Progress in Simulating and Modelling 2D and 3D Turbulence in Free-Surface Flows

Terug naar overzicht

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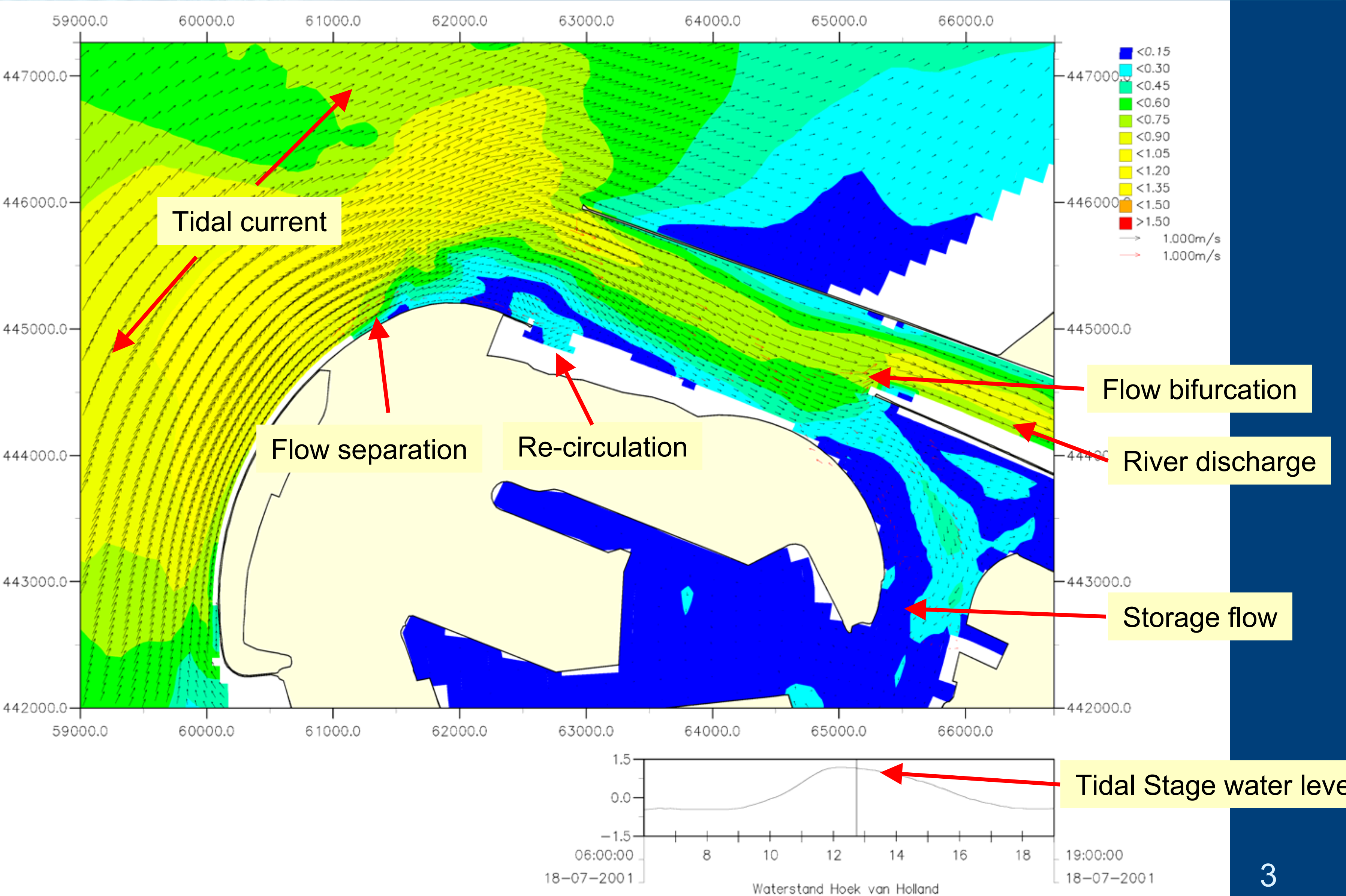
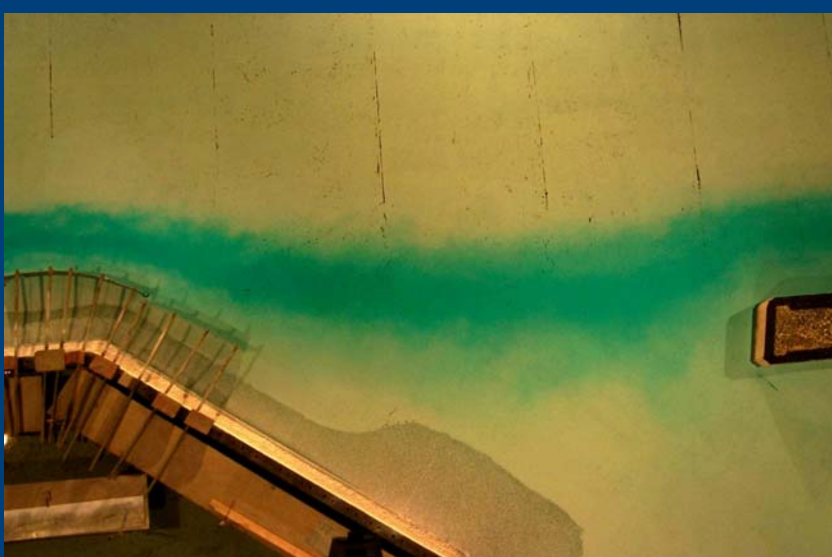
Colloquium Numerical Modelling
Flanders Hydraulics Research

Growing Interest in Simulating Horizontal Eddy Motions :

- **Navigation** (unpredictable flow direction)
- **Structural Stability** (scour , resonance)
- **Dredging Operations** (flow direction , mixing waste material)
- **Biochemical Processes** (upwelling - mixing nutrients - algae)
- **Sediment Transport** (variability bed stress and vertical transport)
- **Harbour Design for low-cost Maintenance** (e.g. mud transport)


Tidal current
Flow separation
Re-circulation
Flow bifurcation
River discharge
Storage flow
Tidal Stage water leve
59000.0
60000.0
61000.0
62000.0
63000.0
64000.0
65000.0
66000.0
447000.0
446000.0
445000.0
444000.0
443000.0
442000.0
<0.15
<0.30
<0.45
<0.60
<0.75
<0.90
<1.05
<1.20
<1.35
<1.50
>1.50
1.000m/s
1.000m/s
1.5
0.0
−1.5
06:00:00
18−07−2001
8
10
12
14
16
18
19:00:00
18−07−2001
Waterstand Hoek van Holland

Courtesy : Cees Kuyper



Horizontal Large Eddy Simulations (HLES) ?

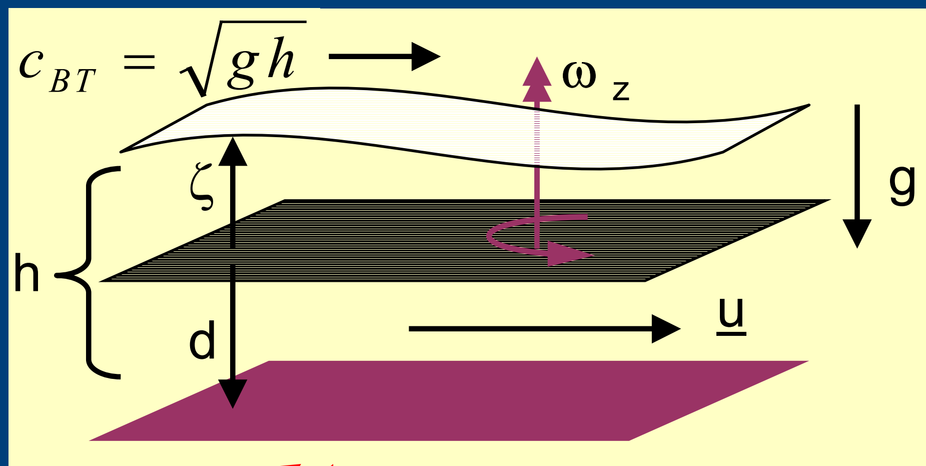
Choice:

- Simulating Quasi-2D turbulence or
- Turbulence Closure for Quasi 2D ?

HLES preferred:

- Subgrid-scale (SGS) model for Quasi-2D turbulence and
- model for 3D turbulence

The Role of the Free Surface (inviscid flow)



$$\frac{D}{Dt}\left(\frac{\omega_z}{h}\right) = 0$$

Poisson Equation

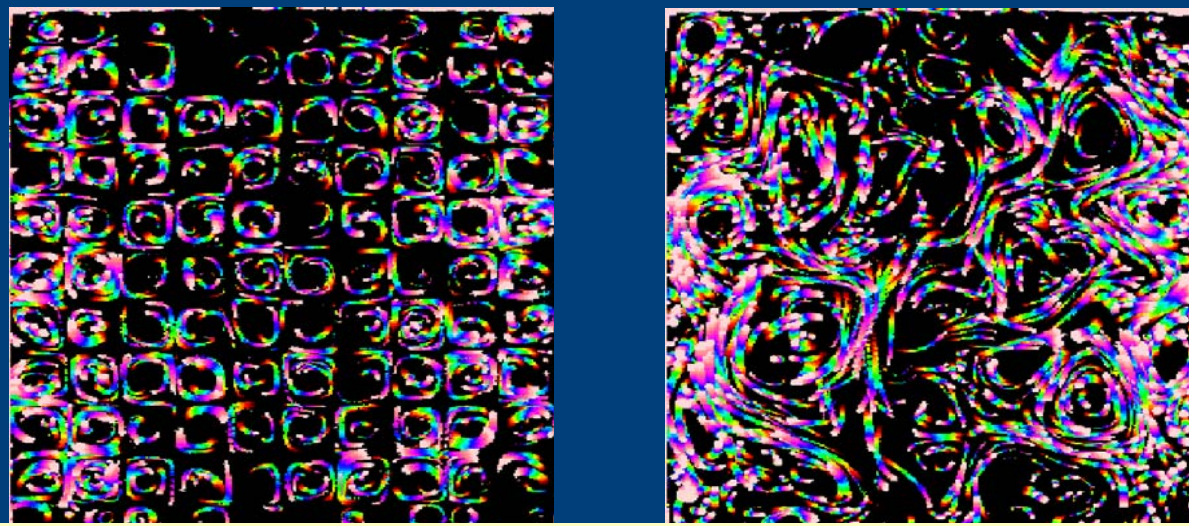
$$\frac{D^2\zeta}{Dt^2} - gh\nabla^2\zeta = h\left(S_{ij}S_{ij} - \omega_z^2\right) + h\left(\frac{1}{h}\frac{D\zeta}{Dt}\right)^2$$

Wave Propagation

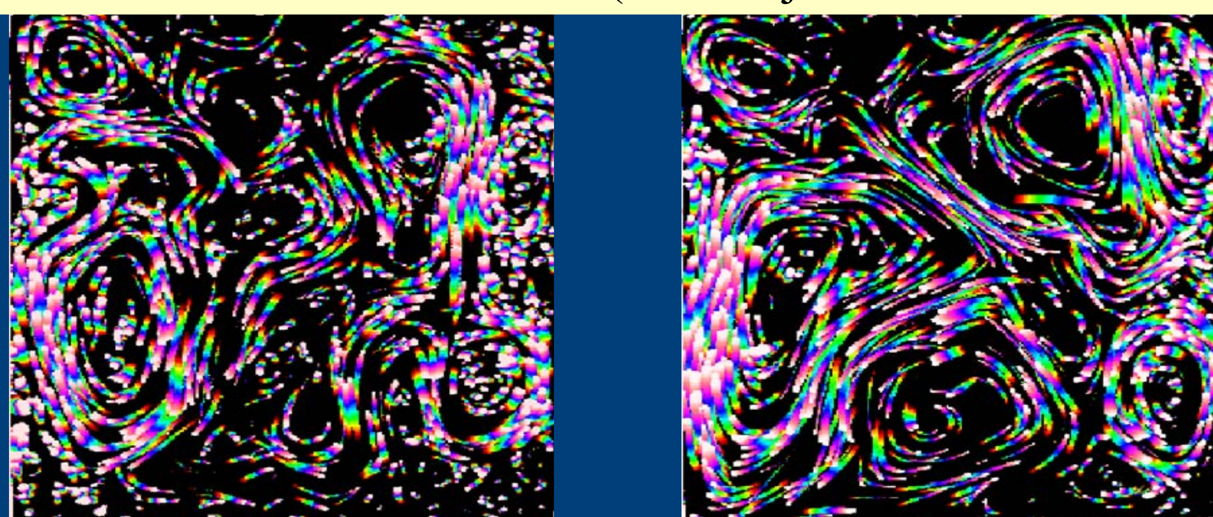
Weiss Function

Vertical Strain

Turbulence in Shallow Water evolving from Counter-Rotating Vortices

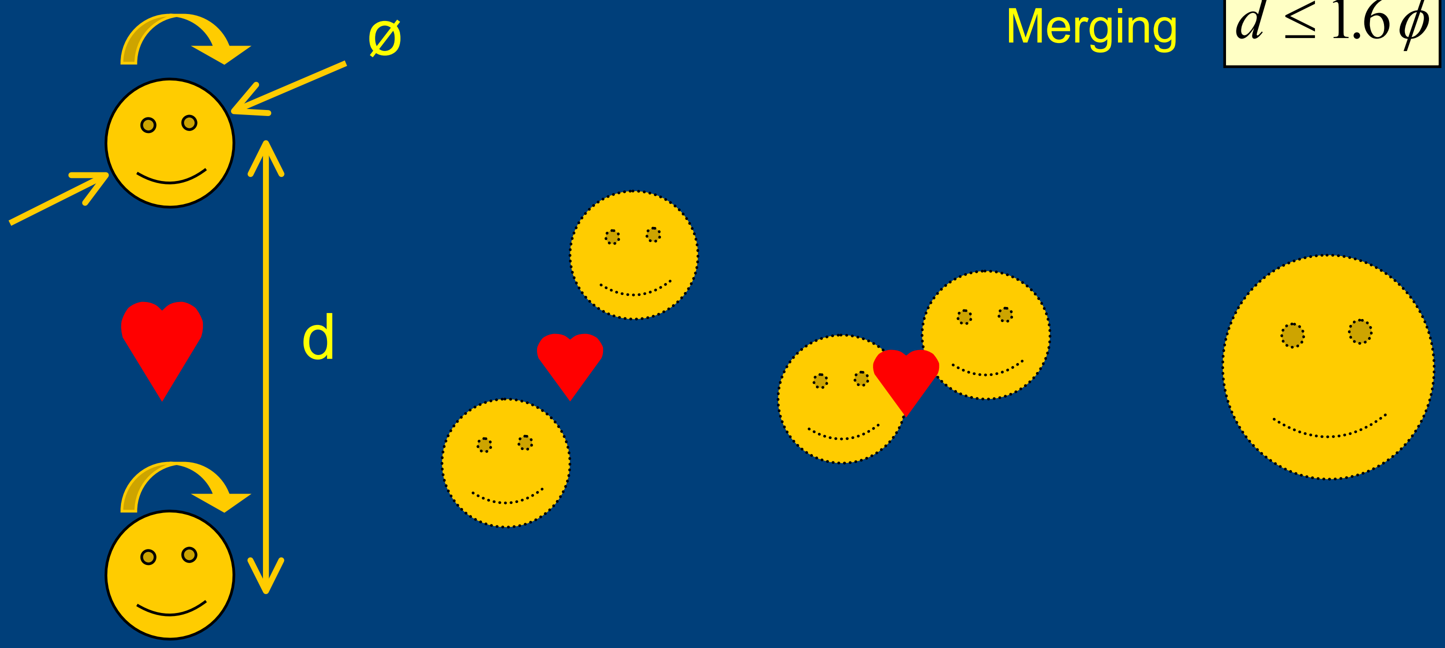


Particle tracks in 1*1*0.01 m^3 (Van Heijst et al. TU Eindhoven)



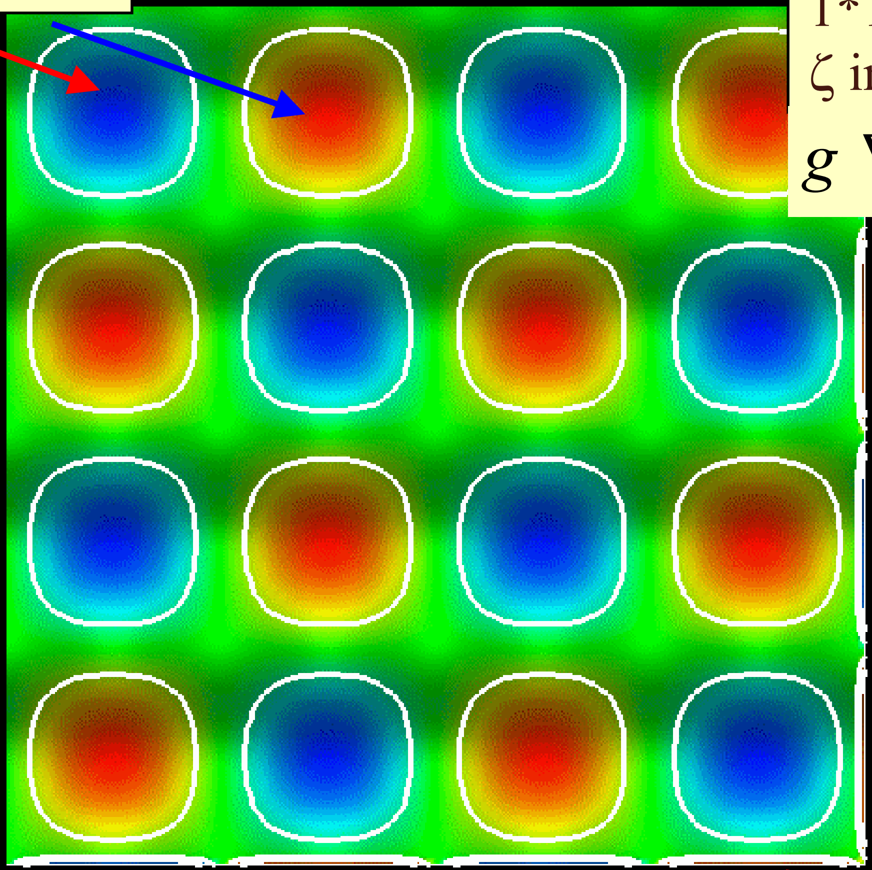
Co-Rotating Vortices can Merge

Merging $d \leq 1.6\,\phi$



Demonstration of Dominance of Poisson Equation for ζ

Counter-rotating Vortices

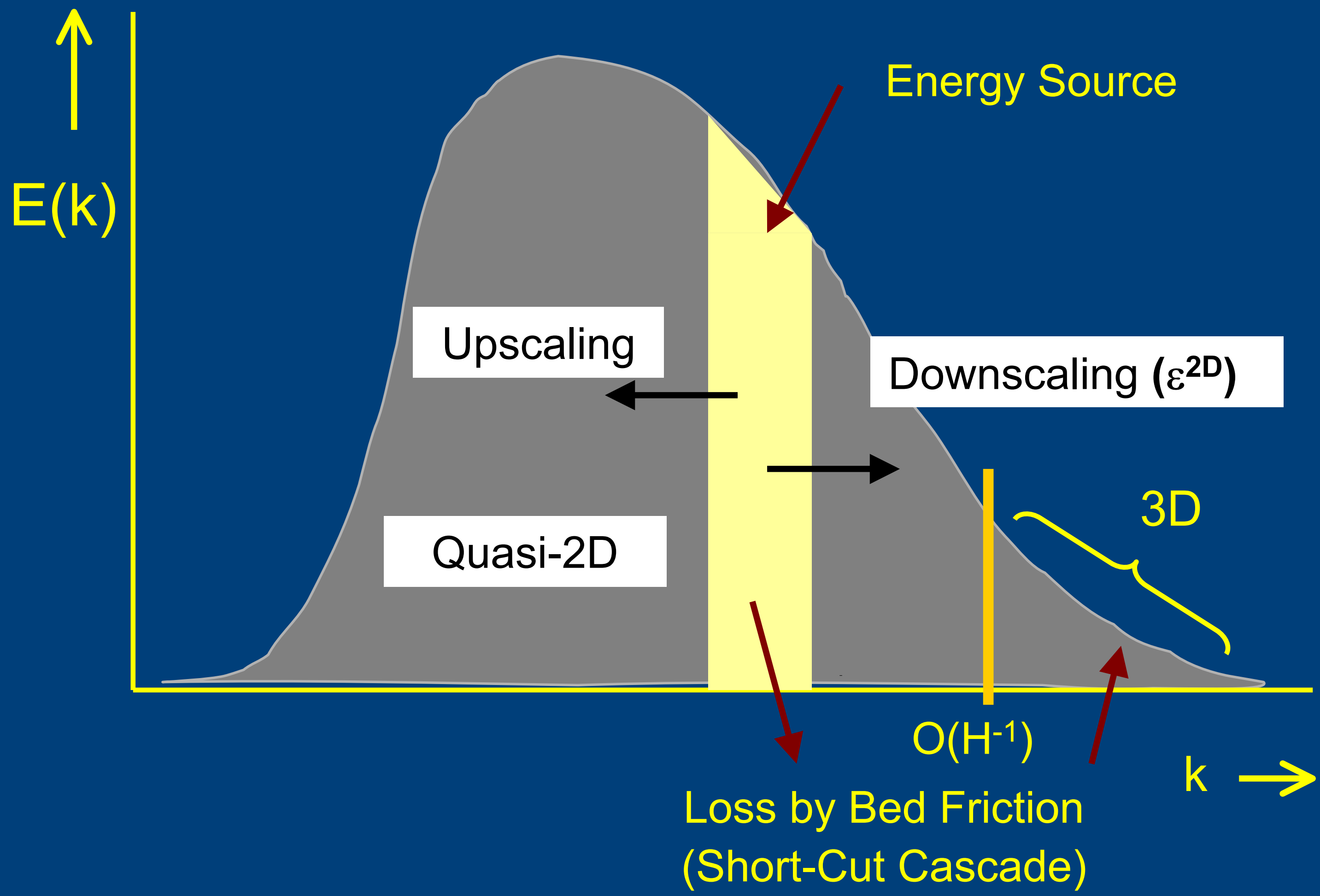


$1*1*0.01\ m^3$; $|u'| = 4$ mm/s
ζ initialized by:

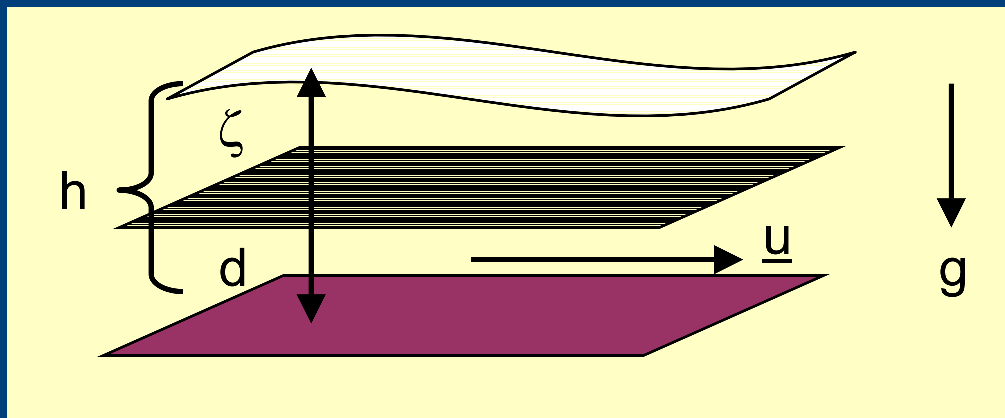
$$g\,\nabla^2\zeta = \omega_z^2 - D_{ij}\,D_{ij}$$

Rigid Walls, Free Slip , inviscid

Spectral - Energy Fluxes



The Shallow-Water Equations
Depth-Averaged and 3D



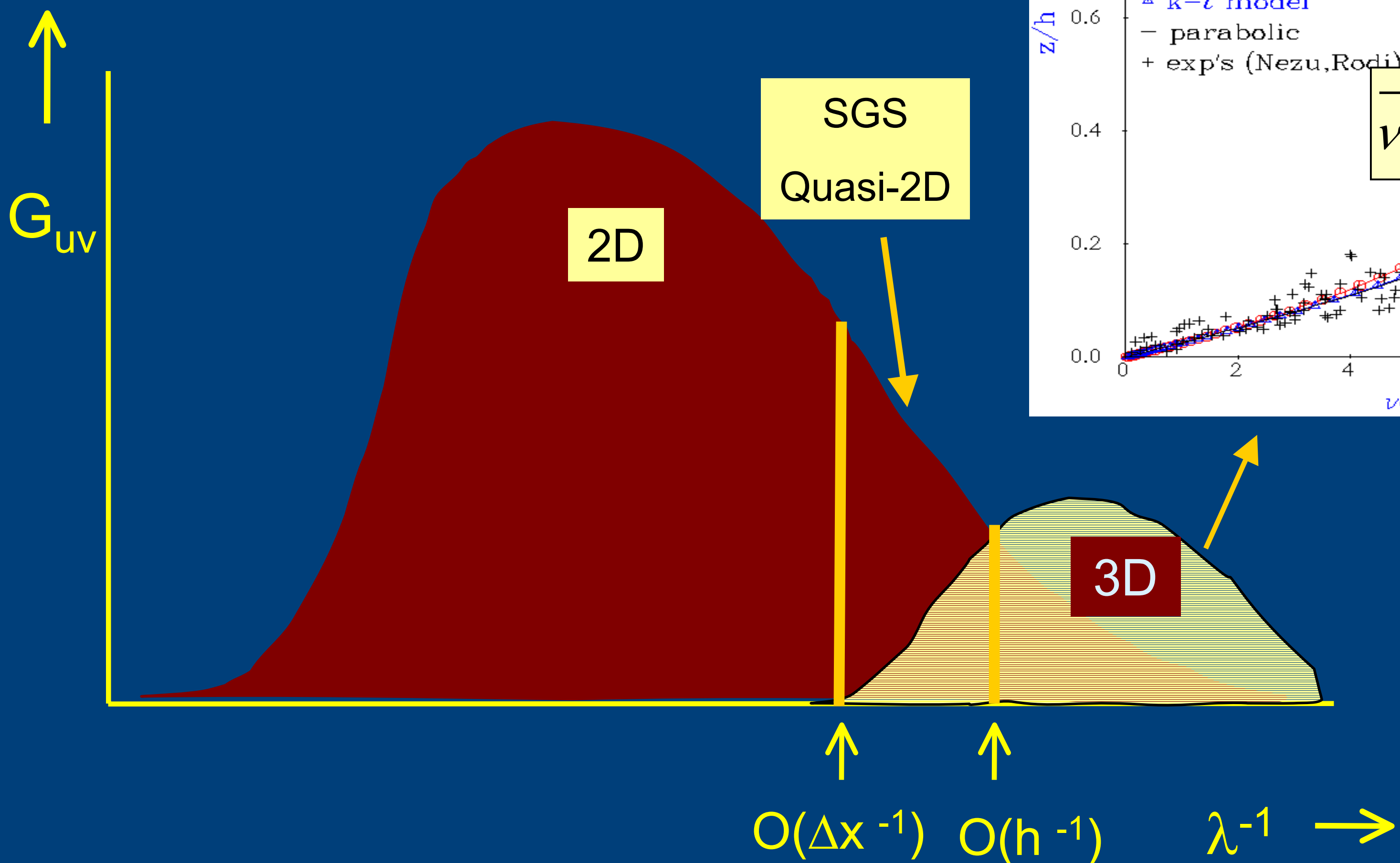
$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \left((d + \zeta)\, \underline{u} \right) = 0$$

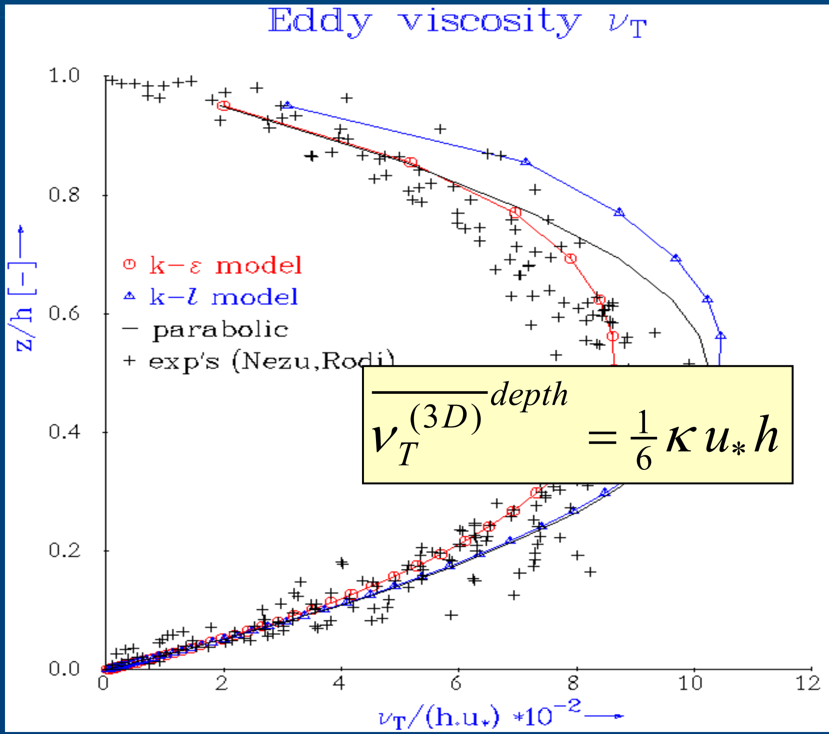
$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} + g \nabla \zeta = \nu \nabla^2 \underline{u} + \underline{T}$$

$$\underline{T} = \nabla \cdot \left(2 \nu_T^{(3D)} \underline{\underline{S}} \right) + \nabla \cdot \left(2 \nu^{(SGS)} \underline{\underline{S}} \right)$$

$$\underline{\underline{S}} = \tfrac{1}{2} \left\{ \nabla \underline{u} + \left(\nabla \underline{u} \right)^T \right\}$$

Shear-stress spectrum G_{uv}





Design Criteria :

- **Drainage of Energy to Subgrid Scales**
- **Energy loss through Short-Cut Cascade**
- **Truncation Wavenumber - Numerical Method**
- **No Mean-Flow Contributions to SGS model(*)**

(*) Smagorinsky closure responds to mean flow even without turbulence

(*) but not the Dynamic Model (Germano *et al.*, 1991) and its novel variants

Recursive High-Pass Filter :

Filter time τ_f ; time step Δt

$$\psi^* = \psi_{n+1} - \overline{\psi}^t_{n+1} \quad ; \quad \overline{\psi}^t_{n+1} = (1-a)\psi_{n+1} + a\overline{\psi}^t_n$$

$$a = \exp\left(-\Delta t / \tau_f\right)$$

$$\underline{\underline{R}}^{(SGS)} - \tfrac{1}{3}\underline{\underline{I}}\, tr\left(\underline{\underline{R}}^{(SGS)}\right) = 2\nu^{(SGS)}\underline{\underline{S}}^*$$

$$\underline{T}^{(SGS)} = \nabla \cdot \left(2\nu^{(SGS)}\underline{\underline{S}}^*\right) \quad ; \quad \underline{\underline{S}}^* = \tfrac{1}{2}\left\{\nabla \underline{u}^* + \left(\nabla \underline{u}^*\right)^T\right\}$$

Loss by bed friction (3D - turbulence generation) yields "Leaky" Spectral-Energy Cascade (homogenous turb.) :

$$\frac{d\,TKE^{(SGS)}}{dt} = \left\langle \underline{\underline{R}}^{(SGS)} : \underline{\underline{S}}^{*} \right\rangle - H^{-1} \left\langle \underline{u}^{(SGS)} \cdot \underline{\tau}_{bed} \right\rangle - \varepsilon^{(2D)}$$

$$\left\langle \underline{\underline{R}}^{(SGS)} : \underline{\underline{S}}^{*} \right\rangle = \nu^{(SGS)} Q(k_s)$$

$$Q(k_s) = Q_{tot} - \int_{k_s}^{\infty} k^2 E(k)\, dk$$

$$k_s = f_{\ell p} \frac{\pi}{\Delta x}$$

$$\underline{\tau}_{bed} = c_f \left| \underline{u} \right| \underline{u}$$

$$B = \frac{3}{4} \frac{c_f \left| \underline{u} \right|}{d + \zeta}$$

Loss by bed friction (3D - turbulence generation) yields "Leaky" Spectral-Energy Cascade:

$$\frac{d}{d\,k_s}\left\{\nu^{(SGS)}Q\left(k_s\right)\right\}+2B\,E\left(k_s\right)=0$$

$$B=\tfrac{3}{4}\frac{c_f\left|\underline{u}\right|}{d+\zeta}$$

$$\frac{d\nu^{(SGS)}}{d\,k_s}=-\left(\gamma\,\sigma_T\right)^2\frac{E\left(k_s\right)}{k_s\,\nu^{(SGS)}}$$

$$E\left(k\right)\propto k^{-\alpha}$$

$$\nu^{(SGS)}=k_s^{-2}\left(\sqrt{\left(\gamma\,\sigma_T\right)^2\left(\underline{\underline{S}}^*:\underline{\underline{S}}^*\right)+B^2}-B\right)$$

$$k_s=f_{\ell p}\frac{\pi}{\Delta x}$$

$$\gamma=0.422\sqrt{1-\alpha^{-2}}$$

$$\sigma_T\approx 0.7$$

Checking the Design Criteria :

- **Drainage of Energy to Subgrid Scales**
- **Truncation Wavenumber - Numerical Method**
- **No Mean-Flow Contributions to SGS model**
- **Energy loss through Short-Cut Cascade**

$$\nu^{(SGS)} = k_s^{-2}\left(\sqrt{(\gamma\,\sigma_T)^2\left(\underline{\underline{S}}^*:\underline{\underline{S}}^*\right)+B^2}-B\right)$$

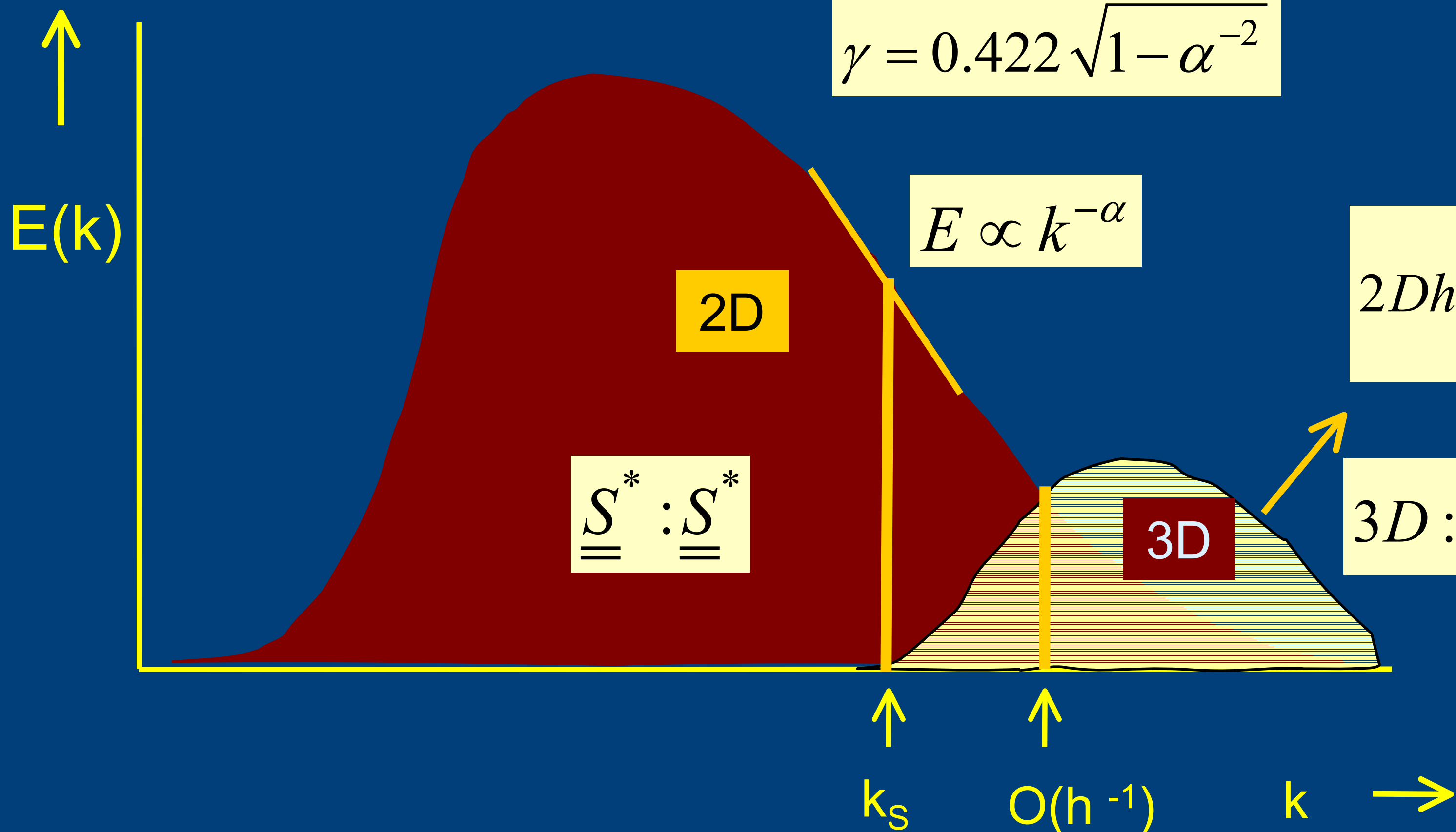
$$B = \tfrac{3}{4}\frac{c_f\left|\underline{u}\right|}{d+\zeta}$$

(1) (2) (3) (4)

$$\nu^{(SGS)} = k_s^{-2}\left(\sqrt{(\gamma\,\sigma_T)^2\left(\underline{\underline{S}}^*:\underline{\underline{S}}^*\right)+B^2}-B\right)$$

$$\gamma = 0.422\sqrt{1-\alpha^{-2}}$$

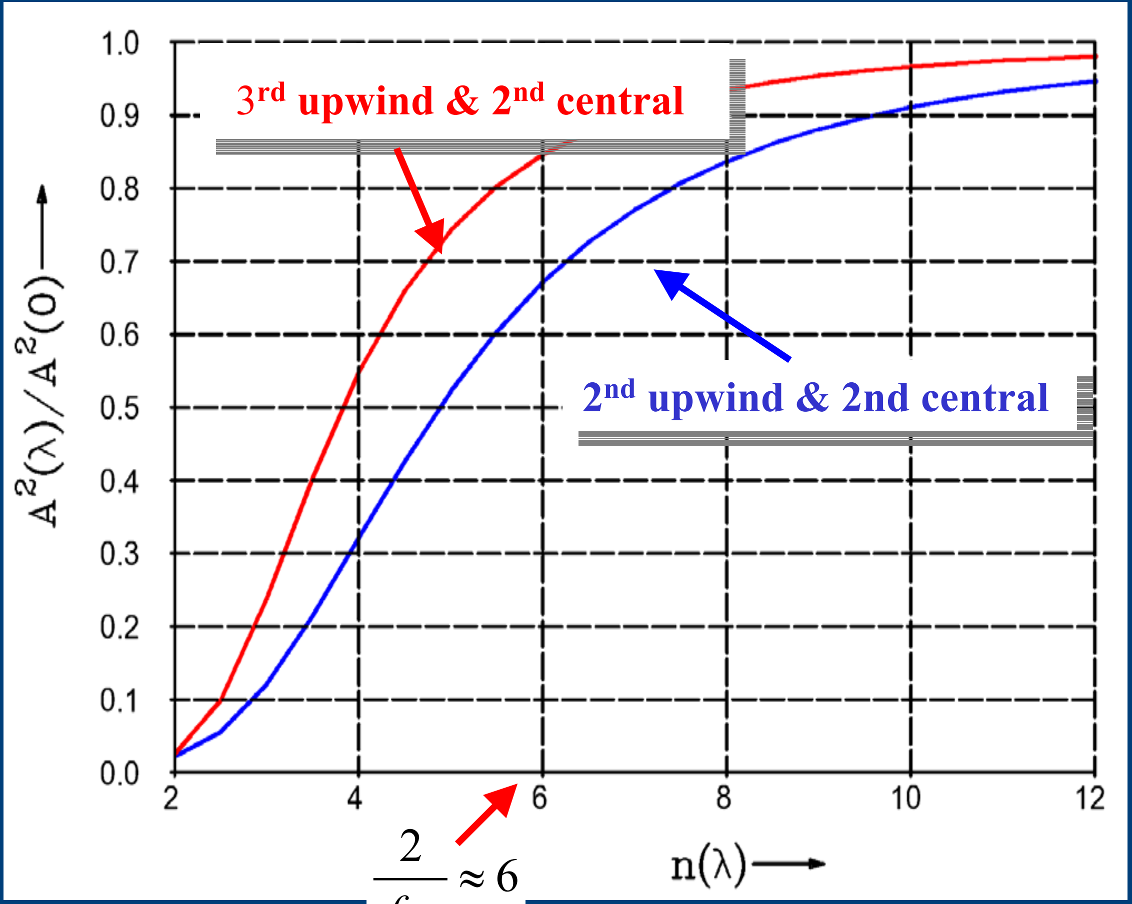
$$E \propto k^{-\alpha}$$



$$2Dh: \quad B = \tfrac{3}{4}\frac{c_f\,|\underline{u}|}{d+\zeta}$$

$$3D: \quad B = k_s^2\,\nu_T^{(3D)}$$

Definition of spatial low-pass wave length

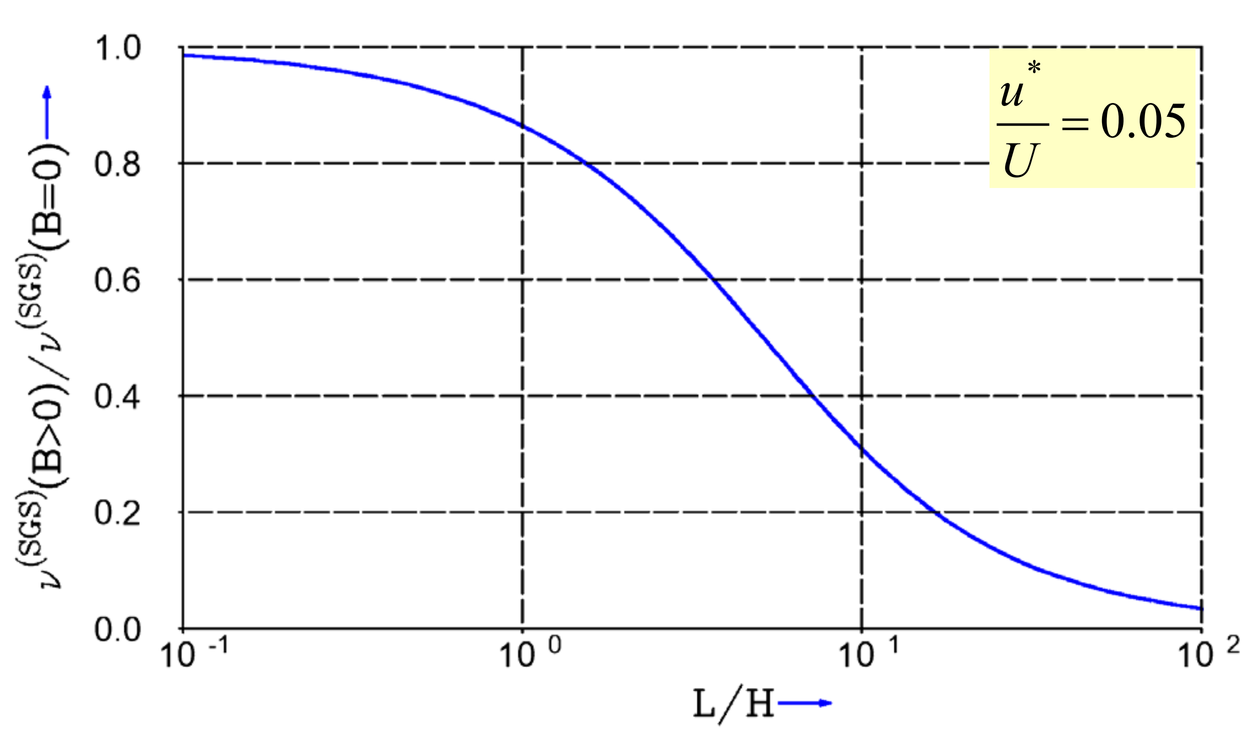


$$\frac{2}{f_{\ell p}} \approx 6$$

≈50% reduction in energy after advection over a single wave length

Bed friction reducing SGS viscosity

$$\nu^{(SGS)} = k_s^{-2}\left(\sqrt{(\gamma\,\sigma_T)^2\left(\underline{\underline{S}}^* : \underline{\underline{S}}^*\right) + B^2} - B\right)$$

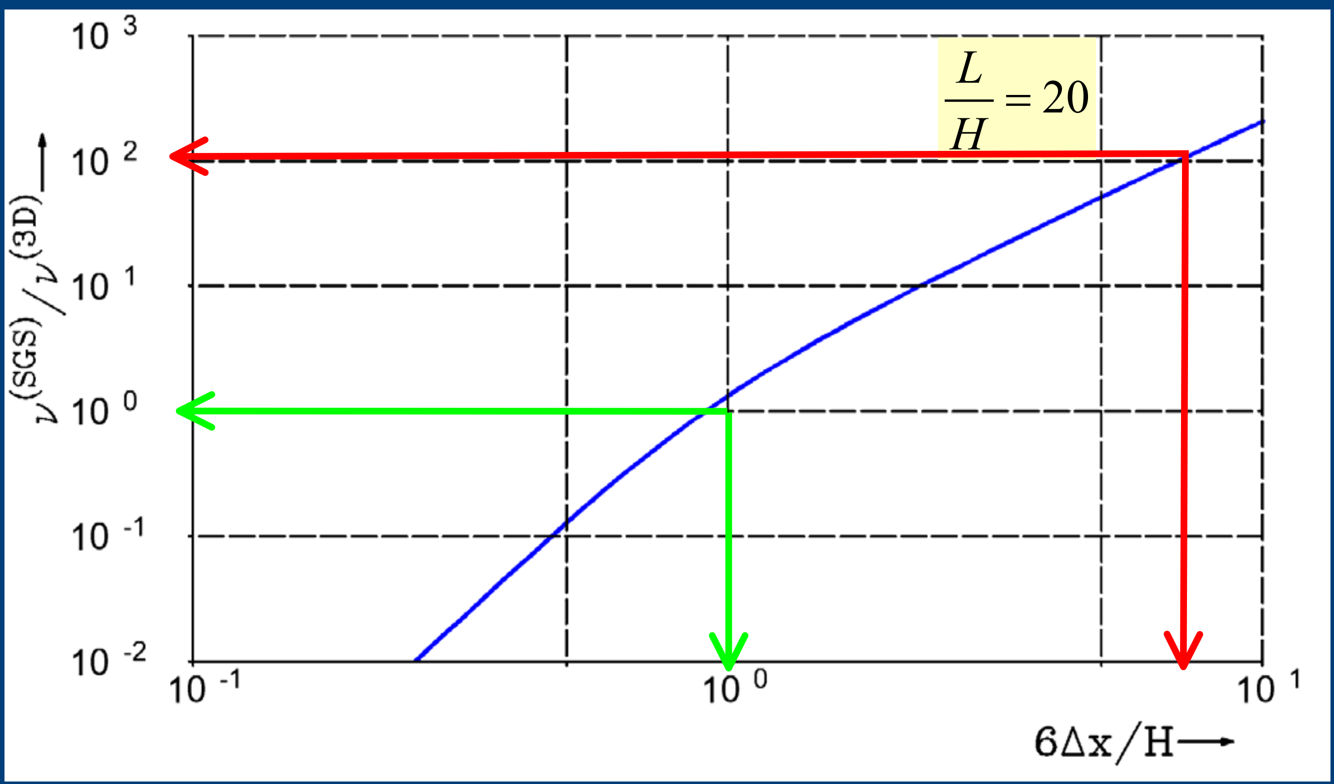


Ratio SGS viscosity – 3D eddy viscosity

$$\nu^{(SGS)} = k_s^{-2}\left(\sqrt{(\gamma\,\sigma_T)^2\left(\underline{\underline{S}}^* : \underline{\underline{S}}^*\right) + B^2} - B\right)$$

$$B = k_s^2\,\nu_T^{(3D)}$$

$$\overline{\nu_T^{(3D)}} = \frac{1}{6}\kappa\,u_*\,h$$



Conclusions :

- **3D Turbulence remains Completely Subgrid**
- **SGS Model dedicated to Shallow Flows** (bed friction)
- **Applicable to 2Dh as well as to 3D** (shallow flow eq's)
- **Modest Role provided $\Delta x/H < O(2)$**
- **Modest Role for very Large Eddies $L/H > O(100)$**
- **Performs well in Test Cases** (see next examples)

See also our papers in *Shallow Flows Symp.* , Delft , June 2003

Boundary Conditions

Perturbations:

$$\underline{u}^{*}\left(\underline{x},t\right)=\sum_{n=1}^{N}\underline{u}^{(n)}\left(\underline{x},t\right)$$

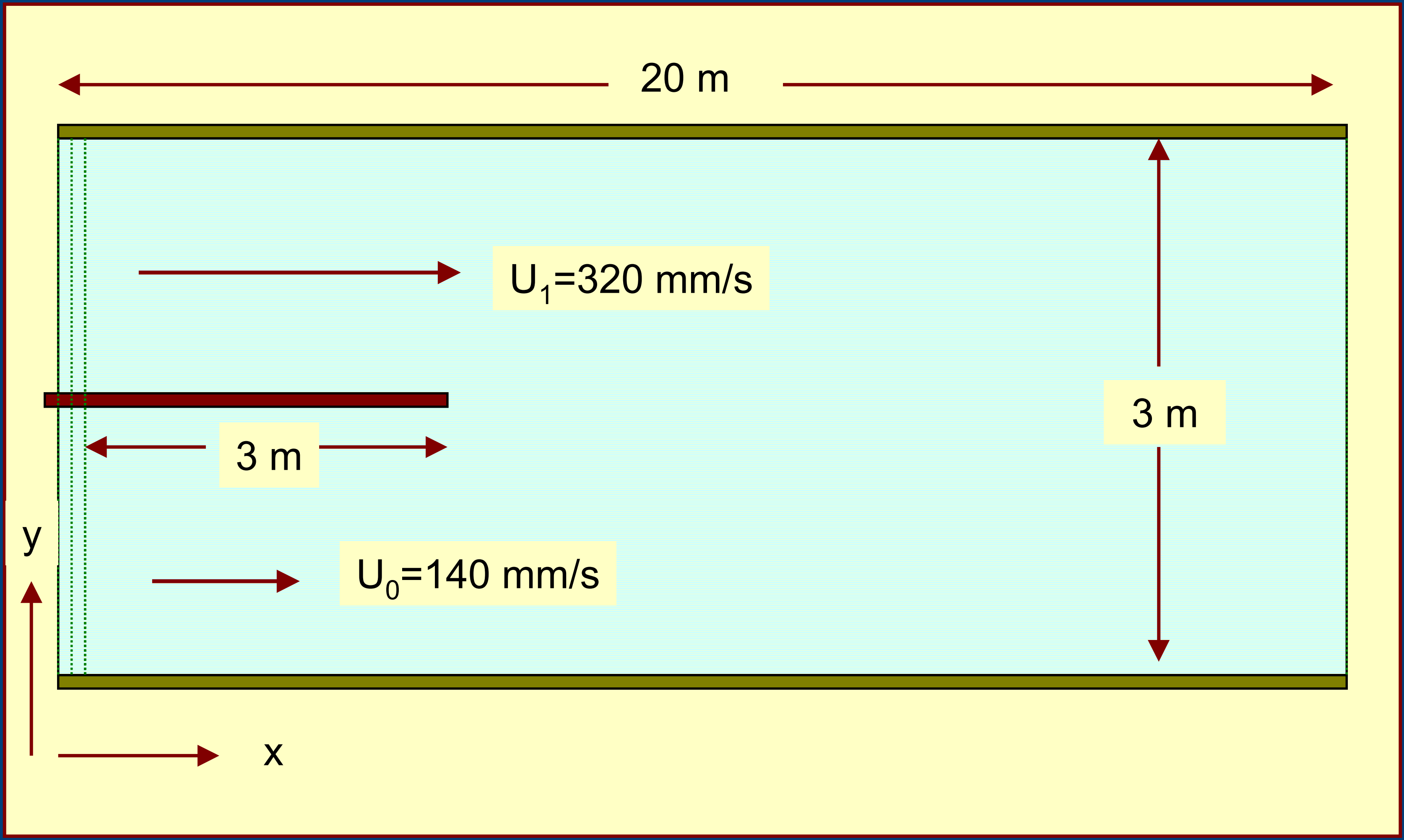
Component:

$$\underline{u}^{(n)}\left(\underline{x},t\right)=\underline{A}^{(n)}*\hat{\underline{k}}^{(n)}\sin\left\{\underline{k}^{(n)}\cdot\left[\underline{x}-\left(\underline{\varpi}+\underline{u}^{(<n)}\right)t\right]+\phi_n\right\}$$

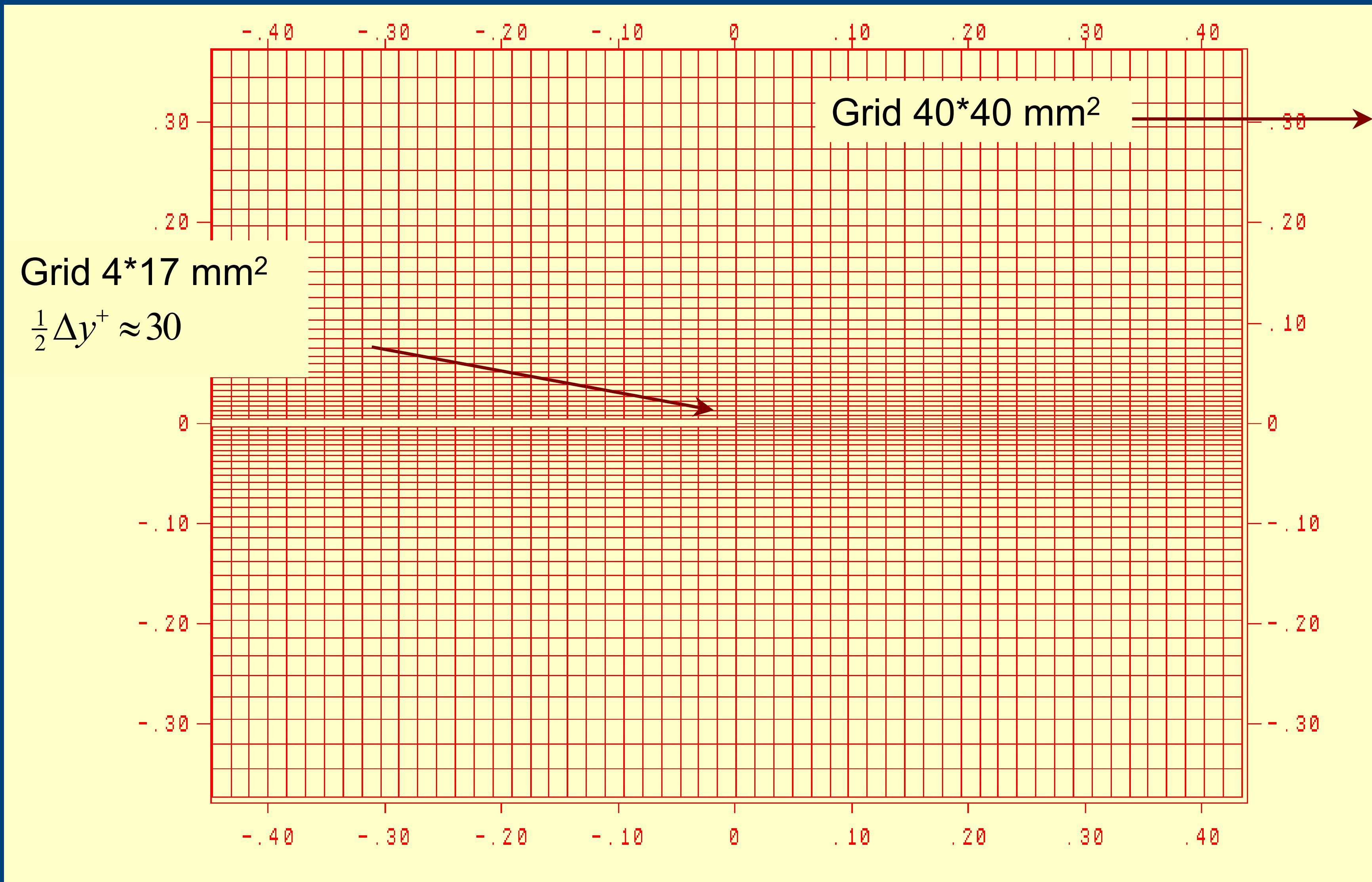
Advection:

$$\underline{u}^{(<n)}=\sum_{n}^{n-1}\underline{u}^{(n)}\left(\underline{x},t\right)$$

A Test Case: Free-Surface Mixing Layer 67 mm Water Depth

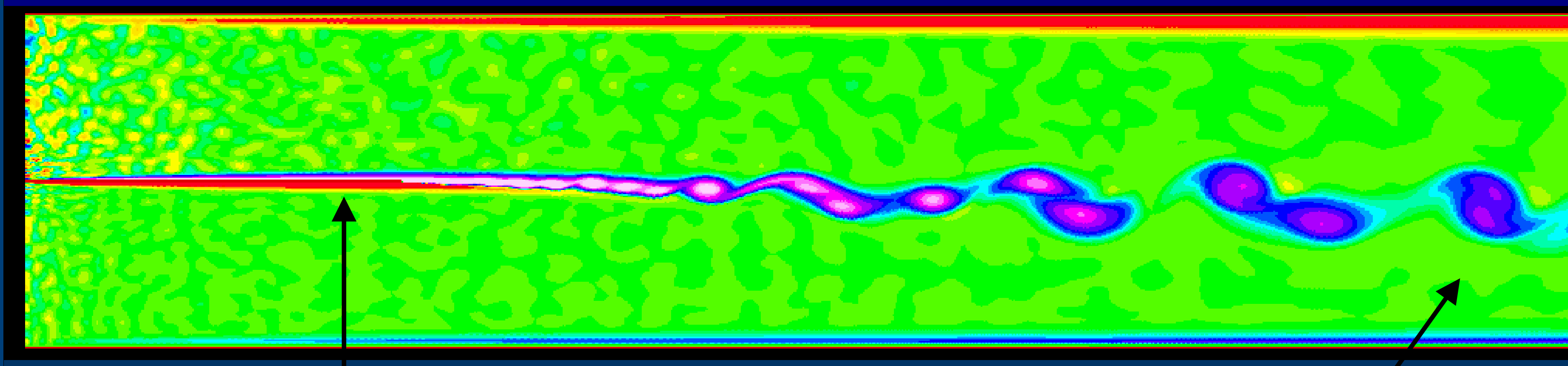


Grid near 8 mm thick splitter plate:



Shallow-Water Mixing Layer

exp's: Uijttewaal & Booij (2000)



Grid $4 * 17\,\mathrm{mm}^2 \left(\frac{1}{2}\Delta y^{+} \approx 30\right)$

Grid 40*40 mm^2

Patterns in water levels

Subgrid-scale eddy viscosity

Turbulence properties

Spectra

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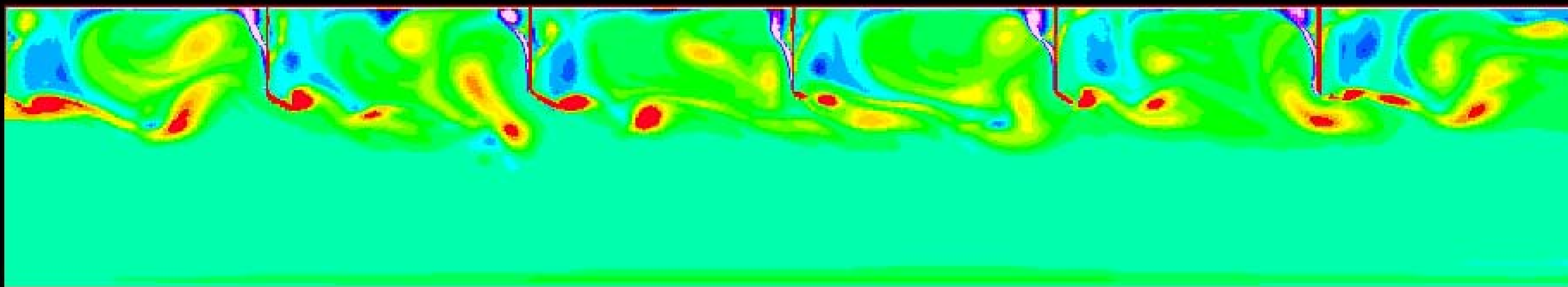
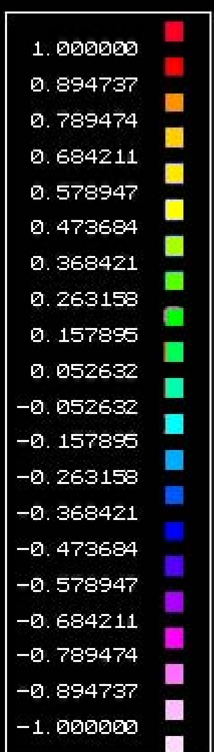
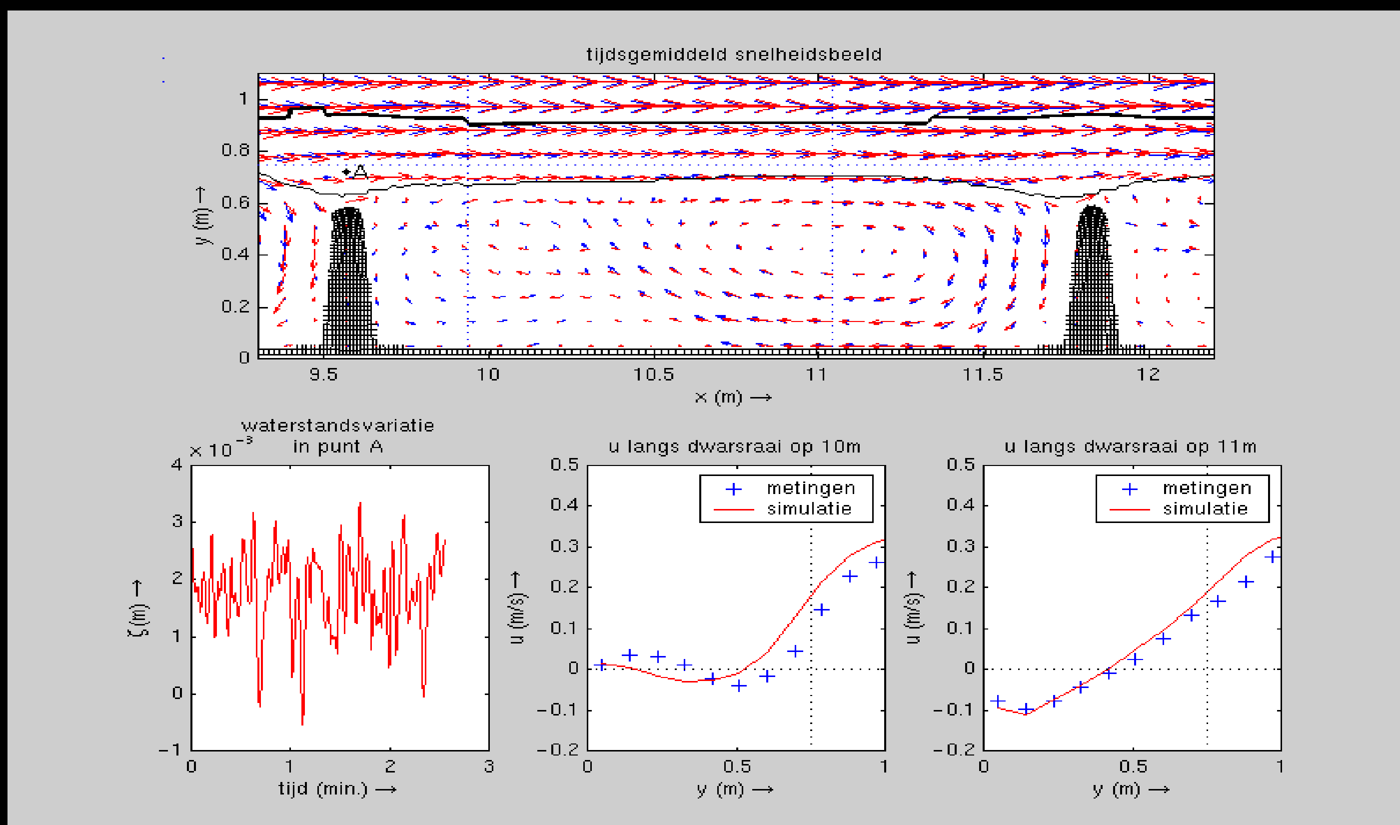
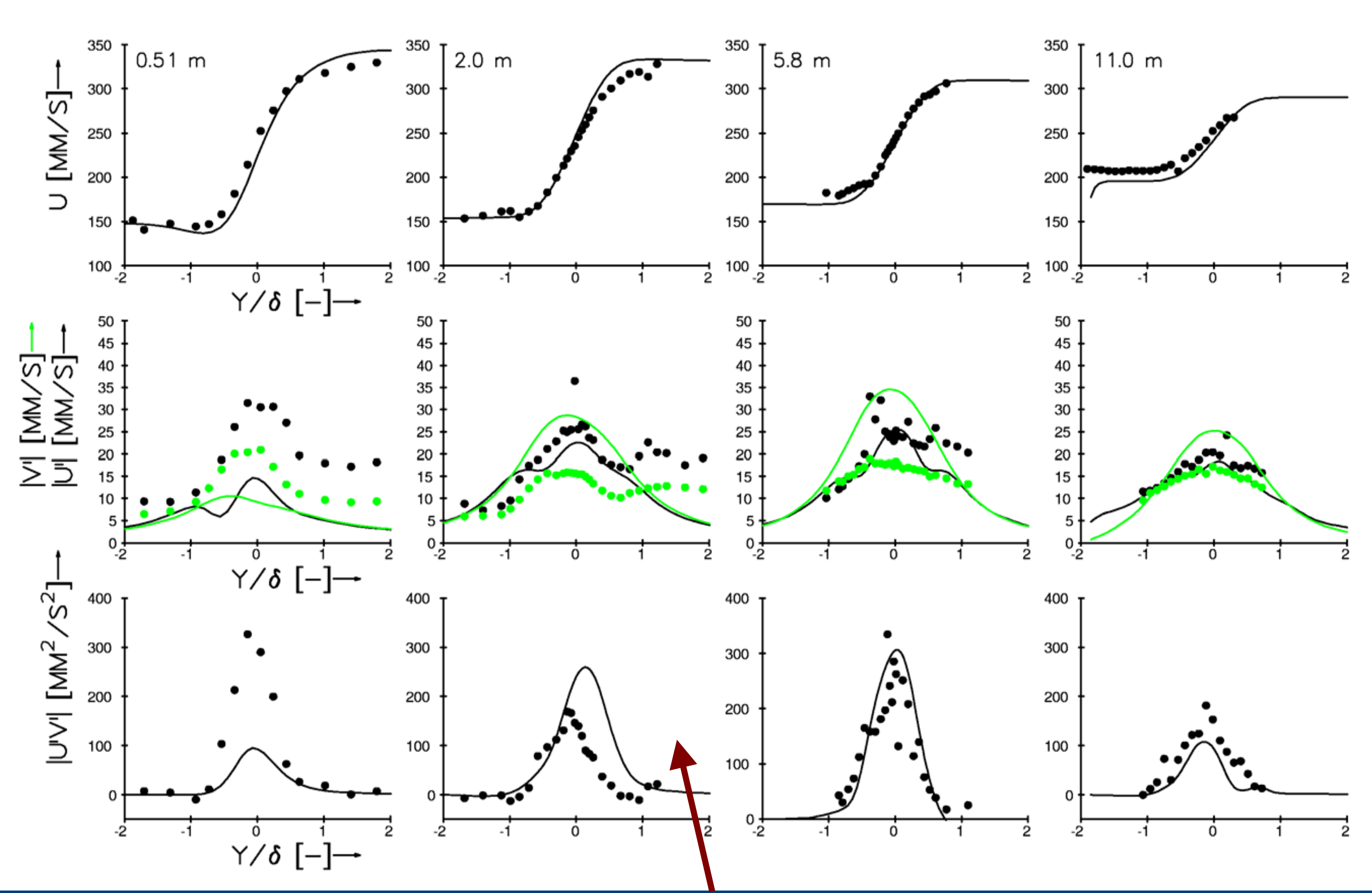


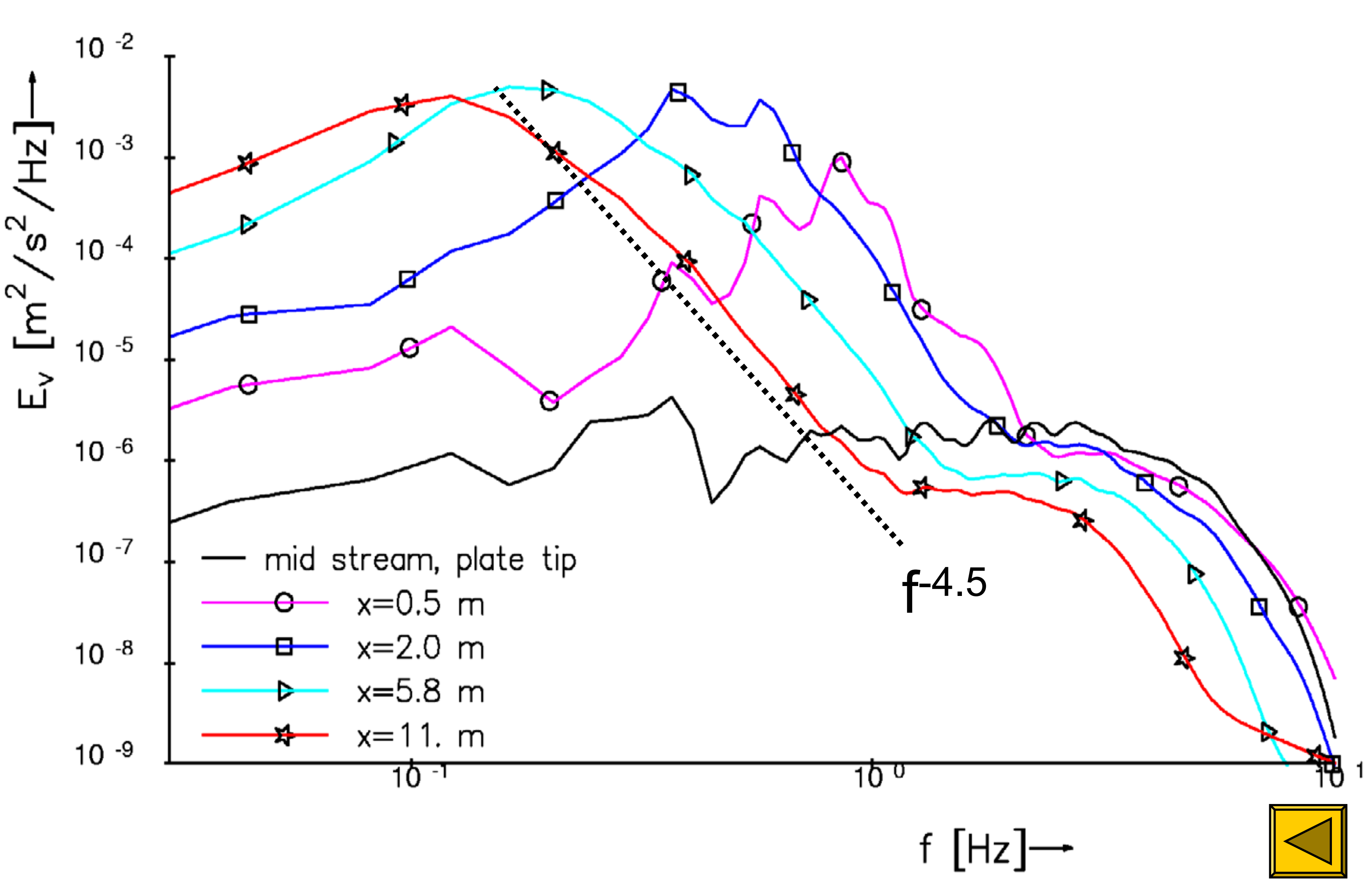






Low level not explained by experimentors

Simulation: Frequency Spectra Spanwise v-component



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