U_{j,j'}$$

$$U_{k,k'}$$



Eindige Volume methode: Hogere orde schema's via MUSCL reconstructie (2)

- Opleggen van een maximum principe door gebruik van slope limiters (TVD, LED schema's)
= slope limited reconstruction

$$\min_{\forall e_{jk} \in \partial T_j} (u_j^n, u_k^n) \leq u_j^{n+1} \leq \max_{\forall e_{jk} \in \partial T_j} (u_j^n, u_k^n)$$

- Gereconstrueerde toestanden invoeren in flux functie

$$H_{j,k} = \frac{H(U_{j,j'}) + H(U_{k,k'})}{2} - \frac{I_{j,k}}{2} (H(U_{k,k'}) - H(U_{j,j'}))$$



Conclusie Eindige Volume methode

- Oorspronkelijk ontwikkeld voor gestructureerde roosters
- Monotone schema's
- 1-dimensionale fysica
- Nauwkeurigheid op onregelmatige willekeurige roosters kan beter
- Grote molecule (wegens reconstructie)



Onderzoek naar meer performante methodes

- Gebruik van eindige elementen interpolatie op willekeurige Roosters: lineaire driehoeken (P1)
- Compacte stencil (vergeleken met FV)
- Consistente formulering (zoals Eindige Elementen)
- Monotoniceits eigenschappen (zoals Godunov FV) voor shock capturing



Discontinuous Galerkin methodes (e.g. Cockburn)
Continue Eindige Elementen methodes (Petrov -G)
Residu distributie schema's



Residu Distributie methode - stationaire behoudswet

$$\nabla \cdot \mathbf{f}(u) = 0 \Leftrightarrow \mathbf{v}(u) \cdot \nabla u = 0, \mathbf{v}(u) = \partial \mathbf{f}(u) / \partial u$$

Iterative scheme

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{V_i} \sum_{E \in \Delta_i} \phi_E^i$$

Element Residual

$$\phi_E = \int_E \nabla \cdot \mathbf{f} \, dV$$

Residual Distribution \Rightarrow Local Nodal Residual

$$\phi_E^i = \beta_E^i \phi_E$$

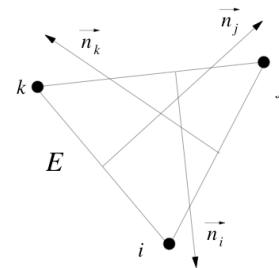
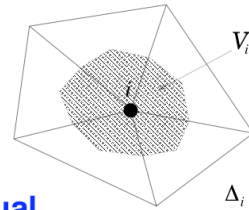
Basic Properties

- Consistency $\sum_{i \in E} \phi_E^i = \phi_E$
- MultiD Upwinding $\phi_E^i \propto k_E^{i+}$

Upwind Parameters

$$k_E^i = \mathbf{v}(\bar{u}) \cdot \vec{n}_i / 2$$

$$k_E^{i-} = \min(0, k_E^i), k_E^{i+} = \max(0, k_E^i)$$



Residu Distributie methode

$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{f}(u) = 0 \quad \text{or} \quad \frac{\partial u}{\partial t} + \vec{\lambda}(u) \cdot \nabla u = 0$$

to be discretised with a piecewise linear solution representation u^h on triangular meshes (P1 finite element representation).

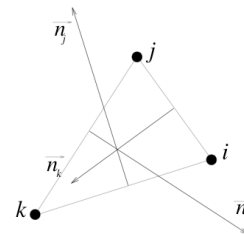
Basic Residual Distribution scheme:

- The residual (or fluctuation) ϕ^T is constructed as

$$\phi^T = \int_T \vec{\nabla} \vec{f}(u^h) \, d\Omega \equiv |T| \vec{\lambda} \cdot \vec{\nabla} u^h$$

where $\vec{\lambda}$ is given by a conservative (Roe) linearisation:

$$\vec{\lambda} = \frac{1}{\Omega_T} \int_T \vec{\lambda}(u^h) \, d\Omega.$$



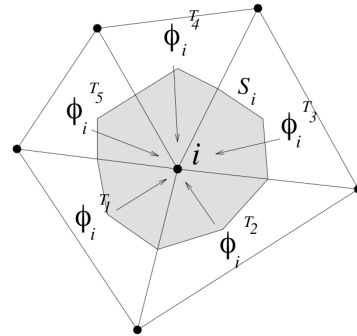
Residu Distributie methode - stationaire behoudswet

- ✓ Het residu van elk element wordt gedistribueerd naar de knooppunten met een gewicht β_i^T
- ✓ Fractie gedistribueerd naar knoop i

$$\phi_i^T = \beta_i^T \phi^T$$

- ✓ Na assemblage in de knooppunten:

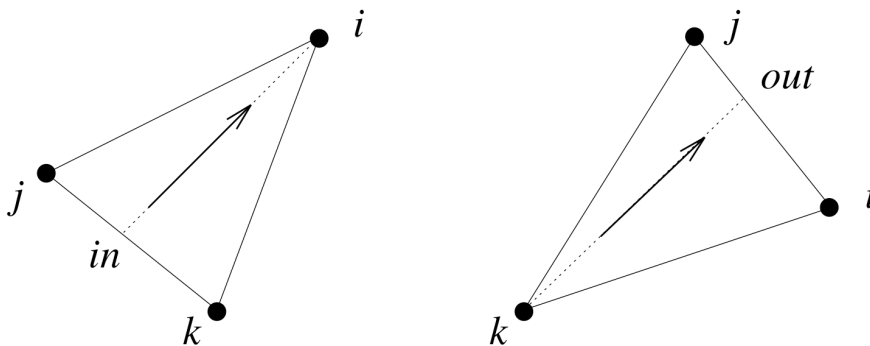
$$u_i^{n+1} = u_i^n - \frac{\Delta t}{S_i} \sum_T \beta_i^T \phi^T$$



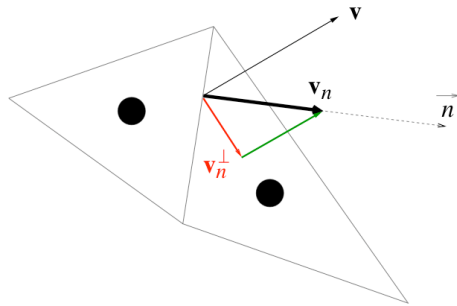
Residu Distributie methode - multiD upwinding

Multidimensional upwinding can now be defined as follows:

- 1 target case: send entire residual to the unique downstream node
- 2 target case: distribute the residual over the 2 downstream nodes
- conclusion: require $\beta_i^T = 0$ for upwind nodes, i.e. nodes for which $k_i \leq 0$



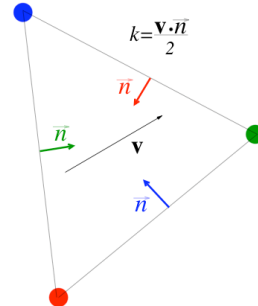
Residu Distributie: van randen naar elementen



FV Upwinding

Along Edge Normals

→ Cross-Wind Propagation of Information
i.e. **Unphysical Cross-Wind Dissipation**



RD MultiD Upwinding

All the Directions

→ Minimum Cross-Wind Propagation
i.e. **Minimum Cross-Wind Dissipation**

RD are **Less Diffusive** → **Higher Resolution**

RD are **Residual Schemes** → **Better Accuracy Properties**



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Residu Distributie: 2de orde monotone schema's

For a linear (or linearized) problem:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\mathcal{V}_i} \sum_{E \in \Delta_i} \phi_E^i = \sum_{E \in \Delta_i} c_{ij}^E u_j^n$$

Positivity

$$c_{ij}^E \geq 0 \rightarrow u_{\min}^n \leq u_i^{n+1} \leq u_{\max}^n$$

2nd order of accuracy

$$\phi_E^i = \mathcal{O}(h^3) : \phi_E^i = \beta_E^i \phi_E \rightarrow \beta_E^i \text{ bounded, } \phi_E = \mathcal{O}(h^3)$$

No linear scheme can satisfy both constraints (Godunov)

Positive Scheme: β_E^i unbounded (only first order accurate)

2nd Order Scheme: $c_{ij}^E < 0$ for some j



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Residu Distributie: 2de orde monotoon schema (niet linear)

We want to design schemes having the properties:

$$(1) \quad \phi_E^i = \alpha_E^i \phi_M^i \quad \text{with } \alpha_E^i \geq 0 \quad \rightarrow \quad c_{ij}^E = \alpha_E^i c_{ij}^M \geq 0$$

for some monotone (positive) scheme M

$$(2) \quad \phi_E^i = \beta_E^i \phi_E \quad \text{with } \beta_E^i \text{ bounded}$$

Combining (1) and (2) we obtain:

$$\phi_E^i = 0 \quad \text{if } \phi_E = 0$$

otherwise:

$$\phi_E^i = \beta_E^i \phi_E = \beta_E^i \frac{\phi_E}{\phi_M^i} \phi_M^i = \frac{\beta_E^i}{\beta_M^i} \phi_M^i = \alpha_E^i \phi_M^i$$

Monotonicity (positivity) Constraint

$$\beta_E^i \beta_M^i \geq 0 \quad \left(\sum_j \beta_E^j = 1, \text{ for consistency} \right)$$



Residu Distributie: 2de orde monotoon schema (niet linear)

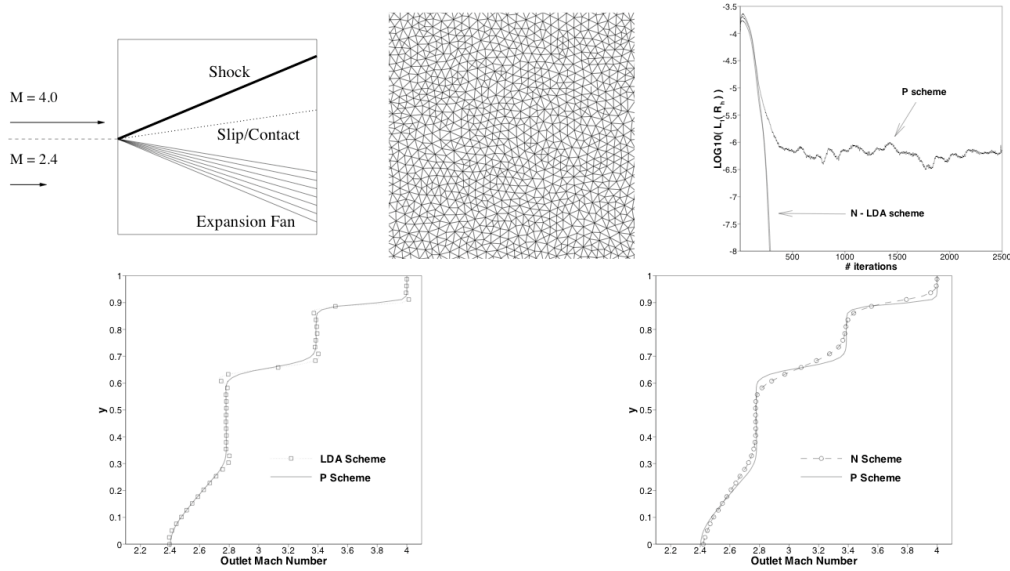
Example of Scheme

$$\beta_E^i = \frac{\max(0, \beta_M^i)}{\sum_j \max(0, \beta_M^j)} \geq 0$$

1. Very effective for steady homogeneous scalar advection (PSI scheme of Struijs)
2. Can be extended to unsteady homogeneous and inhomogeneous advection
3. **Extension to systems via wave decomposition (Abgrall 2000)**



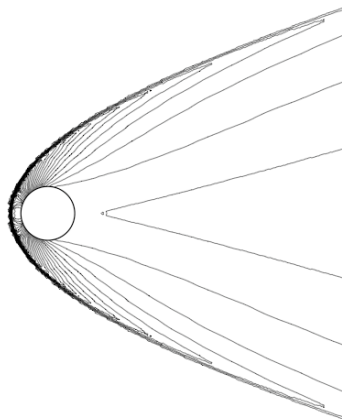
Voorbeeld: Supersone jet interactie



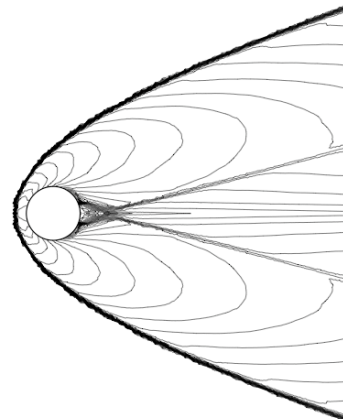
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Voorbeeld: Mach 10 stroming rond cylinder

Pressure contours

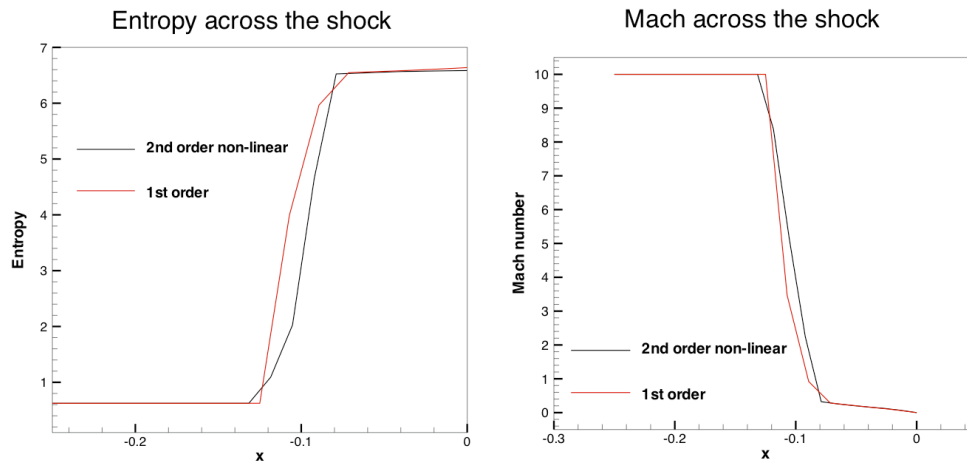


Mach number contours



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Voorbeeld: Mach 10 stroming rond cylinder



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Residu distributie schema's voor Instationaire stromingen

1. **Space and Time Discretization cannot be written in a Decoupled way**
2. **Consistent** Finite-Elements formulation

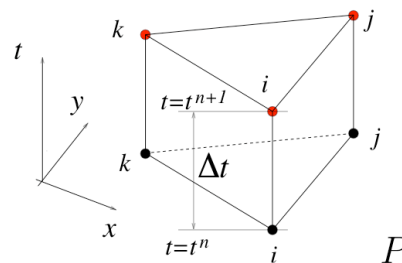
$$\phi_E^i = \int_{t^n}^{t^{n+1}} \int_E \omega_i [u_t + \nabla \cdot \mathbf{f}(u)] dV dt$$

For some schemes becoming:

$$\phi_E^i = \beta^i \phi_P = \beta^i \int_{t^n}^{t^{n+1}} \int_E [u_t + \nabla \cdot \mathbf{f}(u)] dV dt$$

The final result is a system of (non-linear) algebraic equations:

$$\mathcal{A}(u^n, u^{n+1})u^{n+1} = \mathcal{B}(u^n, u^{n+1})u^n$$



The schemes are inherently implicit !!



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Residu distributie schema's voor Instationaire stromingen

Accuracy The condition for 2nd order of accuracy becomes

$$\phi_E^i = \mathcal{O}(h^3, \Delta t^3) : \phi_E^i = \beta^i \phi_P \rightarrow \beta^i \text{ bounded, } \phi_P = \mathcal{O}(h^3, \Delta t^3)$$

Positivity The concept of positivity is extended to the space-time schemes:

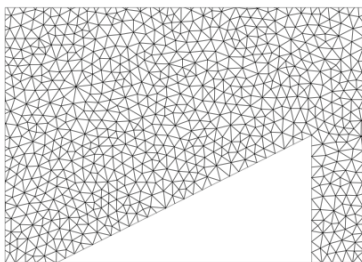
$$\mathcal{A}(u^n, u^{n+1})u^{n+1} = \mathcal{B}(u^n, u^{n+1})u^n$$

1. \mathcal{A} must be invertible
 2. \mathcal{A}^{-1} must have all positive entries
 3. \mathcal{B} must have all positive entries
- Positive 2nd accurate schemes are built exactly like in the steady case.
 - The constraints on the matrices \mathcal{A} and \mathcal{B} translate in a limitation on the size of Δt
 - This limitation is related **only to positivity**, the monotone schemes are in general linearly unconditionally stable
 - **This limitation can be overcome**



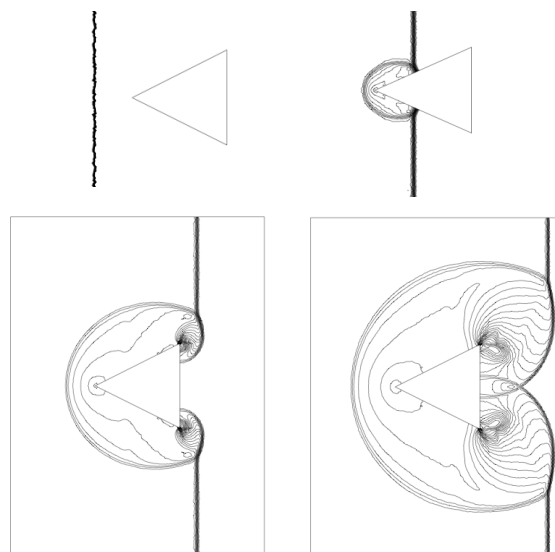
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Voorbeeld: Mach 1.6 shock hitting a finite wedge



Course mesh: $h = 1/50$

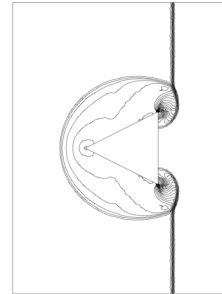
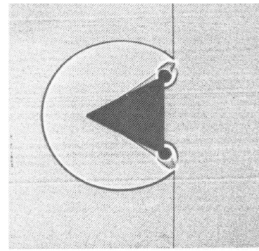
Solutions on real mesh:
 $h = 1/100$
at
 $t = 0, 0.25, 0.5, 0.6875$



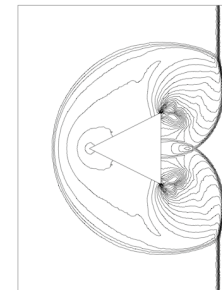
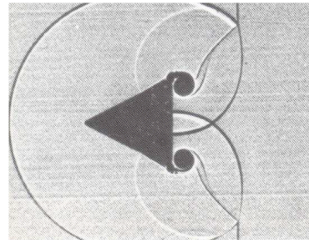
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Mach 1.6 shock hitting a finite wedge Comparison with schlieren

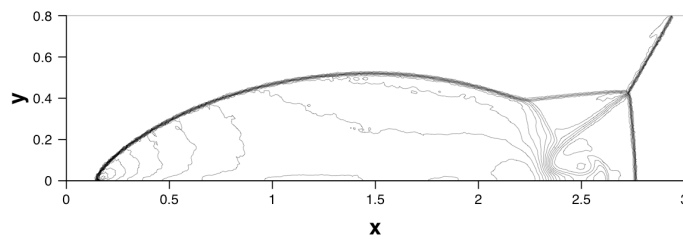
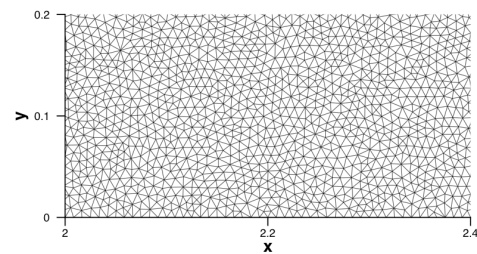
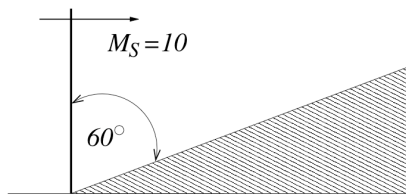
t=0.5



t=0.6875



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$t = 0.2, h = 1/120$

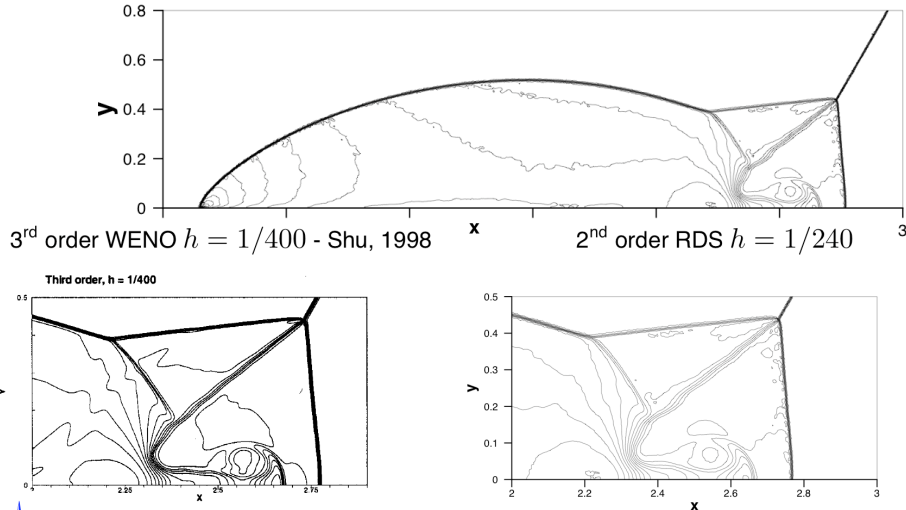


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Double Mach Reflection (2)

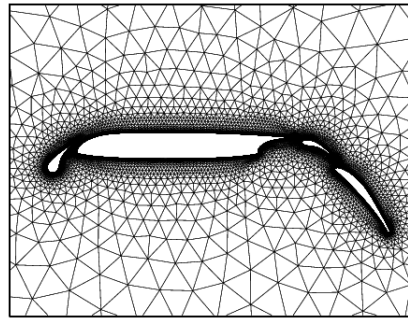
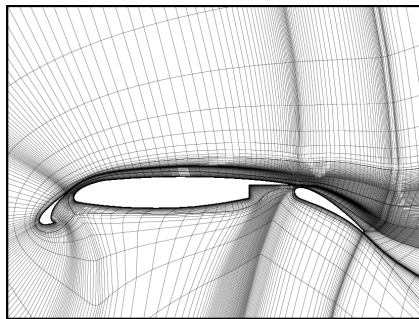
$t = 0.2, h = 1/240$

Contour Integration Based Schemes



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Ongestructureerde roostergeneratie



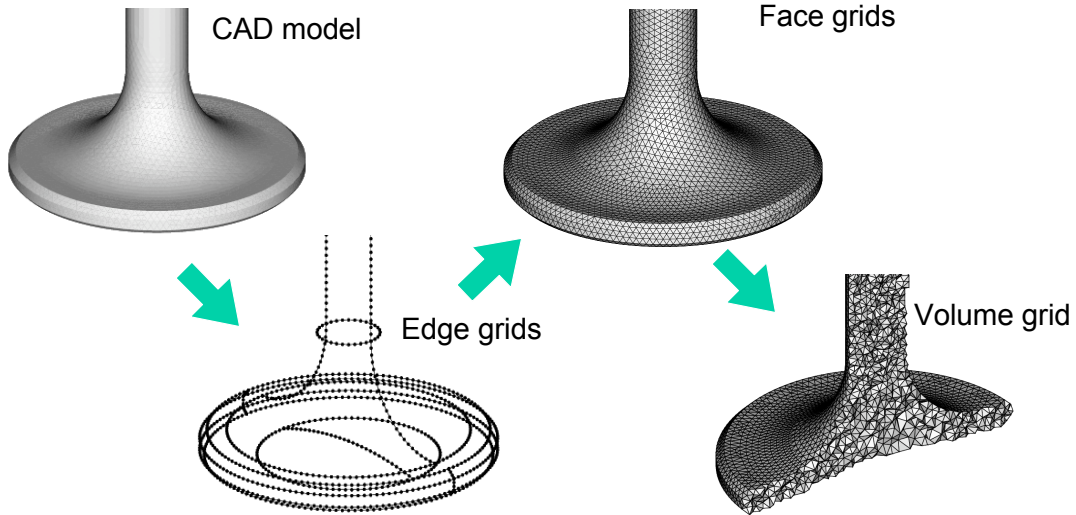
Ongestructureerde versus multi-block gestructureerde roosters:

- Hogere graad van automatisering voor complexe geometriën
- Mogelijkheid voor adaptieve verfijning
- Probleem voor niet gestructureerde roostergeneratoren:
genereren van grenslaag roosters (anisotroop)



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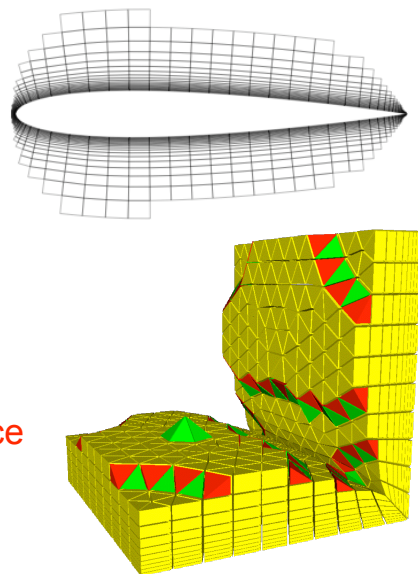
Hierarchische aanpak



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Anisotrope semi-gestructureerde rooster generatie

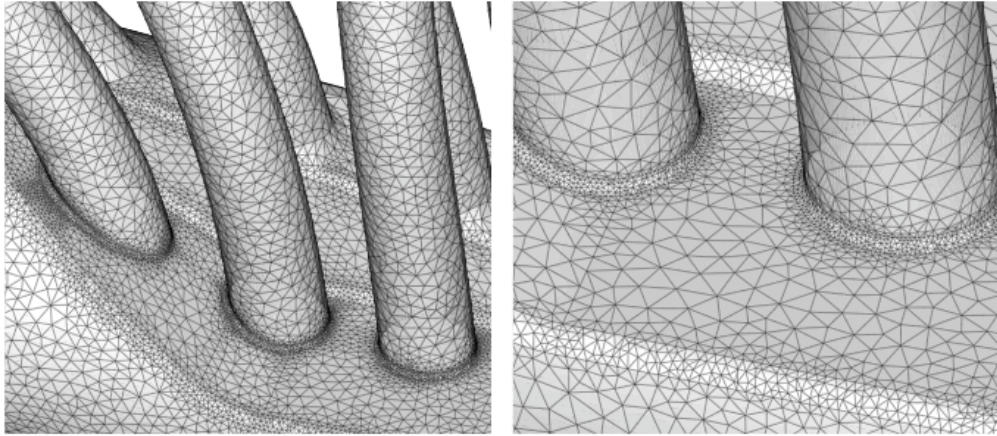
- **Semi-structured prismatic layers near boundaries**
 - Stop if normal spacing reaches isotropic limit
 - Invalid prisms deleted
 - Intersecting prisms deleted
 - Interface cells (prisms, pyramids) generated for isotropic part
- **Isotropic tetrahedra fill remaining space**



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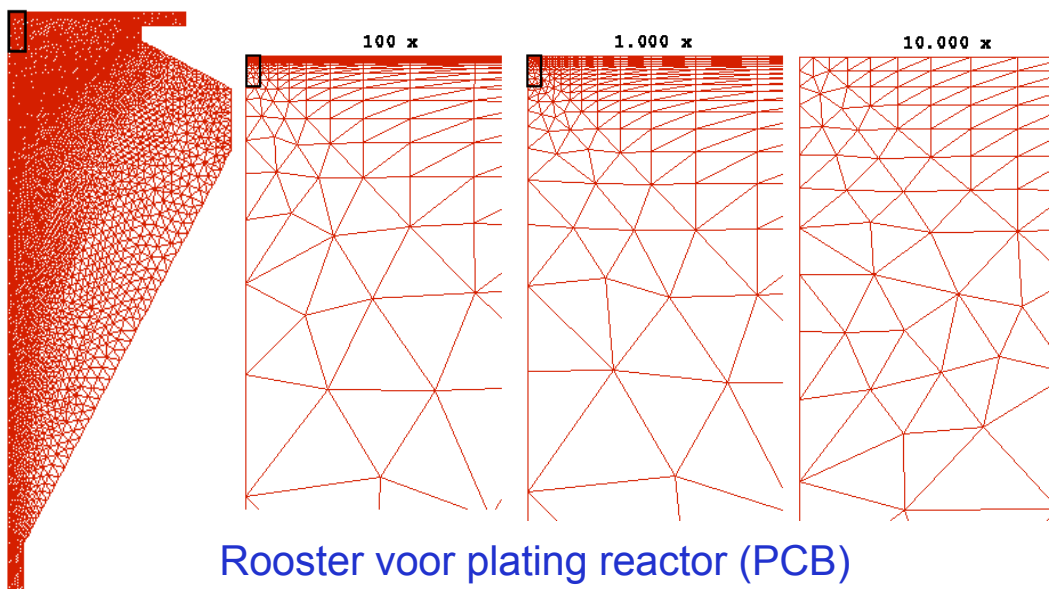
Automatische resolutie van kromming

Surface grid automatically generated from two global parameters



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Semi-gestructureerde rooster generatie

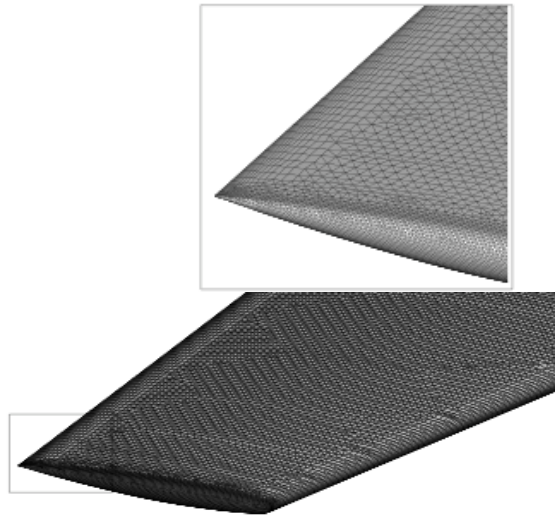


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Hybriede oppervlakteroosters

Advantages

- Align semi-structured grid with the main flow gradients.
- Hexahedral elements have potentially better accuracy.
- Good resolution of leading/trailing edge with minimum number of elements.
- Needed for the new algorithm for the semi-structured grid generation.



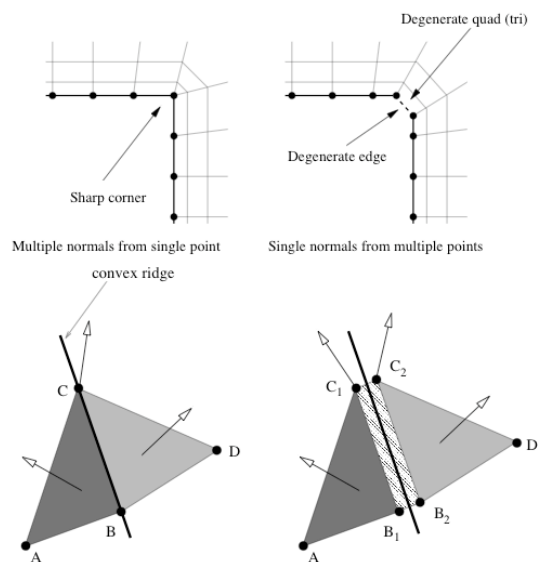
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Front folding/unfolding algoritme

Front unfolding

Create the effect of multiple normals in convex ridges. Keep same data-structures and main semi-structured generation procedure .

- Detect “sharp” convex ridges of the domain based on given angle criterion.
- Modify semi-structured generation front with degenerate (virtual) faces.
- Generate new layer of elements.
- Identify degenerate elements and new faces in the top layer.
- Repeat unfolding in next layer if needed.



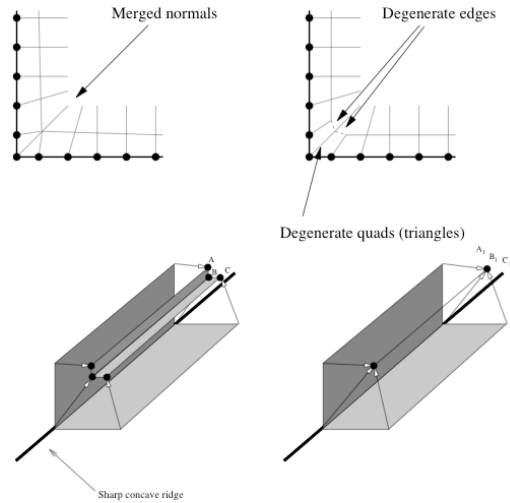
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Front folding/unfolding algoritme

Front folding

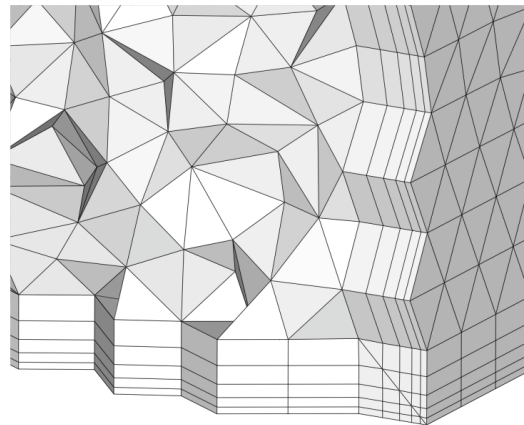
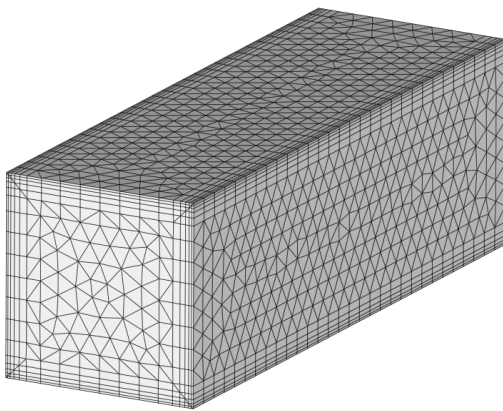
Create the effect of collapsing normals for concave ridges of the domain.

- Detect concave ridges with semi-structured surface grids.
- Create layer of elements.
- Find “thin” quad faces in the top layer.
- Collapse thin faces and update elements and top front.
- Repeat in new layer if needed.



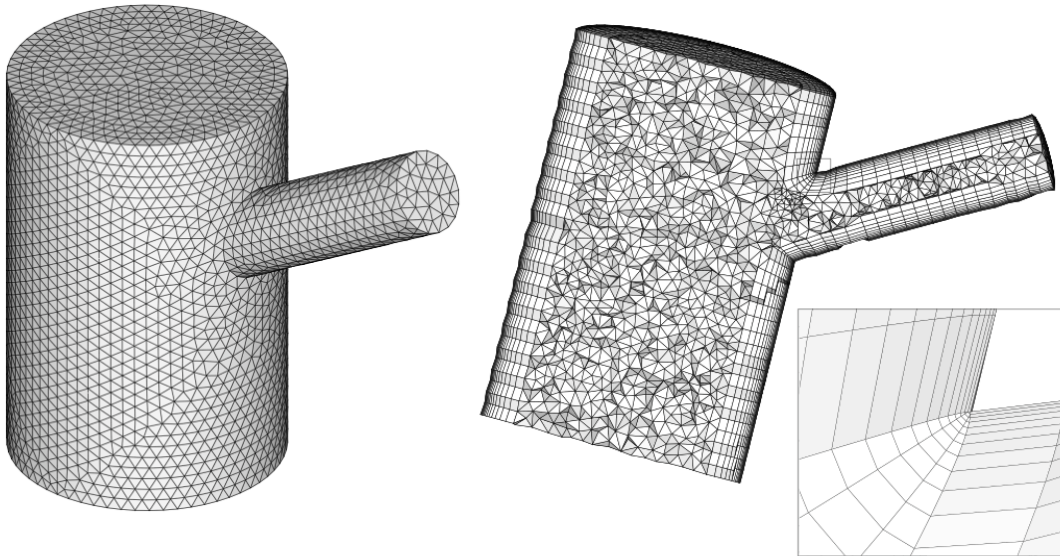
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Voorbeeld van front folding



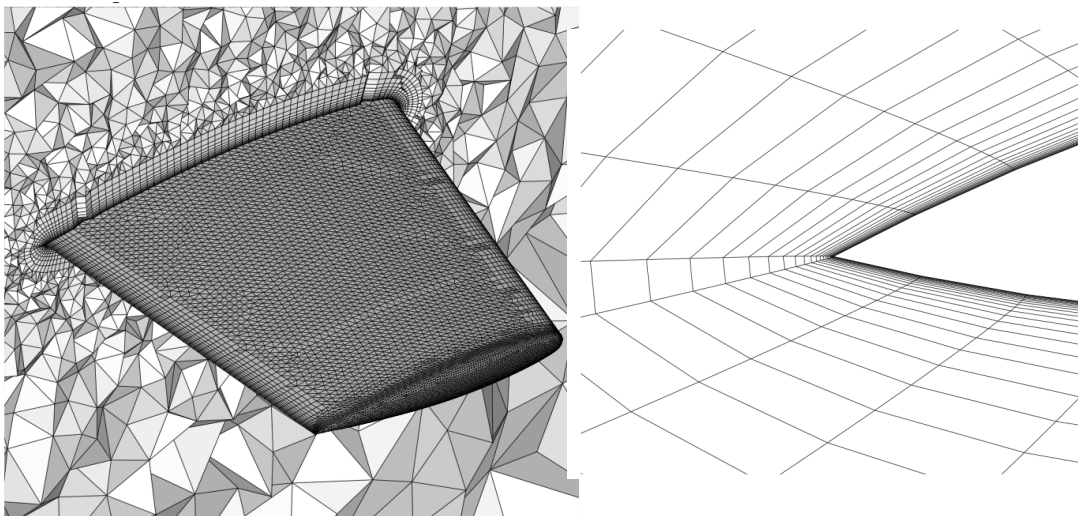
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Voorbeeld van front unfolding



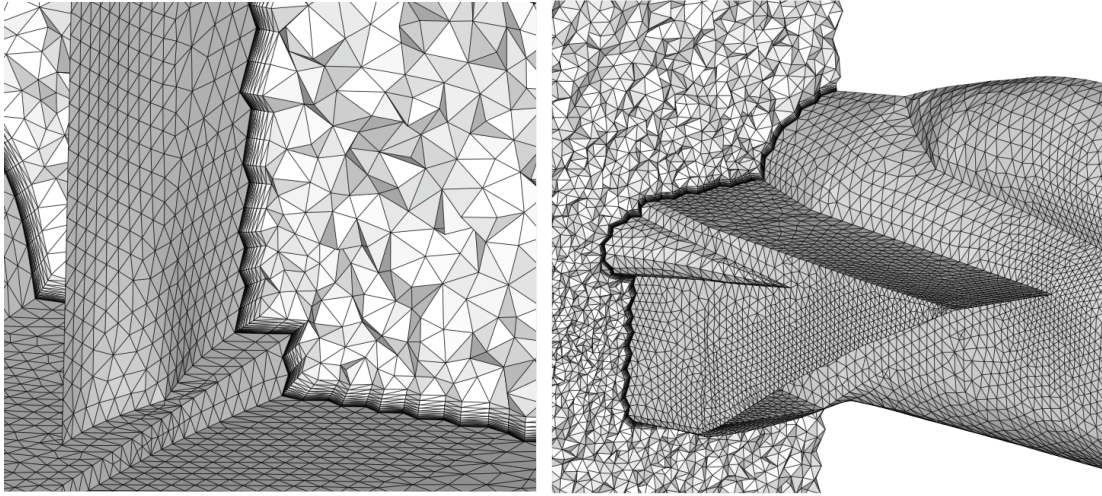
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ONERA M6, hybrid grid, 810k nodes, 1.8M elements



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Airplane, hybrid grid, 2.5 M nodes, 8.5 M elements



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Anisotrope oplossingsafhankelijke roosters

- Generation based on a metric map, analytical or discrete.

$$\mathcal{M}(x, y, z) = \begin{bmatrix} e & f & g \\ f & h & i \\ g & i & j \end{bmatrix}$$

- Unified treatment of boundary spacing, sources, remeshing.
- Isotropic grid: $f = g = i = 0$, $e = h = j = 1/h^2$
- Metric map can be extracted from the solution through the Hessian of one or more solution variables.

$$\mathcal{H}(\rho) = \begin{bmatrix} \frac{\partial^2 \rho}{\partial x^2} & \frac{\partial^2 \rho}{\partial x \partial y} & \frac{\partial^2 \rho}{\partial x \partial z} \\ \frac{\partial^2 \rho}{\partial x \partial y} & \frac{\partial^2 \rho}{\partial y^2} & \frac{\partial^2 \rho}{\partial y \partial z} \\ \frac{\partial^2 \rho}{\partial x \partial z} & \frac{\partial^2 \rho}{\partial y \partial z} & \frac{\partial^2 \rho}{\partial z^2} \end{bmatrix}$$



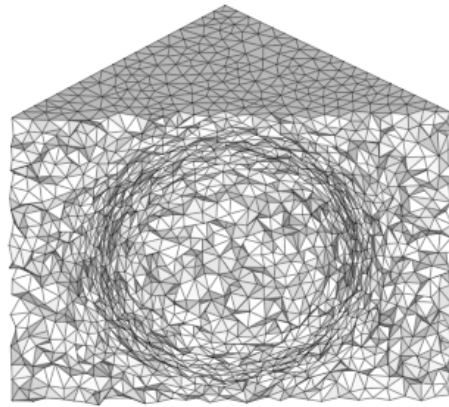
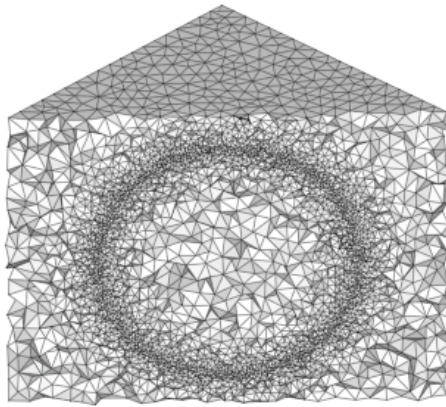
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Oplossingsafhankelijke roosters: Belang van anisotrope verfijning

Analytical spherical function inside a cube

Isotropic – 1M elements

Anisotropic – 150K elements



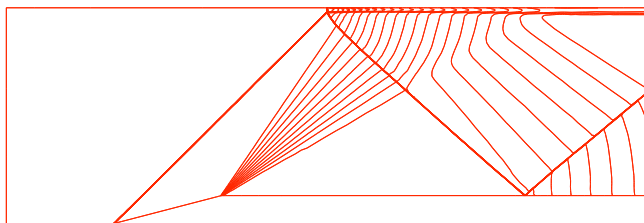
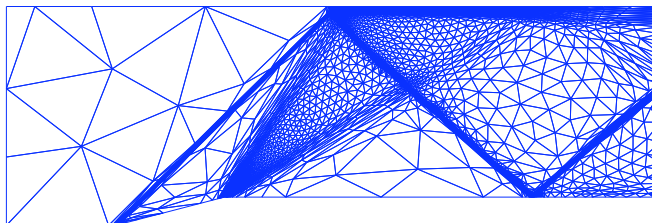
Same grid resolution normal to the spherical surface!



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Oplossingsafhankelijke roosters

Doel: resolutie te verhogen in zones met sterke gradienten



Anisotrope verfijning



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Conclusies

Enkele Ingredienten voor toekomstige complexe simulaties

- ✓ Schema's gebaseerd op eindige elementen interpolaties
- ✓ Monotoon shock capturing voor robuustheid
- ✓ Nauwkeurig op niet gestructureerde roosters
- ✓ Compacte stencils voor eenvoudige parallelisatie
- ✓ Ruimte-tijd schema's bvb voor bewegende randen
- ✓ Hybride roostergeneratie

