A new hypothesis and exploratory model for the formation of large-scale inner-shelf sediment sorting and “rippled scour depressions”

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Abstract

Recent observations of inner continental shelves in many regions show numerous collections of relatively coarse sediment, which extend kilometers in the cross-shore direction and are on the order of 100 m wide. These “rippled scour depressions” have been interpreted to indicate concentrated cross-shelf currents. However, recent observations strongly suggest that they are associated with sediment transport along-shore rather than cross-shore. A new hypothesis for the origin of these features involves the large wave-generated ripples that form in the coarse material. Wave motions interacting with these large roughness elements generate near-bed turbulence that is greatly enhanced relative to that in other areas. This enhances entrainment and inhibits settling of fine material in an area dominated by coarse sediment. The fine sediment is then carried by mean currents past the coarse accumulations, and deposited where the bed is finer. We hypothesize that these interactions constitute a feedback tending to produce accumulations of fine material separated by self-perpetuating patches of coarse sediments. As with many types of self-organized bedforms, small features would interact as they migrate, leading to a better-organized, larger-scale pattern. As an initial test of this hypothesis, we use a numerical model treating the transport of coarse and fine sediment fractions, treated as functions of the local bed composition—a proxy for the presence of large roughness elements in coarse areas. Large-scale sorted patterns exhibiting the main characteristics of the natural features result robustly in the model, indicating that this new hypothesis offers a plausible explanation for the phenomena.

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1. Introduction

The shoreface and inner continental shelf, which together form the interface between the land and the continental shelf, can behave as a source, barrier, filter, or conduit for the exchange of
materials between the land and the sea. Oceanographic and geologic processes in this environment determine how a shoreline will respond to storms, to sea-level rise and to changes in sediment supply over time scales from hours to years to millennia (Swift, 1976; Nummedal, 1991; Wright, 1995). A fundamental understanding of shoreface processes is needed for predicting the behavior of beaches and the nearshore zone for coastal management, and for developing facies models (Walker, 1984) that can be used in understanding and interpreting both modern and ancient shoreface environments (e.g. Walker, 1985). However, such an understanding is lacking, as an examination of the state of knowledge concerning some common inner-shelf morphological features shows.

Observations off California and seven other locations in North America (see review by Cacchione et al., 1984, and references cited therein) have identified previously what more recent studies (Table 1) find are ubiquitous surficial sedimentary features of shoreface and inner shelf environments worldwide, particularly on sediment-starved margins. These enigmatic features have slight topographic expressions (total relief on the order of a meter) and are composed of coarse sand (and in some cases shell hash and gravel) that is arranged into large wave-generated ripples, with wavelengths on the order of a meter. These features, termed “rippled scour depressions” by Cacchione et al. (1984), are typically 100–200 m wide, and extend hundreds to thousands of meters in the cross-shore direction from the outer surf zone to the inner shelf. Examples of this form of spatially-grain-size-sorted feature also exist in the rock record, where they are present in inner shelf settings as cross-shore “sand tongues” within a finer-grained matrix (Aigner, 1985).

Cacchione et al. (1984) attributed their formation to areas of intensified cross-shore flow (during storms with winds that generate near-shore set-up) that preferentially winnow fine material, leaving a coarse lag elongated parallel to the flow. Although the near-bottom flows they measured were directed predominantly in the alongshore direction, these authors speculated that protruding rock outcrops may have turned the flow offshore locally in the vicinity of the rippled scour depressions. However, the location of rock ledges mapped by Cacchione et al. (1984) does not obviously correspond with the locations of the sedimentary features.

More recent observations suggest that a downwelling-current-related mechanism does not adequately explain the wide range of environmental settings where “rippled scour depression”—like features are present. For example, Schwab et al. (1997, 2000) speculated that rippled scour depressions on the relatively high-energy shelf off southern Long Island, NY are the result of late Holocene reworking of Pleistocene deposits primarily by along-shelf processes. Similarly, Twichell and Paskevich (1999) suggested that along-shelf flows are the only plausible mechanism to explain the morphology and stratigraphy of comparable features on the low-energy southwestern Florida inner shelf.

An extensive data set from the shoreface and inner shelf off Wrightsville Beach, North Carolina,
including seismic-reflection profiles, repeated sidescan-sonar surveys (e.g. Figs. 1 and 3), diver vibracores, and diver-based seafloor mapping, indicates that rippled scour depression-like features dominate the shoreface and inner shelf there (Thieler et al., 1995, 2001). These data also show that these features exhibit some of the attributes of flow-transverse bedforms. For example, they are asymmetric. The coarse sediment domains form bathymetric lows on their northern edges—the updrift side, relative to the dominant alongshore transport (Jarrett, 1977; Thieler et al., 2001). However, on their southern sides they exhibit positive relief (Fig. 2). The coarse domains are not centered on the bathymetric low, as the term “rippled scour depression” suggests. The features off the southwestern Florida coast also exhibit this pattern, with the coarse domains extending to the topographic crest (Twichell and Paskevich, 1999; A.C. Hine, personal communication, 2002), as do features off of Long Island, New York (Schwab et al., 2000). However, all these features differ significantly from classic bedforms such as ripples and dunes; their bathymetric expressions are very subtle, with relief on the order of one meter across widths on the order of 100 meters.

The northern edges of the coarse domains form a sharp boundary with the fine sediment domains, which we speculate is indicative of southward, shore-parallel sediment transport resulting in minor infilling at the edges of the bathymetrically negative northern sides (Fig. 2). Sidescan sonar images show a ragged or “wispy” edge on the downdrift side (Fig. 2), and diver observations indicate that this represents drapes of fine material deposited on the downdrift sides of the features. This general morphology is present in sidescan surveys completed in 1992, 1994, and 1995 (Thieler et al., 1995; 2001). However, recently analyzed sidescan sonar data collected after hurricanes Bertha (July 1996), Fran (September 1996), and particularly Bonnie (August 1998) show much more regular southern edges (Thieler, unpublished data). In addition, comparisons of the positions of some of the features before and after hurricane Bonnie indicate that the features migrated by approximately half their widths (Fig. 3). Prior to hurricane Bonnie, feature migration was not noted. Bonnie produced a period of alongshelf forcing that was unusually prolonged relative to other storms, which may have caused the migration of the features.

The sorted features extend across the physiographic base of the shoreface at 10 m water depth, which at Wrightsville Beach corresponds to a sedimentologic boundary between the shoreface and inner shelf as defined by the roundness, and degree of polish of the gravel-sized carbonate fraction (Thieler et al., 2001). The significance of rounding due to energetic nearshore processes has been previously discussed by Pilkey et al. (1967, 1969). There is little well-rounded, gravel-sized material seaward of the shoreface at Wrightsville Beach. Thus, cross-shore sediment movement is probably not as important as alongshore transport in this setting (Thieler et al., 2001).

Current data from inner shelf locations during storms typically shows dominantly alongshelf flows, with minor upwelling/downwelling components. In
addition, current data collected in two depths within the largest of the rippled scour depressions at Wrightsville Beach (Fig. 1) revealed no significant offshore-directed currents (Thieler et al., 1998). Those observations, however, were made during the spring season, and did not capture any large storms. It is possible that some unrecognized mechanism that operates during storms creates the heterogeneous pattern of cross-shelf flows postulated in the prevailing interpretation.
Fig. 3. Geographically and geometrically corrected sidescan sonar images of the shoreface off Wrightsville Beach. (A) Imagery obtained in 1995. Lines are drawn along the northeastern edges of the coarse-grained (light- to white-colored) bedform features for reference. (B) Sidescan sonar image obtained shortly after hurricane Bonnie in August 1998. Note the absence of the “wispy” southwestern edges of the coarse-grained features, as well as apparent feature migration downdrift (to the SW) relative to the 1995 reference lines. This magnitude of movement exceeds the navigational and technical uncertainty (order 10 m) inherent in the processed data. Note also the change from wispy to sharp downdrift (SW) edges of the coarse features.

Taken together, these observations, while not conclusive, are more consistent with the dominance of along-shelf transport, rather than of speculative cross-shelf flow and transport, in the creation and maintenance of these rippled features. Because the term “rippled scour depression” is
associated with the genetic, cross-shelf process interpretation of Cacchione et al. (1984), and because the most recent surveys have shown that they are not simple depressions, we adopt the term "sorted bedforms" to describe the features off Wrightsville Beach and elsewhere.

Swift and Freeland (1978) referred to similar features off the northern North Carolina coast as bedforms, but suggested no general mechanism to generate the marked sorting of sediment sizes associated with them. They referred to an analytical model for the growth of topographic bedforms in a unidirectional current (Smith, 1970). However, inner-shelf sorted features are primarily compositional phenomena, exhibiting very subtle topographic expression with relief on the order of a meter over widths on the order of hundreds of meters. If topographic interactions drove the development of the features, they would be expected to attain more than subtle relief in at least some locations.

The widespread occurrence of sorted bedforms on the shoreface and inner continental shelf suggests that oceanographic processes are responsible for their formation and evolution. They are apparently independent of geologic factors, such as underlying stratigraphic framework or sediment supply, although both factors may play an important role in the overall settings in which they occur. Studies off Wrightsville Beach suggest that these features are not ephemeral, and indeed persist through a wide range of storm and fairweather conditions over interannual and longer time scales.

In the next section we describe observations that, along with those above, motivate a hypothesis for a mechanism that could lead to the self-organization of sorted bedforms. ("Self-organization" refers to pattern formation caused by interactions within a system, as opposed to being dictated by the patterns of the forcing or the initial conditions.) This conceptual model postulates that sorted bedforms are a robust result of the interaction of waves, mean currents, and poorly sorted bed material, in a moderately high-energy environment (outside the surf zone). In Section 3 we describe numerical modeling experiments that provide an initial test of this hypothesis.

2. Qualitative hypotheses

2.1. Boundary layer processes

Once sediments are lifted from the bed, they are maintained in suspension by a balance between upward diffusion due to turbulence and settling due to gravity. During this time, the particles in suspension are advected downstream with the mean current. Sedimentary particles are carried to locations with potentially different turbulence intensities. If the turbulence in the new location is not sufficiently rigorous to counter gravity, sediments will be deposited on the seabed. The overall sediment concentration is generally expected to increase when turbulence intensity increases, and vice versa. The bottom stress depends not only on the strength and properties of waves and currents, but also on the physical roughness of the seabed, which plays an important role in the determination of sediment response (e.g., Grant and Madsen, 1986; Vincent and Hanes, 2002). Assuming the same wave and current forcing and no or similar levels of near-bottom stratification, turbulence levels will be higher over beds with higher physical roughness scales (i.e., coarse bed versus fine or rippled versus flat bed) (Grant and Madsen, 1986).

Natural sand beds always consist of a mixture of different particle sizes with correspondingly different settling velocities. Hence, re-suspension processes (i.e., pick-up rates) and concentrations generally consist of contributions from the different grain sizes present in the seabed. Traditionally, scientists and engineers consider the bulk quantities (total concentrations and total pick-up rates), and try to model them in terms of representative sediment parameters. Many models do not consider the concentrations and pick-up rates for the individual size fractions—quantities that are of great importance to sediment sorting and differential transport mechanisms. Under energetic flow conditions, numerical simulations show that representations of the particle size distribution in the bed are of primary importance in both the pick-up function and the vertical distribution of the suspended sediment (Wallbridge and Voulgaris, 1997). The coarser particles will be deposited as soon as the flow turbulence intensity, as
parameterized by the eddy diffusivity, reduces sufficiently that for the particular size fraction the settling due to gravity dominates vertical diffusion. Thus, coarser particles tend to be deposited in high-energy environments while finer particles remain in suspension until they reach areas where on average the turbulence decreases.

2.2. Observations

Abrupt boundaries separate the coarse domains from the fine material in the features described here and elsewhere (Fig. 4; Table 1). Significant changes in boundary layer structure have been measured over an abrupt transition from fine to coarse substrates (Sleath, 1987; Fredsoe et al., 1993) or have been simulated numerically (Laursen et al., 1994). This phenomenon has also been observed in the field (Allen, 1969; Flood, 1981). These studies suggest an abrupt increase in shear stress and turbulence at the sediment textural boundary. Diver observations off Wrightsville Beach suggest that the change in ripple size from the fine to the coarse domains has a much greater effect on the scale of turbulence arising from near-bed wave orbital motions than would be expected from the change in grain size alone, consistent with physical and model studies. Over the large ripples, fine material has been observed suspended approximately 0.75 m above the bed, while above the fine bed, this visual indication of the scale of turbulent eddies extends only on the order of 0.1 m where ripples have wavelengths and heights approximately an order of magnitude smaller. Wave-generated ripples in the fine material can be reformed by small waves with small near-bed orbital excursions that produce small ripples (Cacchione et al., 1984; Fredsoe and Deigaard, 1992). Ripples composed of coarse material, however, are not as easily reformed, and at most times reflect the scales of previous larger waves.

2.3. Sorted bedform hypothesis

These grain- and ripple-size effects described above are hypothesized to cause the formation and maintenance of sorted bedforms. Starting from a bed of mixed coarse and fine grains, any perturbation that forms a concentration of coarse material

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Fig. 4. (A) Enlarged section of sidescan-sonar imagery showing the meter-wavelength ripples that floor the sorted bedforms. Location is shown as a black circle on Fig. 1. Ripples in the fine sand (wavelength ~ 10 cm) are below the resolution of the sidescan system. Note the sharp contact between coarse and fine sediments. Note also the small package of finer sediments within the coarse sediment (upper left). The crests of the larger ripples can be “seen” through this thin deposit. (B) A diver photo from the same location (looking east) shows the sharp contact between coarse and fine sediments at the northern boundary of the bedform. Ripple wavelengths in the coarse sediment domain (right side of the photo) are approximately 75–100 cm, with heights about 20–30 cm. The ripples in the finer sediments on the left of the photo have wavelengths of 10–20 cm and heights 3–5 cm. Diver observations indicate that the contact is sharp, occurring over a few centimeters. Coarse ripple crests can be traced under the fine sand for several meters, suggesting that fine material periodically infills the northeastern (updrift) edges of the feature (After Thieler et al., 2001).
should initiate a feedback. The increase in turbulence and shear stress above such a concentration would tend to inhibit the deposition of fine sediment locally, and further winnow more of the fine sediment. The coarse lag then protects the material beneath from entrainment. This occurs in eolian megaripples that develop armored upflow-facing flanks in areas where coarse and fine sand are mixed (Sharp, 1963). In the eolian environment, sorting is further enhanced during bedform migration, because any coarse grains distributed in the body of the bedform collect on the upflow-facing side as the fine grains are removed. Subaqueous transport processes are considerably different than those in the eolian environment, but the armoring and collection-during-migration effects will still occur. In the case of the very large-scale subaqueous sorted bedforms, recent observations suggest that significant migration, requiring the transport of the coarse grains, occurs during large storms (Fig. 3). Although the wave-generated ripples may be destroyed during very strong storms, the fine material will not settle out on a coarse plane bed during such high-energy conditions. In the development of many kinds of bedforms, migration causes initially isolated, small, disorganized features to collide and merge, leading to organization on a larger scale (e.g. Anderson, 1990; Landry and Werner, 1994; Murray et al., 2003). In this hypothesis, the feedback will create initial coarse and fine domains on scales already much larger than the wave-generated ripples; the role of these ripples is only to increase the turbulent energy over a coarse sediment domain through their interaction with wave-orbital motions.

The wave-generated ripples will have crests oriented perpendicular to the propagation direction of the waves that formed them, which typically will be sub-parallel to the shoreline. This orientation—sub-perpendicular to the orientation of the much-larger-scale features (Fig. 4)—does not necessarily reflect the direction of the suspended-load transport that we hypothesize to be responsible for the origin and maintenance of the much-larger-scale sorted pattern (although it likely indicates the direction of bedload transport, as we discuss in Section 4).

3. Numerical modeling methods

If these large-scale sorted features have evolved in the manner hypothesized above, their formation presumably occurs over a long time scale (decades?). In addition, sorted bedforms span a range of depths (4–16 m in the Wrightsville Beach area) and therefore a range of wave and current conditions. In this extended, spatially heterogeneous domain, with temporally varying forcing, it is far from clear a priori what sort of large-scale pattern would evolve from the merging of coarse sediment domains created by the feedback described in Section 2. Numerical modeling, however, provides a practical means to investigate whether the hypothesized interactions could lead to the origin and evolution of sorted bedforms like those on inner shelves.

Rather than constructing a model that attempts to explicitly simulate all the processes hypothesized to be involved in as much detail as is practical, and on scales as small as is possible, we are using a complementary approach to modeling that is appropriate when the goal is specifically to explicate a puzzling natural phenomenon. In constructing what have been called “exploratory models” (Murray, 2002, 2003), a modeler leaves out as many processes as possible, and strives for the simplest representations of the processes included, in an attempt to determine the mechanisms that are essential for causing the behavior in question. The modeling endeavor described here is not intended to represent the only way to model this system, but rather to investigate as directly as possible what aspects of the relatively large-scale interactions could produce the observed grain size and bedform patterns.

Details of the vertical structure of the flow and sediment concentration are not treated in our model, nor are dynamics on the time scale of wave orbital motion. We parametrically treat the effects on the sediment-concentration profile resulting from the interaction between orbital motions and wave-generated ripples. A three dimensional grid of cells represents the bed, with grain size composition (proportions of each of two size classes) defined in each cell, although only sediment in cells nearest to the bed-surface at each
horizontal location participate in sediment transport. We calculate fluxes of the two size classes of sediment separately. We use cells with a horizontal width of five meters and a vertical extent of five centimeters.

We base the sediment-transport treatment partly on a relatively simple formula (Bowen, 1980) appropriate for relatively weak currents passively advecting wave-suspended sediment, in which the sediment flux is assumed to be predicted by local hydrodynamic conditions:

\[ \text{Flux}_{\text{local}} = \alpha(U_o)u + \beta(U_o)S, \]

where \(\alpha(U_o)\) is a term representing the suspension of sediment by wave-orbital motions, \(u\) is the mean current velocity, and \(S\) is bed slope. Following Bailard (1981) and Ribas et al. (2003), we use \(\alpha(U_o) = C_1C_fU_0^2\) and \(\beta U_o = C_1C_fU_0^3/(5W_{sj})\), where \(U_o\) is the maximum wave orbital velocity.

We assume shallow-water waves (which implies long-period waves in inner-shelf depths), so that \(U_o = (Ht/2)\sqrt{g/D}\), where \(H_t\) is the wave height, \(D\) is the depth, and \(g\) is the acceleration of gravity.

\(C_f = (16\epsilon_s\rho/3\pi W_{sj})\), where \(\epsilon_s\) is an efficiency factor, \(\rho\) is water density, and \(W_{sj}\) is the fall velocity of sediment in size class \(i\). We treat the drag coefficient, \(C_f\), as a function of bed composition, \(B_c\) (the fraction of the bed material that is coarse in the top 15 centimeters of the bed surface) as a proxy for the presence of large wave-generated ripples. In the model presented here,

\[ c_f = 0.01(1 + aB_c), \]

with \(a = 10\) except where noted. Eq. (2) is a heuristic, simple representation of the dependence of bed roughness on bed composition, formulated to reduce to a typical roughness value over a fine bed.

To model the sediment-transport effects of spatially heterogeneous hydrodynamic conditions (Fig. 5), we adapt an advection-diffusion approach, with source and sink terms representing exchange with the bed, for depth-averaged suspended-sediment concentrations (immersed weight per bed area), \(C\) (implicitly averaged over a time longer than a wave period) (Galappatti and Vreugdenhil, 1985; Wang, 1992; Verbeek et al., 1999). In one dimension:

\[ \partial C/\partial t + \partial(uC)/\partial X - \mu \partial^2 C/\partial X^2 = G(U_o, U) - \gamma C, \]

where \(\mu\) is the diffusivity, and \(G(U_o, U)\) is an entrainment function. Deposition is treated as proportional to \(C\), and \(\gamma\) is a proportionality constant with the units of \(s^{-1}\). This approach allows non-local sediment transport because the divergence of sediment flux is determined not only from the spatial derivative of the local hydrodynamic conditions, but also from the interaction of a flux of sediment advected into a control volume that may be different from that predicted from the local conditions. \(\gamma\) can be interpreted as the inverse of a characteristic sediment settling time, \(T_{\text{settle}}\). In the oversimplified one-dimensional case of suspended sediment mixed uniformly over a height \(H_p\) above the bed, and settling at a...
uniform velocity,

\[ T_{\text{settle}} = H_p / W_{sj} \]  \hspace{1cm} (4)

More generally for each size fraction, \( T_{\text{settle}} \) will depend on a more complicated vertical distribution of suspended sediment, on the local turbulence that influences the settling, and on settling velocity. In this model, we parameterize these effects by constructing an expression for an effective concentration-profile height, \( H_{\text{eff}} \), which will be greater over coarse domains, simulating the effect of the large wave-generated ripples:

\[ H_{\text{eff},i} = (U_o / W_{sj})(0.001 + b B_c). \]  \hspace{1cm} (5)

The first term produces an effective profile height over fine sediment of 0.1 m for suspended fine sediments consistent with diver observations, and \( b \) is a dimensional constant, 0.01 m s\(^{-1}\) in this initial model except where noted.

We neglect the diffusion term in Eq. (3), assume a quasi-steady state, use \( G(U_o, u) = \alpha(U_o)\gamma \), and add a slope term:

\[ \partial(uC) / \partial x = \alpha(U_o)\gamma - \gamma C - \partial(\beta(U_o)S) / \partial x. \]  \hspace{1cm} (6)

Using these forms for \( G(U_o, u) \) and the slope term make this approach consistent with Eq. (1); setting the entrainment and deposition terms equal (assuming sediment flux in equilibrium with local conditions) defines an equilibrium concentration, \( C_{eq} = \alpha(U_o) \). Multiplying this concentration by the advective velocity produces the flux predicted by Eq. (1). Discretizing Eq. (6), and expressing it in terms of this equilibrium flux produces:

\[ \text{FluxOut} - \text{FluxIn} = \{ \text{EquilibFluxOut} - \text{FluxIn} \} P_{sj,i}, \]  \hspace{1cm} (7)

where \( \text{FluxIn} \) and \( \text{FluxOut} \) are the respective fluxes entering and exiting a cell, and \( \text{EquilibFluxOut} \) is the flux that would be predicted to leave the cell, based on the local conditions in that cell and Eq. (1). The difference between the equilibrium flux exiting a cell and the flux that entered the cell determines the maximum exchange between the suspended sediment and the bed. \( P_{sj,i} = T_{\text{advect}} / T_{\text{settle},i} \) is the proportion of this potential exchange that occurs, where \( T_{\text{advect}} = Xu_t \) (\( X \) is the cell width) is the time it takes for the sediment to be advected through the cell:

\[ P_{sj,i} = \text{MIN}\{ (X/u)(W_{sj,i}/H_{\text{eff},i}), 1 \}, \]  \hspace{1cm} (8)

where \( \text{MIN} \) is the minimum of the two quantities.

In Eq. (7), \( \text{EquilibFluxOut} \) given by Eq. (1) for each size class is modulated by the proportion of the material in the top 15 cm of the bed that is of that class. In other words, the \( \text{EquilibFluxOut} \) for the coarse material is given by Eq. (1) multiplied by \( B_c \), and for the fine material, Eq. (1) is multiplied by \( (1 - B_c) \), consistent with numerical results showing that the particle size distribution in the bed determines the size distribution entrained (Wallbridge and Voulgaris, 1997).

We adapt this one-dimensional treatment to two horizontal dimensions in a straightforward way: the suspended sediment will be advected through a cell at a speed of \( (u^2 + v^2)^{1/2} \) (where \( u \) and \( v \) are the horizontal velocity components in the \( x \) and \( y \) directions, respectively), and will travel a distance greater than \( X \). If \( u > v \), then during the time it takes the sediment to traverse a distance of \( X \) in the \( x \) direction, it will have been advected a distance of \( (X/u)v \) in the \( y \) direction, so that the distance traversed as the sediment is advected through the cell is \( (X^2 + X^2 v^2/u^2)^{1/2} = (X/u)(u^2 + v^2)^{1/2} \). Then:

if \( u > v \), \[ P_{sj} = \text{MIN}\{ (W_{sy}/H_{\text{eff},i})(X/u) \times (u^2 + v^2)^{1/2}/(u^2 + v^2)^{1/2}, 1 \}, \]  \hspace{1cm} (9)

or,

if \( v > u \), \[ P_{sj} = \text{MIN}\{ (W_{sj}/H_{\text{eff},i})(X/u), 1 \}. \]  \hspace{1cm} (10a)

Similarly,

if \( v > u \), \[ P_{sj} = \text{MIN}\{ (W_{sj}/H_{\text{eff},i})(X/v), 1 \}. \]  \hspace{1cm} (10b)

In two dimensions, it is clearer to express Eq. (7) in an equivalent form involving the amount of suspended sediment interacting with the bed (eroded or deposited), \( \text{BedExchange} \):

\[ \text{BedExchange} = P_{sj}(\text{FluxExcess}), \]  \hspace{1cm} (11)

\[ \text{FluxExcess} = (\text{EquilibFluxIn} + \text{FluxExcessIn}) - \text{EquilibFluxOut}, \]  \hspace{1cm} (12)
\[ \text{FluxExcessOut} = (1 - P_s) \text{FluxExcess}, \quad (13) \]

where \( \text{EquilibFluxIn} \) and \( \text{EquilibFluxOut} \) are the sums of the fluxes that would be entering and exiting the cell in question from both directions, predicted by local conditions (Eq. (1)). \( \text{FluxExcessOut} \) is passed on to the next cells in both directions, and \( \text{FluxExcessIn} \) is sum of the excesses passed into the cell from both directions. The excess flux advected out of the cell in question, \((x, y)\), is divided in the \( x \) and \( y \) directions in proportion to the relative magnitudes of \( u \) and \( v \).

If \( u > 0 \)
\[ \text{FluxExcessIn}(x + 1, y) = (1 - P_s) \text{FluxExcess}(x, y)(|u|/(|u| + |v|)), \quad (14a) \]

If \( v > 0 \)
\[ \text{FluxExcessIn}(x, y + 1) = (1 - P_s) \text{FluxExcess}(x, y)(|v|/(|u| + |v|)), \quad (14b) \]

where \( \text{FluxExcessIn}(i,j) \) is the flux of excess suspended sediment into cell \((i,j)\). (If \( u \) is negative, the sediment is advected into cell \((x, y - 1)\), and similarly in the \( y \) direction.)

In the initial model experiments described here we impose spatially uniform current and wave conditions, and employ periodic boundary conditions. The initial condition consists of a flat bed with the composition in each cell (proportions of coarse and fine material) chosen randomly and independently from a range of possible compositions. For each run, we choose a time step that is small enough to ensure numerical stability. (This time step is a function of the maximum potential sediment flux, which is in turn a function of the wave, depth, current and sediment characteristics.) Numerical experiments have shown that the results presented below are not sensitive to changes in the time step, as long as it is short enough to avoid numerical instability. Each iteration of the model consists of: (1) a sweep through each row of cells in the \( x \) direction, applying this algorithm in each cell for coarse-sediment transport, using a fall velocity for coarse material; (2) another sweep in the \( x \) direction for fine-sediment transport, using a fall velocity for fine sediment; and (3) and (4), sweeps through the columns of cells in the \( y \) direction for coarse and fine sediment transport. The amount of sediment in the top bed cell is then adjusted according to \( \sum \text{BedExchange} \). (If the top cell becomes overfilled, the excess sediment is added to the cell above, which then becomes the top cell. If the top cell is emptied, sediment is taken from the cell below, which then becomes the top cell.)

The algorithm described above is very similar to using a central finite difference solution scheme for Eq. (3), waiting for a steady state, neglecting diffusion, and adding the slope term in (6). In the limit of small cell size, this algorithm reduces to such a solution, applied to the two grain-size fractions separately, with the entrainment term modulated by the availability of each grain-size fraction in the bed locally, coupled with the conservation of mass determining bed changes. Numerical experiments have shown that the results are not sensitive to the spatial discretization.

4. Initial model results

4.1. Basic results

Sorted bedforms develop spontaneously under virtually all combinations of model parameters and wave, current and sediment characteristics we have tried (e.g. Figs. 6–8). As we discuss below, however, these characteristics and parameters do affect the details of the shapes and sizes of the features. Plan-view and profile patterns can match the scale and main aspects of natural features (Fig. 2). In plan view, the pattern development through time can strongly resemble other kinds of bedforms, such as eolian ripples (Figs. 6–8). However, these seabed features are not primarily topographic features; their relief is subtle (on the order of decimeters over horizontal scales on the order of 100 m). These bedforms are created by the differences in sediment transport over zones of coarser and finer bed material, through the feedback described in Section 2.

4.2. Qualitative analysis of sorted bedform behavior

Observing animations of model results and considering the interactions in the model leads to
Fig. 6. Output of the numerical model. Gray scale shows the size composition in each cell; white indicates purely coarse material, black purely fine material. The cells nearest the surface show the average composition of the top 0.15 m of the bed (the quantity that affects the sediment-transport calculations). The image shows 200 × 200 cells (1 km × 1 km). Conditions were uniform across the domain, with 1 m wave heights, 10 m water depth, and a 0.1 m/s current directed from lower left to upper right. Initial conditions consisted of a percentage of coarse material assigned independently for each cell randomly from a range of 10–20% coarse, and a flat bed. Initial conditions appear as an even gray tone in plan view. Boundary conditions are periodic. Fall velocities for fine and coarse sediments were 0.01 and 0.5 m/s, respectively. (A) Plan view and profile patterns after a simulated time of approximately 2.5 days. (Simulated times cannot be compared directly to actual times; current and wave conditions in the model represent only the times that a natural system experiences the combination of energetic wind-driven currents and waves that have sufficiently large periods and heights to significantly affect the bed in inner-shelf depths.) (B) Patterns after a simulated time of approximately 5 days. (C) Patterns after a simulated time of approximately 10 days. (D) Patterns after a simulated time of approximately 20 days. Bedform merging will continue, eventually leaving only one feature in the domain.

Some explanations of the appearance and behaviors of finite-amplitude, well developed sorted bedforms in the model (as opposed to the initial feedback described in Section 2). We start with the characteristics of the profile patterns: the advective part of the sediment flux (in Eqs. (1) and (6)) creates the relief. Fig. 5 depicts schematically an approximately horizontal coarse-sediment domain. As the flow moves over the upstream part of the coarse domain, the sediment flux increases (Eqs. (1) and (2)), and the divergence of flux (spread over a finite region; Eq. (6)) lowers the bed in this area. As the flow moves past the coarse domain, deposition in the downstream region (mostly of fine sediment since the coarse material settles out almost immediately) raises the bed there. As the coarse domain migrates, because of transport of the coarse material, the downstream edge will be deposited on top of this hill of fines. The combination of lowering of the upstream portion and raising of the downstream portion of a coarse domain...
Fig. 7. Output of the numerical model. Conditions were the same as in Fig. 6, except that the current reversed direction once per model day. Images show snapshots from just before a reversal. (A) Plan view and profile patterns after a simulated time of approximately 2.5 days. (B) Patterns after a simulated time of approximately 5 days, showing a time mid-way through a period of current directed toward the upper right. Wavelength at this stage is less than 100 m. (C) Patterns after a simulated time of approximately 5 days, showing a time shortly after a period of current toward the upper right began. (D) Patterns after a simulated time of approximately 20 days, showing a time near the end of a period of current directed toward the upper right. This wavelength does not change appreciably after this time step: the dislocations in the plan view pattern will anneal, eventually leaving seven sorted bedforms along the diagonal. This corresponds to a wavelength of 200 m.

Inclines that domain, producing the asymmetry of the sorted bedforms.

The slope-dependent terms in Eqs. (1) and (6) tend to diffuse the bed topography, creating a negative feedback that limits the inclination of the coarse domain. Transport of fine material is enhanced on the downhill slope just upstream of a coarse domain. Transport of fine material is enhanced on the downhill slope just upstream of a coarse domain (e.g., Fig. 6 profile), and is retarded on the uphill-sloping coarse domain. If the uphill slope is too steep, fine sediment will accumulate in the trough. Similarly, if the flux of fines out of the downstream edge of the coarse domain is too small to match the flux of fines on the downhill slope in the fine domain, erosion will occur just downstream of the migrating coarse domain. Both deposition in the trough and erosion ahead of the crest of the feature will tend to lower the slope of the coarse domain if it becomes too great. Thus, relief and the slopes of the coarse and fine domains self-organize into a configuration that allows an approximately steady state.

In detail, the self-organization is more complex than this simplified description indicates. Because essentially all the coarse material above trough level collects on the surface as a coarse patch migrates (e.g., Fig. 6 profiles), the concentration of coarse material in the active surface layer (set to be
15 cm in the model) depends on the relief and the horizontal width (and therefore the slope) of the coarse patch. The concentration of coarse sediment affects the sediment fluxes through the coarse domain, chiefly because of the dependence of the flux on bed composition (Eqs. (1) and (2)), and the fact that entrainment of fine (coarse) sediment is limited by the proportion of the bed that is fine (coarse). The interplay between advective and diffusive processes described above, and thus the self-organization of the relief and slopes, will be affected by these concentration effects. In addition, sediment fluxes decrease slightly in lower areas (because the wave-orbital velocities decrease with increasing depth), and this effect will also contribute to the negative feedback that limits relief of the sorted bedforms.

The plan-view behaviors are somewhat simpler. Although the fluxes of coarse sediment are smaller than those for fine sediment (because of the dependence on fall velocities in Eq. (1)), transport of coarse sediment will cause a coarse domain to slowly migrate. The rate of migration depends on the flux of coarse sediment at the downstream edge of the coarse domain, and this flux is determined primarily by the coarseness of the sediment in the domain (Eq. (1)). (Although the slope of the coarse domain will have an effect, this effect is less important for coarse material than for fines; the slope-dependent term in Eq. (1) is multiplied by the inverse of the square of the fall velocity. Thus, the slope of the coarse domain will be limited by the transport of fine sediment, and will not strongly affect coarse transport.) The coarser a
domain is, the faster it tends to migrate. This contrasts strikingly with topographic bedforms, where the migration speed is determined by the height of the feature (holding the flux of sediment across the crest constant), with smaller features migrating faster. However, the evolution of the plan-view pattern resembles those exhibited by other kinds of bedforms, because some features move faster than others, and therefore features collide and merge. This merger process leads to an increase through time in the scale and degree of organization of sorted bedforms, as it does for other bedforms.

4.3. Dependence on current characteristics

The model behaviors have proven surprising in several regards, the most important of which involve the effects of different mean-current regimes. If the advecting current always moves in the same direction, model bedforms increase in wavelength perpetually (limited only by the size of the model domain) (Fig. 6). As is the case for eolian-ripple models (Anderson, 1990; Werner and Gillespie, 1993; Landry and Werner, 1994), although the frequency of mergers decreases as the merger process reduces the range of migration speeds of surviving features, the merger process never stops completely.

When the current direction reverses periodically, however, a well-defined steady-state wavelength emerges (Fig. 7(D)). This occurs because, during the time the current moves in one direction, a coarse domain can only migrate so far. It then sweeps back and forth within a limited region. After each coarse domain merges with any others within its range, the wavelength essentially stops changing. However, dislocations—\(J\)-junctions in the plan view pattern (Fig. 7)—continue to merge in plan view, transversely merging two features into one or vice versa, or shifting longitudinally. As is the case for other bedform patterns (Anderson, 1990; Werner and Gillespie, 1993; Landry and Werner, 1994), as these dislocations annihilate, the average wavelength increases. In profile view, these dislocation migrations appear as a coarse domain entering the region swept out by another, and the resulting merger of the two.

Increasing the frequency of current reversals reduces the range that can be swept out in a current-reversal cycle, therefore decreasing the steady-state wavelength. The steady-state wavelength should increase linearly with the period of current-reversal.

In a regime of reversing currents with an asymmetry in the duration or strengths of the currents in the two directions, features show a net migration in the dominant direction. In this case, features with different migration speeds can continue to merge, although the resulting increase in average wavelength over time becomes very slow as the asymmetry is reduced.

In a symmetric reversing-current regime, even after the pattern has reached a long-term statistically steady state, the widths of the coarse domains change during each half cycle. Immediately after a reversal, the part of a coarse domain that has become the downstream end is buried by fine sediment. As the coarse domain migrates in the new direction, the temporarily buried coarse material is again integrated into the surficial coarse domain. The coarse domain widens progressively as the more deeply buried sediment closer to the previous trough location is excavated. The domain reaches a maximum width just before the next reversal (Figs. 7(C) and (D)). Such behavior could possibly explain why the Wrightsville Beach coarse domains widened as they apparently migrated during the prolonged southerly alongshelf currents caused by hurricane Bonnie (Fig. 3).

With a single current direction, the coarse domains tend to be very wide (Fig. 6) relative to those in the reversing-current case (Fig. 7). In plan view, the single-direction pattern looks like patches of fine sediment moving across a coarse seafloor. Some sorted bedforms in nature have this appearance, such as those off of the southwestern Florida coast (Twichell and Paskevich, 1999).

The plan-view patterns in reversing- and single-current-direction cases differ in another striking way. With reversing directions, the pattern evolves into a well-organized set of continuous flow-transverse coarse and fine domains (similar to a well-developed eolian ripple pattern). However, with a single direction, the domains are less continuous in the transverse direction (e.g.,
Fig. 6(D)). This aspect of the single-current-direction pattern also roughly resembles that of the sorted bedforms off of the southwestern Florida coast (Twichell and Paskevich, 1999), although the currents there are not restricted to a single direction. We will return to this point in the Discussion section.

Changing the current velocity affects the plan-view and profile patterns (Fig. 8(A)). Increasing the velocity increases the sediment flux (Eq. (1)), and therefore the migration rate. In a reversing-current regime, this tends to increase the wavelength. In addition, increasing the velocity decreases the relative importance of the diffusive part of the sediment transport (Eqs. (1) and (6)). Steeper slopes and greater relief result (Fig. 8(A)).

4.4. Dependence on Wave Height and Water Depth

Holding the water depth constant and increasing wave heights increases the near-bed orbital velocity, $U_0$, which increases sediment fluxes (Eq. (1)). With higher fluxes, migration speeds increase, leading in a reversing current regime to a larger wavelength (compare Figs. 8(B) and 7(D)).

Although both the advective and diffusive aspects of the sediment transport increase with increasing wave height, the diffusive term in Eq. (1) depends on a higher power of $U_0$ than does the advective term. Thus, increasing the wave height increases the relative importance of the diffusion, leading to lower slopes (Fig. 8(B)).

Holding the wave height constant, reducing the depth also increases $U_0$, and has the same effects as increasing the wave height.

4.5. Dependence on sediment grain sizes

Reducing the fall velocity of the fine sediment, $W_{sf}$, increases the relative importance of the diffusive part of the fine sediment transport (Eqs. (1) and (6)), reducing the slopes, especially in the fine domains (compare Figs. 8(C) and 7). The relatively low slopes in the fine domains make the coarse domains relatively narrow. In other words, increasing the contrast between grain sizes (fall velocities) tends to make the plan-view pattern look more like isolated stripes of coarse material in a fine matrix.

Reducing the fall velocity of the coarse sediment, $W_{sc}$, increases the flux of coarse sediment (Eq. (1)), which also increases the migration rate and therefore the steady-state wavelength. Reducing $W_{sc}$ also reduces the contrast between the fall velocities, and increases the relative importance of the diffusive aspect of coarse sediment transport. These changes lead to lower slopes in the coarse domains, and a plan-view pattern that tends to look more like a generally coarse seabed with fine-sediment features superimposed (Fig. 8(D)).

4.6. Dependence on average bed composition

The effects of changing the range of bed compositions in the initial conditions are relatively subtle. When the average proportion of coarse sediment is higher, the patterns initially develop faster, since the initial sediment fluxes are generally higher (Eqs. (1) and (2)). However, the self-organization of the sorted bedforms eventually creates coarse domains that have compositions ($B_c$) that are independent of the average bed composition. (The coarseness of coarse patches appears to be determined by the other factors discussed above.) This means that the migration rates of coarse patches are also essentially independent of the average composition, and therefore the average wavelength is essentially unaffected. With the same number of sorted bedforms with coarse domains of the same composition, when the average bed composition is coarser, the coarse domains are wider.

4.7. Sensitivity to changes in model parameters

Within limited ranges, changing the poorly constrained model parameters ($a$ in Eq. (2) and $b$ in Eq. (5)) does not qualitatively affect model results; the basic phenomena described in Section 4.1 occur robustly. However, quantitatively, the details of the patterns (slopes, wavelengths, etc.) are affected. Changing $a$ affects the dependence of the friction coefficient, $c_f$, and therefore the sediment fluxes (Eq. (1)), on local bed composition. Changing this parameter by an order of
magnitude (between 2 and 20, where the standard value = 10) principally affects the slopes. A lower value leads to lower slopes, especially in fine domains, and to lower relief. In plan view, these changes translate into narrower coarse domains. Changing \( b \) affects the dependence on local bed composition of the effective height of the suspended-sediment concentration profile, \( H_{\text{eff}} \). Changing this parameter within an order of magnitude (0.005–0.05, standard value = 0.01) has little effect.

We have also performed experiments with different algorithms for treating non-local sediment transport (variations on Eq. (7)), and the results are not affected significantly. We have tried using sediment-entrainment functions (Eqs. (3) and (6)) that are not related to Eqs. (1) and (2), but are based on a near-bed reference concentration that is a function of a Shields parameter modified for the presence of ripples (Green and Black, 1999), and assuming a diffusive profile (Fredsoe and Diegaard, 1992). We used empirical formulae for ripple dimensions as a function of hydrodynamic conditions and grain size (e.g. Styles and Glenn, 2002). As long as the resulting bundle of parameterizations is consistent with our main hypothesis that sediment flux increases with bed coarseness, the results are qualitatively and quantitatively similar to those shown here, as can been seen by comparing Fig. 9 with Figs. 8(B) and (D).

5. Discussion, conclusions and future work

The parameterizations in this model represent simple first attempts to explore the pattern formation effects of the interactions hypothesized to be responsible for the formation and maintenance of sorted bedforms (Section 2). Clearly, field measurements of hydrodynamic conditions and sediment-concentration profiles over adjacent coarse and fine domains, and using these data to derive empirically based parameterizations of \( c_r \), \( H_{\text{eff,}r} \), and \( P_{s,i} \) generally would be helpful. Little weight should be put on the quantitative details of the present model results, since the scales and details of the pattern vary somewhat with changes in model parameters (Section 4.7).

However, the initial results presented in Section 4 do show that the hypotheses discussed in Section 2 offer a plausible explanation of the sedimentary features described in Section 1. For example, Figs. 7(D) and 8(D) show that the wavelength and widths of well developed features are within the range those of the observed pattern off Wrightsville Beach (Fig. 1). Cross sections in well-developed model patterns (Figs. 6(D), 7(D), 8 and 9) and in nature (Figs. 2(B) and (C)) both show relief on the order of 1 m, and in both cases the surficial coarse domain extends from trough to crest, rather than being centered on the topographic depression. In the model, the coarse domains are inclined upward in the direction of the most recent current.

Wind-driven currents from the northeast to the southwest, driven by mid-latitude cyclones, are dominant at Wrightsville Beach. The sense of asymmetry shown in Figs. 2(B) and (C) is...
consistent with forcing in this direction, when compared to the cross sections in Figs. 6–9. However, wind-driven currents at this location also occur in the opposite direction. The stratigraphy in Fig. 2(B) seems to indicate that the coarse domain has remained or reoccurred in the same location, suggesting that perhaps it experiences oscillatory migration, as occurs in the model with a symmetric reversing current. The asymmetric current regime at Wrightsville Beach (dominated by currents to the southwest) does not seem consistent with this conclusion, since in the model asymmetric currents lead to a net migration in the direction of the dominant current. However, the interpreted stratigraphy in Fig. 2(B) is based on a limited number of sediment cores, and does not conclusively establish whether the Wrightsville Beach sorted bedforms exhibit a net migration.

The stratigraphy in model cross sections, with coarse domains draped on top of well-sorted fine domains underlain by unsorted sediment (Figs. 6–9), is simpler than that shown in Fig. 2(B). This may reflect a longer history of more complex forcing sequences in the natural setting than in the model.

In the model runs presented here, wave conditions are held constant through time, and current conditions are either constant or periodically varying. Currents are always directed along a single axis. Such experiments show how the spacing, widths, and cross-sectional characteristics depend on environmental variables. In nature, however, wave and current conditions vary frequently and irregularly. If the natural features behave similarly to those in the model, their appearance likely does not reflect a steady state in equilibrium with the environmental forcing. This may be why the Wrightsville Beach features changed in appearance so drastically during the prolonged strong forcing from Hurricane Bonnie (Fig. 3); perhaps the “wispy” edges of the features pre-Bonnie reflect an approach to a smaller wavelength that would be in equilibrium with relatively less energetic conditions, and the larger, more clean-edged appearance post-Bonnie reflects the more-rapid reestablishment of larger scales created by rare, very high-energy events. We plan to investigate the effects of temporally varying conditions in future modeling experiments.

We plan to further develop the model by adopting non-periodic boundary conditions in the cross-shelf direction, so that the effect of the range of wave and current conditions across the domain in the natural system can be explored. If extended flow-transverse features still form in such a heterogeneous model environment, their appearance will likely be different. Near-bed orbital velocities and possibly mean current velocities will increase in shallower water, likely creating variations in plan-view and cross-sectional characteristics across the domain. Figs. 1 and 2 suggest such variation in the Wrightsville Beach features. In addition, the migration velocities will likely vary from shallow to deep water. Any features spanning a range of depths will therefore likely have an orientation that deviates from the shore-normal orientation that alongshelf currents would form in the present model. (Swift and Field (1981) suggested such a mechanism for the orientation of sand ridges.) Many natural examples of sorted bedforms exhibit such a deviation from a shore-normal orientation (see references in Table 1), including those at Wrightsville Beach (Fig. 1).

Constraining the current directions to one axis makes sense for nearshore “rippled scour depressions” such as those off of Wrightsville Beach and other locations (Table 1). However, large-scale sorted features also exist in other environments, such as on the shallow shelf environment off southwestern Florida (Twichell and Paskevich, 1999). In this area, farther removed from shore, currents are not constrained to move predominantly along a single axis. Even variations in a regime of linearly constrained currents cause a surprising variety of patterns (Section 4.3), and we plan to investigate the effects of more complex current patterns in future model experiments.

In the modeling effort presented here, we have included only suspended-load sediment transport, leaving out as many processes as possible in an effort to determine the minimum set of interactions that could cause the formation of sorted bedforms. We plan to add bedload transport, as predicted by the commonly used
Bagnold-Bowen-Bailard formula (e.g. Bailard and Inman, 1981). This formula includes the effects of wave asymmetries that will tend to enhance the cross-shelf component of the transport. Because bedload transport is more likely to be significant compared to suspended transport for coarse material than for fine, including this model of transport could lead to appreciable cross-shelf motions of coarse sediment (Cacchione and Drake, 1990). We will explore the possible effects of this process in future work.

The results presented here suggest that pre-existing cross-shelf-extended collections of coarse material, such as coarse outcrops of sediment from paleo-fluvial channels (e.g., Schwab et al., 2000; Thieler et al., 2001) are not necessary for sorted bedforms to develop. However, any such initial concentrations of coarse sediment will be reinforced by the mechanisms discussed here; our hypothesis is not in conflict with such origins for the features. However, our results do suggest that the initial conditions are not important; the characteristics of the flow-transverse bedforms in the model result not from the form of the initial perturbations, but from the subsequent self-organization processes.

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