

A discrete correction scheme for envelope approach of wave group statistics

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Abstract

The existence of empty envelope excursions (EEE) brings error to the envelope approach of wave group statistics, which identifies wave group by envelope upcrossing of a critical level. A group number correction scheme is suggested in this paper to exclude EEE from wave group statistics. To this end, the Ditlevson and Lindgren [J. Sound Vib. 122 (1988) 571] theory about the fraction of empty excursion envelopes (FEEE) is examined to see if it fits for ocean waves. The sea waves are simulated with Monte Carlo method and with P-M and JONSWAP spectrums. The values of FEEE of the simulated waves are investigated and compared with the theory of Ditlevson and Lindgren. The comparison shows that, at the second-order approximation, theoretical predictions of FEEE are close to those derived from simulations. This approximate analytical expression of FEEE is then employed to form a group number correction scheme. Comparisons between numerical and theoretical results of wave group properties show that this correction scheme is quite effective.

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1. Introduction

Wave grouping is an important behavior of ocean waves, especially of swells. It is known to play a principal part in a multitude of coastal and ocean engineering problems. The statistical properties about wave groups are of interest, among them are the mean run length \bar{H} and the mean group length \bar{G} . The run length is defined as the number of waves in a high run while the group length is defined as the number of waves between the beginnings of two successive high

run. There are mainly two approaches to the analysis of these statistical properties. One may be referred to as the wave envelope approach, in which these properties are studied based on the envelope of wave elevation, as discussed by Ewing (1973), Vanmarcke (1975), Goda (1976), Longuet-Higgins (1984), Ochi and Sahinoglou (1989), Masson and Chandler (1993) among others. The other one may be referred to as the Markov chain method, in which the sequences of wave heights are treated as Markov chain, as discussed by Kimura (1980), Longuet-Higgins (1984), Battjes and van Vledder (1984), Dawson et al. (1996), Dawson (1997) among others.

The mean run length and mean group length derived from continuous wave envelope are different

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from the commonly used parameters based on the series of discrete wave heights. In order to make the envelope approach adequately model group statistics, discrete correction scheme is needed to compensate the difference. Vanmarcke (1975), Goda (1976) and Longuet-Higgins (1984) each suggested a discrete correction scheme. In Vanmarcke (1975) and Goda (1976), it is assumed that an envelope group with $i < H < (i+1)$, where $i=0, 1, 2, 3, \dots$ corresponds to a run length $j=i$ or $j=i+1$ with probability $i+1-H$ or $H-i$. Thus the mean number of waves per group can be estimated as

$$\bar{j} = \frac{\sum_{i=1}^{\infty} (\alpha_{i1} + \alpha_{i2}) i}{\sum_{i=1}^{\infty} (\alpha_{i1} + \alpha_{i2})} \quad (1)$$

with

$$\alpha_{i1} = \int_{i-1}^i (H-i+1)P(H)dH \quad (2)$$

and

$$\alpha_{i2} = \int_i^{i+1} (i+1-H)P(H)dH \quad (3)$$

where $P(H)$ is the probability that a group be larger than H , which is approximated by a Poisson distribution. These assumptions lead to

$$\bar{j} = \frac{1}{1 - e^{-1/\bar{H}}} \quad (4)$$

In Longuet-Higgins (1984), it is assumed that the probability that a high run has a integer run length $j > 0$ is proportional to $\int_{j-(1/2)}^{j+(1/2)} P(H)dH$. With $P(H)$ approximated by a Poisson distribution and truncated at $H=1/2$, the following relationship is obtained,

$$\bar{j} = \bar{H} + 0.5 \quad (5)$$

Although these correction schemes achieved some success, Elgar et al. (1984) and Masson and Chandler (1993) found that (1) these authors had employed incorrect function of $P(H)$ for narrow spectra, hence yield incorrect results for narrow spectra waves and (2) for $\bar{H} \rightarrow \infty$, \bar{H} and j should be identical ($\bar{H} \rightarrow j$), in contrast with Eq. (5).

For a given wave record (sufficiently long), the mean run length \bar{H} and the mean group length \bar{G} of it

may be interpreted as the ratio N/N_g and N_{\max}/N_g , respectively, where N_{\max} is the number of all the waves, N is the number of the waves that exceed the critical level and N_g the number of wave groups. In the envelope approach, the number of wave groups is represented by the number of envelope upcrossings N_u . However, the two are usually not identical. From this point of view, the main error in estimating \bar{H} and \bar{G} through envelope approach arises from taking N_u to be N_g . In order to improve the prediction of wave group statistics, a group number correction scheme is needed to exclude those upcrossings that do not correspond to real high runs. Vanmarcke (1975), Goda (1976) and Longuet-Higgins (1984) among others noted the existence of such envelope excursions above a critical level that without there being any upcrossings of the original processes during the time of these excursions, as shown in Fig. 1. These envelope excursions are named empty envelope excursions (EEE) by Ditlevson and Lindgren (1988). Given the existence of EEE, the group number shall be $N_g = (1 - FEEE)N$, where the FEEE (fraction of empty excursion envelopes) means the proportion of EEE in the number of wave envelope excursions of a given critical level. In Longuet-Higgins (1984), those envelope excursions with run length less than $1/2$ are taken to be EEEs. This assumption corresponds to $FEEE = \int_0^{1/2} P(H)dH$. In Vanmarcke (1975) and Goda (1976), the probability that an envelope excursion with a run length $H < 1$ be associated with an EEE is assumed to be $1-H$ and

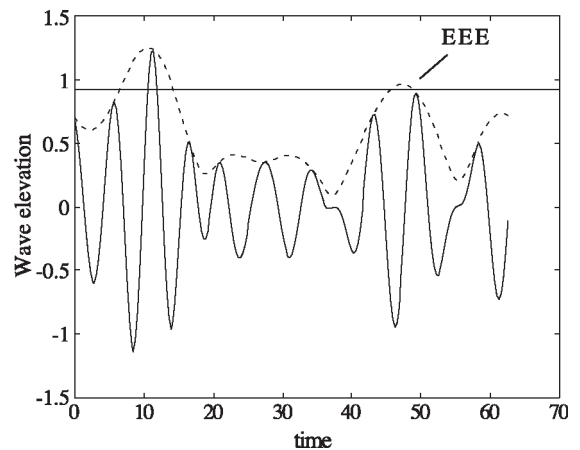


Fig. 1. An example of empty excursion envelope. The straight line herein represents a critical level.

$FEEE = 1 - \alpha_{11}$. An alternative analytical expression for FEEE was derived by Ditlevson and Lindgren (1988). They evaluated FEEE by use of the Slepian model process method. The formula (Eq. (9)) in the next section of this paper obtained by them is an approximation that improves the assessments due to Vanmarcke (1975) and Longuet-Higgins (1984). Lindgren (1989) arrived at the same conclusion as Ditlevson and Lindgren through a simpler way.

In this paper, the Ditlevson and Lindgren (DL) formulae at different order of approximation will be examined to see if they fit the ocean waves well. For this aim, the ocean waves are simulated with Monte Carlo method using Longuet-Higgins linear wave model (see, for instance, Longuet-Higgins, 1984) together with available wave spectrums, and the FEEEs of the simulated waves are investigated and compared with DL theory. Comparison shows that the formula corresponding to second order approximation is suitable. This formula is then employed to improve the prediction of wave group statistics.

It is worth noting that, besides EEE, the splitting phenomena (Masson and Chandler, 1993) also contributes to the difference between N_g and N_u . However, this effect is not considered in this paper. Further work shall be done in the future to know how often the splitting occurs and to take this into the group number correction scheme. Another thing shall be noted is that, in this paper, the linear wave model (Eq. (6)) is used in numerical simulation. Although the sea waves are generally nonlinear, Elgar et al. (1984) demonstrated that the statistical characteristics of wave groupiness derived (based on discrete counting) from linear simulations of ocean waves agree well with field data except for the very shallow water case. In this paper, we consider only the deepwater case. Therefore, it is reasonable to assume that the statistical characteristics derived from simulated waves represent those of the real sea.

2. The FEEE of ocean waves

2.1. Wave envelope definition and DL theory

In order to facilitate the subsequent analysis, the Longuet-Higgins (1984) wave envelope definition

and DL theory are summarized in this section. The Longuet-Higgins model for sea surface elevation process $\zeta(t)$ is,

$$\zeta(t) = \sum_{n=0}^{\infty} c_n \cos(\omega_n t + \varepsilon_n), \quad (6)$$

where ω_n are angular frequencies, ε_n are random phases distributed uniformly over $[0, 2\pi]$, and the amplitudes c_n are such that

$$\sum_{d\omega} \frac{1}{2} c_n^2 = S(\omega) d\omega. \quad (7)$$

Here $S(\omega)$ is the power spectra of process $\zeta(t)$. The envelope process of $\zeta(t)$ denoted by a real function $\rho(t)$ is defined by

$$\rho(t) e^{i\phi(t)} = \sum_{n=0}^{\infty} c_n \exp\{i[(\omega - \bar{\omega})t + \varepsilon_n]\}, \quad (8)$$

where $\phi(t)$ is a phase function, $\bar{\omega}$ is an average angular frequency defined as $\bar{\omega} = m_1/m_0$, m_i is the i th order spectral moment, $m_i = \int_0^\infty \omega^i S(\omega) d\omega$. In the wave envelope theory, it is assumed that $\rho(t)$ corresponds to the continuous change of wave amplitude.

Suppose that the process $\zeta(t)$ is a stationary and ergodic Gaussian process, then we can apply DL theory to it. In DL theory, the linear regression method is employed to estimate the conditional mean of the processes $\rho(t)$ and $\phi(t)$ given that the sample path at time $t=0$ upcrosses the critical level $\rho(u)$ with $\phi(0)=0$ and with given time derivatives $\dot{\rho}(0)$ and $\dot{\phi}(0)$. The two conditional mean processes shortly before and after the time $t=0$ are represented by their Taylor expansions. The coefficients of the terms of order higher than 1 are all estimated through the regression method and expressed in terms of $\rho(0)$, $\phi(0)$, $\dot{\rho}(0)$, $\dot{\phi}(0)$. Therefore, the higher order approximations of the two conditional mean processes are not necessarily more accurate than the lower order ones. By comparing their theory with numerical simulations of random waves, Ditlevson and Lindgren found that the second order approximation worked well on the considered envelope problem. In the next section, this approximation will be found applicable to ocean waves too. The value

of FEEE corresponding to this approximation, which is denoted by r , is

$$r(\tilde{u}, v, \varepsilon) \cong \int_{-\tilde{u}(\sqrt{1+\varepsilon^2/4}-\varepsilon/2)}^{\tilde{u}(\sqrt{1+\varepsilon^2/4}+\varepsilon/2)} f_H(\eta) \times \left[1 - \sqrt{2\pi} \frac{\Phi(v\pi(\tilde{u}^2 - \eta^2 + \varepsilon\tilde{u}\eta)/\tilde{u}) - 1/2}{v\pi(\tilde{u}^2 - \eta^2 + \varepsilon\tilde{u}\eta)/\tilde{u}} \right] d\eta \quad (9)$$

where $\Phi(x)$ is the standard error function, $\tilde{u} = u/m_0$ is the dimensionless critical level, ε and v are the spectra skewness and bandwidth parameter separately, $v = \mu_2^{1/2} m_1^{-1}$, $\varepsilon = \mu_3 m_0^{1/2} \mu_2^{-3/2}$, μ_i is the i th order spectral moment about the mean frequency $\mu_i = \int_0^\infty (\omega - \bar{\omega})^i S(\omega) d\omega$. For $v < 0.4$ and $\tilde{u} \gg 1$, a close approximation to $r(\tilde{u}, v, \varepsilon)$ is

$$r(\tilde{u}, v, \varepsilon) \cong r(\tilde{u}, v, 0) \cong 2 \left(1 - \sqrt{2\pi} \frac{\Phi(v\pi\tilde{u}) - \frac{1}{2}}{v\pi\tilde{u}} \right) \times \left(\left[\Phi(\tilde{u}) - \frac{1}{2} \right] \left(1 - \frac{1}{\tilde{u}^2} \right) + \frac{\varphi(\tilde{u})}{\tilde{u}} \right) \quad (10)$$

where $\varphi(x)$ is the standard normal distribution function.

2.2. Ocean wave simulation and statistical results of FEEE

We simulate sea waves with P-M and JONSWAP spectrum as target spectrums. The two spectrums, denoted by $S_{P-M}(\omega)$ and $S_J(\omega)$ are, respectively

$$S_{P-M}(\omega) = \alpha g^2 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega_0}{\omega} \right)^4 \right] \quad (11)$$

and

$$S_J(\omega) = \alpha g^2 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega_0}{\omega} \right)^4 \right] \gamma^{\exp \left[-\frac{(\omega - \omega_0)^2}{2\sigma\omega_0^2} \right]}, \quad (12)$$

where α is the scale coefficient, ω_0 is the peak frequency, γ is the peak-enhanced factor ranging from

1.5 to 6.0 and σ the peak-shape parameter which satisfies

$$\sigma = 0.07, \omega \leq \omega_0; \sigma = 0.09, \omega > \omega_0 \quad (13)$$

The JONSWAP spectra with different values of γ is used as target spectrum. Following Longuet-Higgins (1984), the spectrums are truncated in the simulation at upper and lower cut-offs $1.5\omega_0$ and $0.5\omega_0$. The simulation is done by use of Monte Carlo method and according to Eqs. (6)–(8). The random phase, which is assumed to be distributed uniformly within $[0, 2\pi]$, are generated by the random number generating program which is incorporated in the software Matlab. The time step is set to be $\Delta t = \pi/(10\omega_0)$ and the overall time length is set to be $2\pi \times 10^5 \omega_0^{-1}$. The number of random phase is 10^5 , which is sufficiently large for the simulation to be adequate for wave group statistics (Tucker et al., 1984; Elger et al., 1985). For each target spectrum, the simulation is repeated for 10 times and the value of FEEE, as well as wave group parameters, is obtained as the means of the corresponding measurements of the 10 processes. For two different critical levels, $\tilde{u} = \sqrt{\pi/2}$ and $\tilde{u} = 2$, results of FEEE measured from these simulated processes r_s are listed in Table 1 together with corresponding theoretical results r_t that are calculated by Eq. (9). Table 1 shows that Eq. (9) predicts FEEE lower than simulated results. The mean error $E[|r_s - r_t|]$ is 2.11% for $\tilde{u} = \sqrt{\pi/2}$ and 2.73% for $\tilde{u} = 2$. These errors will not cause much difference in group number correction. The third order approximation of FEEE of DL theory is also tested. Comparison shows that this approximation gives significantly larger values of FEEE than simulations. Therefore, we take Eq. (9) to represent the FEEE of ocean waves.

Table 1

Comparison between simulated and theoretical results of FEEE

Target spectrum	$\tilde{u} = \sqrt{\pi/2}$		$\tilde{u} = 2$	
	r_t (%)	r_s (%)	r_t (%)	r_s (%)
P-M spectra	4.14	6.46	12.78	16.23
JONSWAP spectra				
$\gamma = 1.5$	4.42	6.61	12.90	15.65
$\gamma = 3.0$	5.18	7.06	13.02	15.07
$\gamma = 4.5$	5.01	7.24	12.74	15.20
$\gamma = 6.0$	5.14	7.54	11.82	14.24

3. The improvement of wave group statistics

For a given critical level u the numbers of upcrossings per unit time by wave elevation $\zeta(t)$ and its envelope $\rho(t)$ are, respectively (Longuet-Higgins, 1984)

$$N = (2\pi)^{-1}(m_2/m_0)^{1/2}\exp(-u^2/2m_0) \quad (14)$$

$$N_u = (\mu_2/2\pi)^{1/2}u/\mu_0\exp(-u^2/2\mu_0) \quad (15)$$

Based on these equations, Longuet-Higgins (1984) gave the following results:

- (1) The mean distance between the beginnings of two successive wave groups, \bar{l}_L , is

$$\bar{l}_L = 1/N_u \quad (16)$$

- (2) The mean wave group length, \bar{G}_L is

$$\bar{G}_L = \bar{l}_L N_{\max} \quad (17)$$

- (3) The mean run length, \bar{H}_L , is

$$\bar{H}_L = \left(\frac{1}{2\pi}\right)^{1/2} \frac{(1+v^2)^{1/2}}{v} \bar{u}^{-1} \quad (18)$$

From Eq. (18) we know that, unless for the case $v \rightarrow 0$, the value of \bar{H}_L can be less than 1 as the critical level u is high enough, and furthermore, $\bar{H}_L \rightarrow 0$ as $u \rightarrow \infty$. These results are illogical and should be remedied by taking $(1-r)N_u$ rather than N_u as the number of wave groups. Therefore, Eqs. (16)–(18) should be corrected to become

$$\bar{l}_c \equiv \frac{\bar{l}_L}{1-r}, \quad \bar{G}_c \equiv \frac{\bar{G}_L}{1-r}, \quad \bar{H}_c \equiv \frac{\bar{H}_L}{1-r} \quad (19)$$

For those simulated ocean waves discussed in the preceding section, wave group characteristics are measured based on discrete wave heights. Compare Eq. (16) and Eq. (19) with these simulations, it is found that the mean relative error $E[|\bar{H}_s - \bar{H}_L|/\bar{H}_s]$ is 4% for $\bar{u} = \sqrt{\pi/2}$ and 13% for $\bar{u} = 2$, whereas $E[|\bar{H}_s - \bar{H}_c|/\bar{H}_s]$ is 1.5% for both critical levels. Another comparison is made among theoretical prediction of \bar{l}_L , \bar{l}_c and simulations. The relative difference is 10% for $\bar{u} = \sqrt{\pi/2}$ and 15% for $\bar{u} = 2$ before correction and 5% for both critical levels after correc-

tion. We can see that the corrected formulae show better consistency with simulated results. Similar work may be done towards the mean number of waves in a wave group. We omit it for simplicity.

Eq. (19) has avoided the illegitimacy of Eq. (18) mentioned above. Numerical calculation shows that for finite magnitudes of \bar{u} , the values of \bar{H}_c are always bigger than 1. For the infinite value of \bar{u} , $\bar{u} \rightarrow \infty$, it is easy to show that $\bar{H}_c \geq 1$ still persists. For sea waves after band-limited filtering, the spectra widths usually satisfy $v < 0.4$. So when $\bar{u} \gg 1$, the value of FEEE can be approximated by Eq. (10). Using the well known approximation (e.g. Feller, 1968)

$$\Phi(x) = 1 - \frac{1}{\sqrt{2\pi x}} e^{-x^2/2} [1 + O(x^{-1})], \quad (20)$$

we easily get

$$\lim_{\bar{u} \rightarrow \infty} \bar{H}_c = \sqrt{1+v^2} \geq 1. \quad (21)$$

4. Conclusions

In this paper, a group number correction scheme is introduced to exclude FEEE from wave group statistics. For this aim, the validity of DL theory to sea waves is examined through comparing it with simulated sea waves. The sea waves are represented by Longuet-Higgins wave model together with the JONSWAP or P-M spectrum and are simulated with Monte Carlo method. The FEEE of the simulated waves is investigated and compared with DL theory. The comparison shows that Eq. (9) is applicable to sea waves. This equation is then used to improve the wave group statistics. Comparisons between simulated and theoretical results of statistical characteristics of wave groupiness show that the correction is effective, especially for high critical levels. Besides that, the corrected formulae have the following advantages:

- (1) The values of the mean number of waves in a high run become never less than 1.
- (2) Since FEEE increases rapidly with the broadening of spectral width and increasing of the critical level, the former theory are not able to give accurate predictions for not very narrow spectrum and high critical level. The group number

correction scheme has reduced the error resulted by EEE, hence broaden the applicability of the envelope approach.

- (3) In the former theory, the characteristics \bar{l}_L , \bar{G}_L and \bar{H}_L all depend only on the spectral width ν at given dimensionless critical level \tilde{u} . But physically they should also depend on other parameters characterizing spectral distribution, say, the spectral skewness, etc. Spectral skewness did appear in Eq. (19).

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