HYDRODYNAMICS OF THE SCHELDT ESTUARY

Jacques C.J. NIHOUL and François C. RONDAY
Université de Liège, Belgium

Jean J. PETERS and André STERLING
Ministère des Travaux Publics, Belgium

I. HYDRODYNAMIC CHARACTERISTICS OF THE SCHELDT ESTUARY

I. 1. Introduction

The Scheldt Estuary is the Southern branch of the Rhine - Meuse - Scheldt delta. The natural evolution of the delta has been, to a large extent, influenced by man's activities : embanking, closing of arms and creation of artificial fresh water lakes.

Since the closing in 1867 of the connection with the Western Scheldt, the Scheur and the Oostgat are the only openings of the Eastern Scheldt to the sea. The mouth of the Rhine is situated 80 km to the North.

The drainage basin of the Scheldt and its tributaries covers some 21.580 km^2 in the North-West of France, the West of Belgium and the South-West of the Netherlands.

The flow of the Scheldt River is generally small while tidal motions in the estuary are large, producing a fairly good mixing of fresh and sea waters.

A map of the Scheldt Estuary (fig. 1) shows, downstream of Walsoorden, a complicated system of channels often referred to as "flood" and "ebb" channels according as the main water motion occurs during flood-tide or ebb-tide, respectively. (Such a classification is, in many cases oversimplified, especially for the Middelgat channel and the Gat Van Ossenisse channel between Terneuzen and Hansweert).

Upstream of Walsoorden, up to Gentbrugge, the river is characterized by a main channel, well defined, with, occasionally embryos of secondary channels upstream.

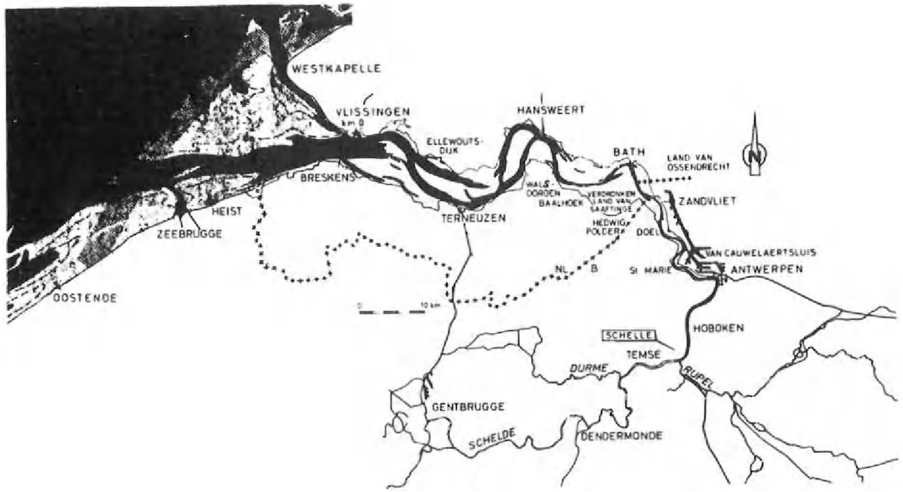


Fig. 1. Map of the Scheldt Estuary.

I. 2. Hydraulic and geometrical parameters of the Scheldt Estuary

Using the "hydraulic parameter" and the "geometrical parameter" introduced by Pritchard (Pritchard 1967) one can distinguish different zones in the Scheldt Estuary, characterized by different water and salt circulations.

The hydraulic parameter is the ratio of the volume of water flowing upstream the estuary through a given section during the flood tide (mentioned as "flood volume" in the figures) to the volume of fresh water flowing into the estuary upstream the section during a complete tidal cycle (mentioned as "fresh water volume" in the figures).

At Schelle, immediately downstream of the mouth of the Rupel and generally the upstream limit of the brackish water zone, the mean fresh water flow is estimated at some 100 m^3/s, corresponding roughly to a volume of $4,5.10^6 m^3$ evacuated over a tidal period. Monthly averaged fresh water flows less than 40 m^3/s or higher than 350 m^3/s at Schelle may be regarded as exceptional (Coen 1974).

The tide in the Scheldt estuary is essentially the lunar semi-diurnal tide M_2 with a period of 12 hours, 25 minutes. The tidal amplitude increases from 4 m in the open sea to 5 m at Hermiksen, upstream of Antwerpen and decreases then to values of the order of 2 m at Gentbrugge (Theuns and Coen 1973).

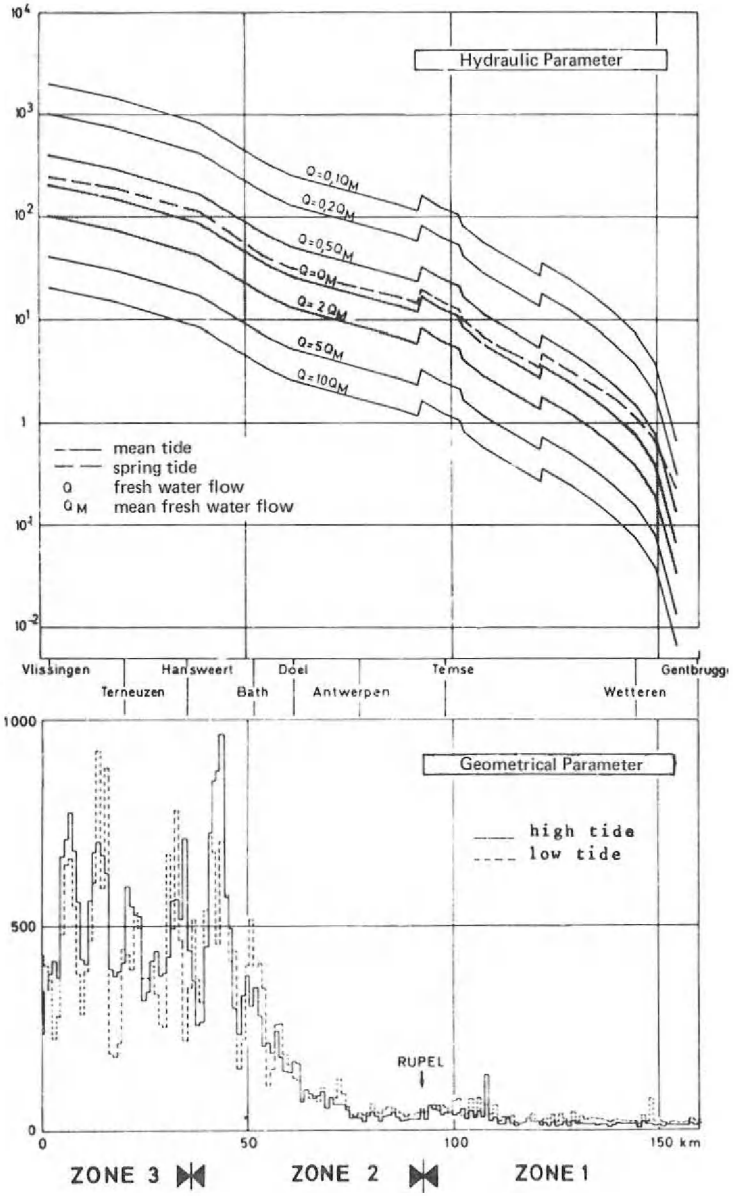


Fig. 2.

The hydraulic parameter varies considerably as a function of fresh water flow and position along the estuary (fig. 2). At a given section, for the same fresh water flow, it varies weakly with the tidal amplitude. For mean values of the fresh water flow and the tidal amplitude, the hydraulic ratio varies from 200 at Vlissingen to 100 near Hansweert (close to Walsoorden), 10 near the mouth of the Rupel and 1 or less at Gentbrugge.

The geometrical parameter is more difficult to define. In a rectangular channel the ratio of width to depth is meaningfull. In an irregular section of a shallow and wide estuary with many channels, sand-banks and tidal flats, the ratio of width to average cross-sectional depth varies with position along the estuary and with the tidal levels, sometimes very abruptly. Therefore the cross-sections were simplified and the variation of the width to mean ratio was considered between extreme water levels (fig. 2).

In the region of multiple channels, downstream of Walsoorden, this ratio oscillates between 100 and 1000 with an approximate mean value about 450. The irregular oscillations observed in the first 50 km from Vlissingen can be attributed to the existence of numerous sand-banks and shallow areas with a fairly complicated flow pattern around and over the banks and the possible formation of zones of "dead water". Upstream of Walsoorden, the width-depth ratio decreases rapidly and remains regularly small, of the order of 10 % of the mean downstreams values. In the region upstream of the Rupel tributary, the width to depth ratio loses much of its meaning. The water there is practically fresh water. The cross section is very small at low tide and the ratio may become artificially large where the depth becomes very small.

According to Pritchard's classification (Pritchard 1967), the Scheldt estuary can thus be divided in three zones.

(i) zone 1, from Gentbrugge to the mouth of the Rupel characterized by low values of both geometrical and hydraulic parameters.

(ii) zone 2, from the mouth of the Rupel to Walsoorden, characterized by intermediate values of the parameters indicating a partial mixing of the water.

(iii) zone 3, from Walsoorden to the sea, characterized by high values of the two parameters indicating a good mixing of the water (except perhaps during exceptional river flood periods in the winter when higher stratification occurs).

The three zones are indicated in fig. 3 which shows the variation of the wet section A(x) of the Scheldt at half-tide as a function of the distance x from Vlissingen.

I. 3. Distribution of salinity in the Scheldt Estuary.

Schematically, the salinity of the Scheldt Estuary varies from low values, characteristic of fresh water, near Gentbrugge to large values, characteristic of sea water, near Vlissingen.

The longitudinal salinity profile is of course a function of the fresh water flow which limits the intrusion of salt, but it also varies over a tidal period, being shifted upstream at flood-tide and downstream at ebb-tide. (For extremely small fresh water flows, the intrusion of salt may extend farther than the mouth of the Rupel; for extremely large fresh water flows, the salinity at Vlissingen may be appreciably smaller than the sea water value, indicating that a considerable amount of mixing still takes place beyond Vlissingen).

This situation is illustrated in fig. 4 : the shaded regions indicate the envelopes of the expected displacements of the longitudinal salinity profile over half a tidal period at respectively a typically large (left) and a typically small (right) flow of fresh water.

Lateral salinity gradients are generally negligible upstream of Walsoorden. The maximum salinity differences between the two banks of the river are observed between Walsoorden (∿ km 40) and the Dutch-Belgian border (∿ km 60) where occasionally, differences up to 4 °/₀₀ are found. In the mean, however, over a tidal period, the differences remain very small.

Fig. 5 shows in illustration the distribution of chlorinity observed on March 31st 1971 at slack water in a cross section of the Scheldt Estuary situated near the Dutch-Belgian border. One can see that there is a small lateral gradient.

Downstream of Walsoorden, the existence of flood and ebb channels separated by large banks and the associated complicated circulation pattern may produce lateral variations of salinity and, in particular, occasional salinity differences from one channel to the other.

Vertical salinity profiles show, in general, an increasing salinity from the surface to the bottom with strong irregular variations over a tidal cycle (fig. 6). The difference between mean surface and mean bottom salinity is generally less than 1 °/₀₀ with maximum values up to 2 $^{\circ}/_{\circ\circ}$ near the Dutch-Belgian border (De Pauw and Peters 1973).

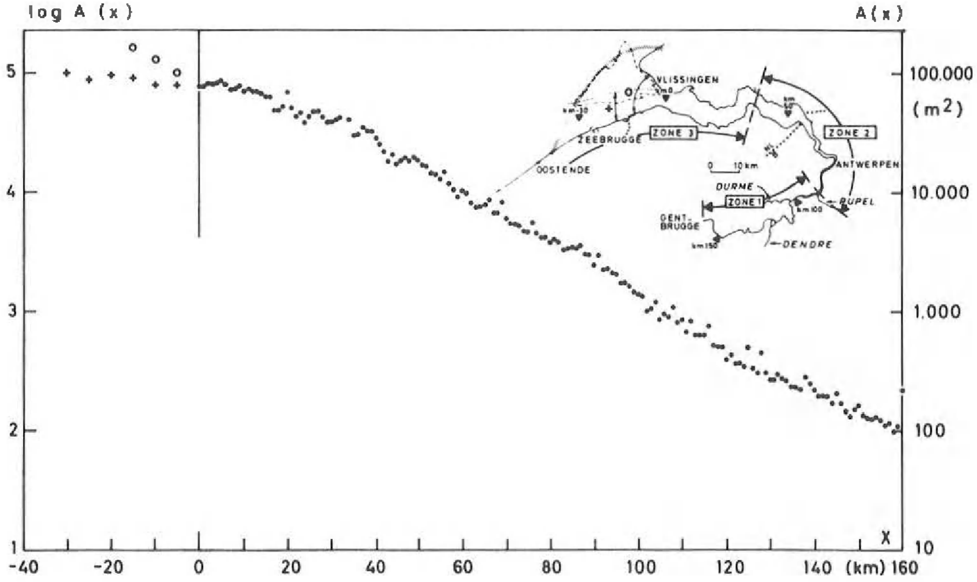


Fig. 3. : Longitudinal profile of the wet section of the Scheldt Estuary.

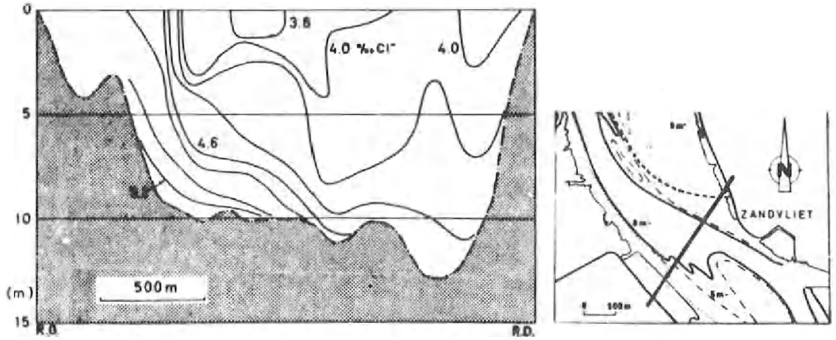


Fig. 4. : Envelopes of longitudinal salinity profiles in the Scheldt Estuary.

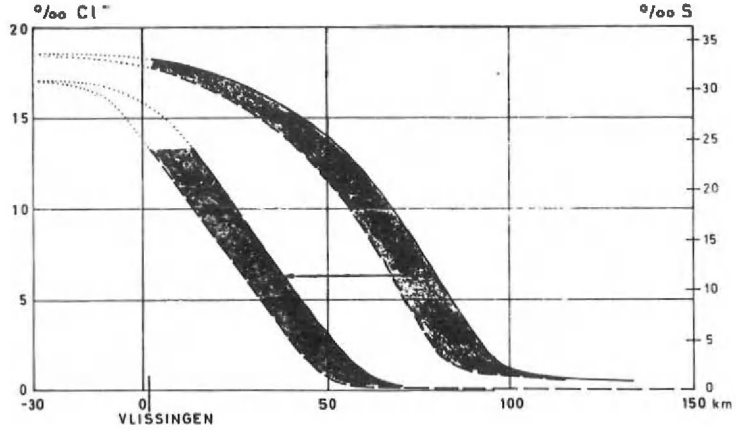
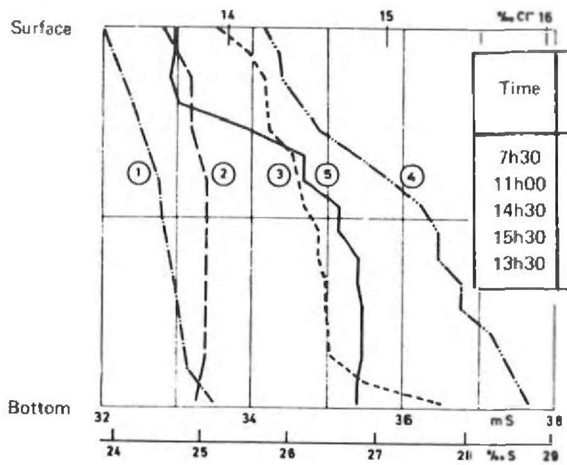


Fig. 5. : Distribution of chlorinity observed on March 31st 1971 at slack water in a cross section of the Scheldt Estuary near the Dutch-Belgian Border.



Time	Tide	Date	Position Longitude	Latitude
7h30	Flood	31.1.73	51°24'30" N	3°54'40"E
11h00	Flood	1.2.73	"	"
14h30	Ebb	31.1.73	"	"
15h30	Ebb	1.2.73	51°24'50" N	3°55'30"E
13h30	Ebb	30.1.73	51°24'30" N	3°54'40"E

Fig. 6. : Vertical salinity profiles in a section of the Scheldt Estuary situated in the Middelgat Channel (km 30).

The observations of the distribution of salinity in the Scheldt Estuary seem to confirm the existence of three distinct zones as suggested in section 2. :

(i) zone 1, upstream of the Rupel, is typically a region of fresh water of the Scheldt and its tributaries.

(ii) zone 2, between the Rupel and Walsoorden, is representative of a partially stratified estuary. The longitudinal and vertical salinity gradients are maximum in that region and more particularly near the Dutch-Belgian border ($\sim$ km 60) where the maximum variations of salinity over a tidal period are also observed.

(iii) zone 3, downstream of Walsoorden, is fairly well-mixed. Variations of salinity are small except occasionally in secondary channels and around sand banks.

I. 4. Tidal and residual currents in the Scheldt Estuary

The water circulation in the Scheldt Estuary is dominated by the tides. Tidal currents can reach values of 1 m/sec or more (fig. 7) while the order of magnitude of the residual currents (defined as the mean currents over one or several tidal periods) varies typically from 1 cm/sec at the mouth of the estuary to 10 cm/sec in the region of the mouth of the Rupel. Vertical variations may be important (Peters 1974).

The residual currents are due to the fresh water inflow upstream, the non-linear residue of tidal oscillations and the presence of salinity gradients. They will be discussed in more details in part III.

II. HYDRODYNAMIC MODELS APPLICABLE TO THE SCHELDT ESTUARY

II.1. Introduction

The dynamics of the Scheldt Estuary is determined by the presence, on the one hand, of strong tidal currents with subsequent turbulence and mixing, and by the existence on the other hand of salinity gradients and subsequent stratification and buoyancy effects.

As mentioned in part I, the estuary is only partially mixed, at least in zone 2, and the tri-dimensional hydrodynamic equations describing the estuary are, to begin with, more complicated that the equations on which mathematical models of well-mixed shallow continental seas are based.

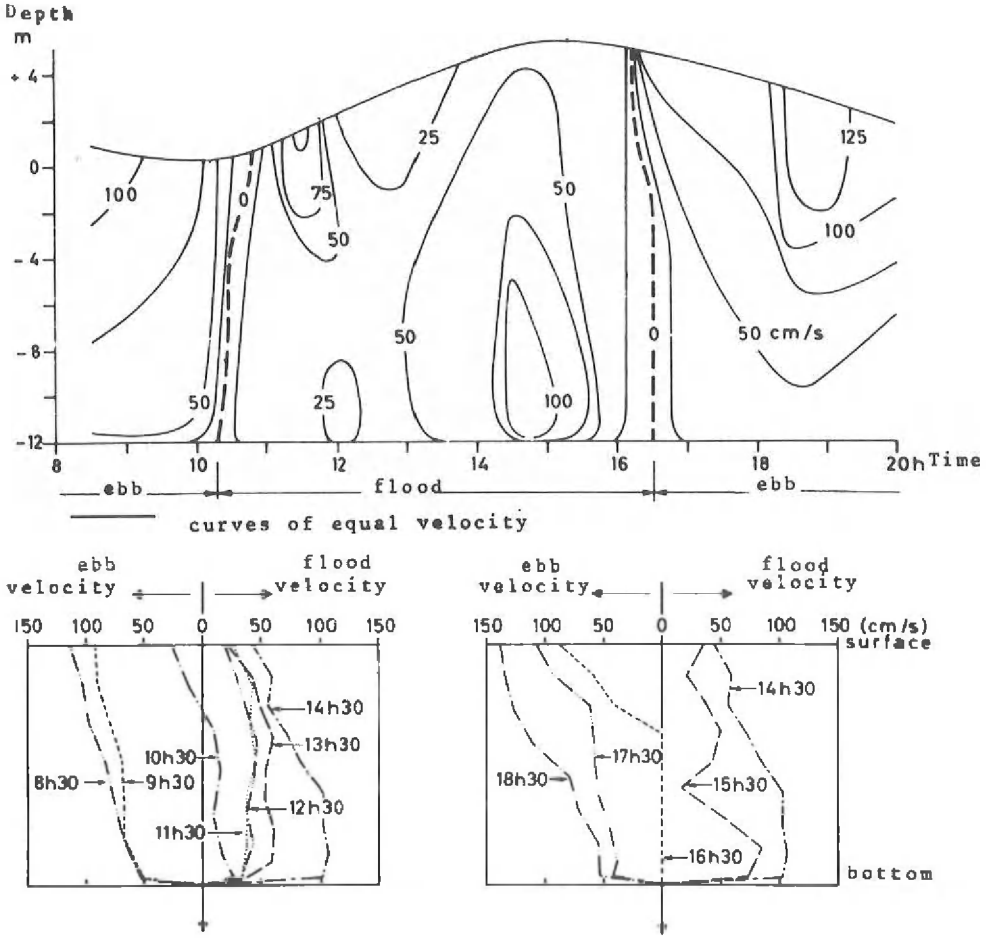


Fig. 7. : Current velocity profiles measured at a fixed station near the Dutch-Belgian border, Oct. 15th, 1970.

However, the morphology of the estuary (limited in width and depth and extending essentially streamwise) suggests different simplifications and it is often justified to work with two-dimensional and one-dimensional models integrated over the width or (and) the depth. Such models may be very different from one estuary to another ; particular characteristics of the river determining the important effects and those which can be neglected.

In the following, one discusses the hydrodynamic models applicable to zone 2 and zone 3 of the Scheldt estuary (the problem of zone 1 is less typical and can be treated by classical river mechanics). Simplifications will be made ; based on the examination of the orders of magnitude provided by the existing data bank for the Scheldt (e.g. Jäger 1973, Ronday 1975).

Clearly, they are amenable to permanent revision as the data bank is completed and refined and as simulation exercises, confronting models with observations, reveal other priorities.

II.2. Boussinesq equations

The Hydrodynamics of the Scheldt Estuary is determined by the coexistence of fresh water flow, strong tidal and wind currents generating turbulent fluctuations and mixing, and small but still significant salinity gradients, sources of density stratification and gravity effects.

It is obviously not possible - and not even desired - to describe the dynamics of the estuary in every detail reproducing micro-turbulent fluctuations, the time and length scales of which are small as compared to the main hydrodynamic processes (say, smaller than time scales of the order of 1 minute and length scales smaller than 1 meter while characteristic time scales of tidal motions are of the order of 1 hour or more and characteristic time scales of residual currents several orders of magnitude larger). The mathematical model is basically concerned with mean values which, from a theoretical point of view, may be defined as stochastic averaged but which, practically, are time averaged over a period of time (a couple of minutes, say, for the Scheldt) sufficiently large to eliminate the micro-turbulent fluctuations and sufficiently small to leave the main processes unaffected (Nihoul 1975).

The micro-turbulent fluctuations are not, however, completely eliminated from the equations by the averaging. They subsist in the nonlinear terms and their effect - which is essentially an enhanced dispersion, similar to the molecular diffusion but many times more efficient - must be parameterized with the help of the so-called "eddy diffusivities".

Although the water density of the Scheldt is not constant, its variations, associated essentially with the salinity gradients, are small and Boussinesq approximation can be applied.

Thus if $\underset{\sim}{u}$ is the mean velocity, p the mean pressure and a the mean (buoyancy being defined as the deviation of water density from a constant reference value, divided by that constant value and multiplied by the acceleration of gravity g) the Boussinesq equations can be written, the vertical axis $\underset{\sim}{e}_3$ pointing upwards (e.g. Nihoul and Ronday 1976)

$$\nabla \cdot \underset{\sim}{u} = 0 \tag{1}$$

$$\frac{\partial a}{\partial t} + \nabla \cdot \underset{\sim}{u}\, a = \frac{\partial}{\partial x_3} \left(\lambda \frac{\partial a}{\partial x_3}\right) \tag{2}$$

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \nabla \cdot \underset{\sim}{u}\underset{\sim}{u} + 2\, \underset{\sim}{\Omega} \wedge \underset{\sim}{u} = - \nabla q - a\, \underset{\sim}{e}_3 + \frac{\partial}{\partial x_3} \left(\nu \frac{\partial \underset{\sim}{u}}{\partial x_3} \right) \tag{3}$$

where λ is the eddy diffusivity, $\underset{\sim}{\Omega}$ the earth rotation vector, ν the eddy viscosity and where, p and ρ_m denoting respectively the pressure and the constant reference density,

$$q = \frac{p}{\rho_m} + g\, x_3$$

In eqs. (2) and (3) horizontal turbulent dispersion has been neglected because characteristic length scales of horizontal variations are always much larger than vertical length scales and this effect is negligible as compared to vertical turbulent dispersion. (One emphasizes here that, with an averaging time of the order of a minute, the turbulent dispersion discussed here is that of microturbulent fairly homogeneous and isotropic eddies and that the corresponding eddy diffusivities are comparable in the horizontal and the vertical directions. There is, of course, an important horizontal dispersion associated with variable and irregular currents but this effect is, in eqs. (2) and (3) included in the advection terms and will appear explicitly later).

II.3. Three-dimensional hydrodynamic models.

Eqs. (1), (2) and (3) form a system of five scalar equations for the five unknowns u_1, u_2, u_3, q and a.

In rectangular coordinates, appropriate to a rectilinear channel, they can be written :

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0 \tag{4}$$

$$\frac{\partial a}{\partial t} + \frac{\partial}{\partial x_1}(u_1\, a) + \frac{\partial}{\partial x_2}(u_2\, a) + \frac{\partial}{\partial x_3}(u_3\, a) = \frac{\partial}{\partial x_3}\left(\lambda \frac{\partial a}{\partial x_3}\right) \tag{5}$$

$$\frac{\partial u_1}{\partial t} + \frac{\partial}{\partial x_1}(u_1\, u_1) + \frac{\partial}{\partial x_2}(u_1\, u_2) + \frac{\partial}{\partial x_3}(u_1\, u_3) + 2\,(\Omega_2\, u_3 - \Omega_3\, u_2) = - \frac{\partial q}{\partial x_1} + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial u_1}{\partial x_3}\right) \tag{6}$$

$$\frac{\partial u_2}{\partial t} + \frac{\partial}{\partial x_1}(u_1 u_2) + \frac{\partial}{\partial x_2}(u_2 u_2) + \frac{\partial}{\partial x_3}(u_3 u_2) \qquad (7)$$

$$+ 2(\Omega_3 u_1 - \Omega_1 u_3) = -\frac{\partial q}{\partial x_2} + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial u_2}{\partial x_3}\right)$$

$$\frac{\partial u_3}{\partial t} + \frac{\partial}{\partial x_1}(u_1 u_3) + \frac{\partial}{\partial x_2}(u_2 u_3) + \frac{\partial}{\partial x_3}(u_3 u_3) \qquad (8)$$

$$+ 2(\Omega_1 u_2 - \Omega_2 u_1) = -\frac{\partial q}{\partial x_3} - a + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial u_3}{\partial x_3}\right)$$

For the Scheldt Estuary, where the radius of curvature R of the bends is always much larger than the characteristic length scale l_2 of transverse variations, it is readily shown (Nihoul et Ronday 1976) that these equations remain valid if the x_1-axis is taken along the central streamline of the river, the x_2-axis perpendicular to it in the transversal direction and the x_3-axis vertical (oriented upwards with the origin at the mean river level) provided the vertical component of the earth rotation vector is corrected to include the effect of curvature.

With a good approximation, it is sufficient to replace, in the equations, $2\Omega_3$ by

$$f = 2\,\Omega_3 - \frac{v_c}{R} \qquad (9)$$

where v_c is the circumferential velocity (Nihoul and Ronday 1976).

For a typical velocity v_c of 1m/sec and a radius of curvature of some 10 km, the two terms in the right-hand side of (8) are of the order 10^{-4} sec^{-1} and thus

$$2\,\Omega_1 \sim 2\,\Omega_2 \sim 2\,\Omega_3 \sim f \sim 10^{-4}\ sec^{-1}$$

In rectilinear sections, R is infinite and the correction vanishes.

One can estimate the orders of magnitude of the different terms of eqs. (4) to (8) for different values of the variables within the range of observations in the Scheldt. One finds that several terms are always much smaller than the others and can be neglected.

This can be shown simply by considering a typical case.

Let, in MKS units, (Jäger 1973) :

$u_1 \sim 1;\ u_2 \sim 3\cdot 10^{-2};\ u_3 \sim 3\cdot 10^{-4};\ a \sim 10^{-1}$

be characteristic values of the velocity components and the buoyancy and

$l_1 \sim 3 \cdot 10^4;\ l_2 \sim 10^3;\ l_3 \sim 10;\ t_c \sim 10^4 \sim f^{-1}$

characteristic length scales and time scales.

The Richardson number (measuring the importance of the vertical stratification) is found to be

$$Ri = \frac{\dfrac{\partial a}{\partial x_3}}{\left\| \dfrac{\partial \underset{\sim}{u}}{\partial x_3} \right\|^2} \sim 1 \qquad (10)$$

The separation between the mean values $\underset{\sim}{u}$, a and the micro-turbulent fluctuations is effected by integration over a period of time τ of the order of one or two minutes ($\tau \sim 10^2$).

According to Kolmogorov theory, the corresponding eddy viscosity, in a non stratified medium, is given by

$$\nu_0 \sim \varepsilon\, \tau^2 \qquad (11)$$

where ε is the rate of turbulent energy transfer from the larger eddies to the smaller ones where dissipation takes place.

In stratified waters, the eddy viscosity ν and the eddy diffusivity λ depend on the Richardson number. One can write (Munk and Anderson 1948)

$$\nu = \frac{\nu_0}{(1 + 10\ Ri)^{1/2}} \qquad (12)$$

$$\lambda = \frac{\nu_0}{(1 + 3{,}33\ Ri)^{3/2}} \qquad (13)$$

Taking $\varepsilon \sim 10^{-5}$ as a typical value for the Scheldt Estuary (Nihoul et Ronday 1976), one finds, using (10)

$\nu \sim 3 \cdot 10^{-2} m^2/sec$
$\lambda \sim 10^{-2}\ m^2/sec$

These values are in good agreement with observations (e.g. Bowden 1965, Fisher 1972).

Comparing orders of magnitude, one finds (Nihoul and Ronday 1976) :

(i) that all terms in the left-hand side of eq.(8) and the vertical turbulent diffusion in the right-hand side are completely negligible as compared to buoyancy which can only be balanced by the vertical gradient of q , i.e.

$$a = -\frac{\partial q}{\partial x_3} \qquad (14)$$

(ii) that the transverse gradient of q is essentially balanced by Coriolis and eventual curvature effects and, to a smaller extent, by turbulent diffusion and time variations of the transverse velocity, yielding

$$\frac{\partial q}{\partial x_2} \lesssim 10^{-4} \quad \text{(MKS)} \qquad (15)$$

(iii) that, in eq.(6), to the contrary, earth rotation and curvature effects can be neglected as compared to turbulent diffusion.

To estimate the longitudinal gradient of q, it is convenient to write

$$q = q_l + q_v + q_t \qquad (16)$$

where q_1 is the cross-section average of q and depends on x_1 and t only ; q_v is the width-average of $q - q_1$ and depends on x_1 , x_3 and t ; q_t is equal to $q - q_1 - q_v$ and depends on the four variables x_1, x_2 , x_3 and t.

Substituting in (14) and (15), one gets

$$\frac{\partial q_t}{\partial x_2} \lesssim 10^{-4} : q_t \lesssim 10^{-1}$$

$$a = -\frac{\partial q_v}{\partial x_3} - \frac{\partial q_t}{\partial x_3}$$

$$\sim -\frac{\partial q_v}{\partial x_3}$$

$$q_v \sim 0\,(l_3\, a) \sim 1$$

An estimate of q_1 can be based on the value of q at the free surface

$$q = \frac{p_a}{\rho_m} + g\,\zeta \qquad (17)$$

where ζ is the surface elevation.

For elevations of the order of 1 m,

$$q_1 \sim 10; \ \frac{\partial q}{\partial x_1} \sim \frac{\partial q_1}{\partial x_1} \sim 3\cdot 10^{-4}$$

To sum up, the equations of a three-dimensional hydrodynamic model of the Scheldt Estuary can be written

$$\nabla \cdot \underset{\sim}{u} = 0 \tag{18}$$

$$a = -\frac{\partial q}{\partial x_3} \tag{19}$$

$$\frac{\partial a}{\partial t} + \nabla \cdot \underset{\sim}{u}a = \frac{\partial}{\partial x_3}\left(\lambda \frac{\partial a}{\partial x_3}\right) \tag{20}$$

$$\frac{\partial u_1}{\partial t} + \nabla \cdot \underset{\sim}{u}u_1 = -\frac{\partial q}{\partial x_1} + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial u_1}{\partial x_3}\right) \tag{21}$$

$$\frac{\partial u_2}{\partial t} + fu_1 = -\frac{\partial q}{\partial x_2} + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial u_2}{\partial x_3}\right) \tag{22}$$

Eqs. (18) to (22) form a complete system of five scalar equations for the five unknowns u_1, u_2, u_3, a, q.

II.4. Two-dimensional models assuming transversal homogeneity

Early models of estuaries traditionally assumed a sufficiently good transversal homogeneity to neglect, in the equations, all terms containing derivatives with respect to x_2 (e.g. Pritchard 1967).

These terms, in eqs. (18), (20) and (21) also contain the transverse velocity u_2 and the hypothesis amounts to considering that, in section II.3., either the velocity u_2 has been overestimated or the characteristic length scale l_2 has been underestimated.

If this is the case, eq. (18) reduces to

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} = 0 \tag{23}$$

and one can define a stream function ψ such that

$$u_1 = \frac{\partial \psi}{\partial x_3} \tag{24}$$

$$u_3 = -\frac{\partial \psi}{\partial x_1} \tag{25}$$

The hydrodynamic equations can then be reduced to 3, i.e.

$$a = -\frac{\partial q}{\partial x_3} \tag{26}$$

$$\frac{\partial a}{\partial t} + \frac{\partial \psi}{\partial x_3}\frac{\partial a}{\partial x_1} - \frac{\partial \psi}{\partial x_1}\frac{\partial a}{\partial x_3} = \frac{\partial}{\partial x_3}\left(\lambda \frac{\partial a}{\partial x_3}\right) \quad (27)$$

$$\frac{\partial^2 \psi}{\partial t\, \partial x_3} + \frac{\partial \psi}{\partial x_3}\frac{\partial^2 \psi}{\partial x_3\, \partial x_1} - \frac{\partial \psi}{\partial x_1}\frac{\partial^2 \psi}{\partial x_3^{\,2}} = -\frac{\partial q}{\partial x_1} + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial^2 \psi}{\partial x_3^{\,2}}\right) \quad (28)$$

for the three variables ψ, a, q ; the latest being also regarded as a function of t, x_1 and x_3 only.

Eq. (22) is ignored in this type of model as it provides a useless relation between uninteresting variables u_2 and q_t. However, a closer examination of that equation shows that one must take such models with some reservation.

Indeed fu_1 reaches typical values of order 10^{-4} and must be balanced by another term at least. If l_2 is larger than estimated (say, 10^4), either q_t is larger ($q_t \sim 1$ instead of 10^{-1}) or u_2 is larger. In the first case, q_t is a non negligible contribution to q in equations like (21), and one may not assume that q does not depend on x_2. In the second case, an increase of u_2 compensating the decrease of l_2 , it is likely that terms containing $\frac{\partial u_2}{\partial x_2}$ or $u_2 \frac{\partial}{\partial x_2}$ cannot be neglected.

The model would remain valid if u_2 was smaller than estimated ($u_2 << 3\ 10^{-2}$) but observations indicate that, in the Scheldt Estuary, this condition is far from being realized in many places.

II.5. Width-integrated models

If one is not primarily interested in transverse variations, it is advantageous to integrate the evolution equations from one bank to the other and to study the longitudinal and vertical variations of mean or integrated values over the width of the estuary.

Let

$$x_2 = D\,(x_1, x_3) \quad (29)$$

and

$$x_2 = -\Delta\,(x_1\ x_3) \quad (30)$$

be the equations of the banks.

The banks must be stream surfaces. Thus

$$u_1 \frac{\partial D}{\partial x_1} + u_3 \frac{\partial D}{\partial x_3} = u_2 \qquad \text{at} \qquad x_2 = D \tag{31}$$

$$u_1 \frac{\partial \Delta}{\partial x_1} + u_3 \frac{\partial \Delta}{\partial x_3} = -\, u_2 \qquad \text{at} \qquad x_2 = -\,\Delta \tag{32}$$

Let

$$V_1 = L\, \bar{u}_1 = \int_{-\Delta}^{D} u_1 \, dx_2 \tag{33}$$

$$V_3 = L\, \bar{u}_3 = \int_{-\Delta}^{D} u_3 \, dx_2 \tag{34}$$

$$A = L\, \bar{a} = \int_{-\Delta}^{D} a \, dx_2 \tag{35}$$

where $\bar{y}$ is the width-averaged of $y (y = u_1, u_2, a)$ and where

$$L = \Delta + D \tag{36}$$

Setting $\tilde{y} = y - \bar{y}$ $(y = u_1, u_2, a)$, one writes

$$u_1 = \overline{u_1} + \tilde{u}_1 \tag{37}$$

$$u_3 = \overline{u_3} + \tilde{u}_3 \tag{38}$$

$$a = \bar{a} + \tilde{a} \tag{39}$$

Integrating eqs. (18) and (20) from $-\,\Delta$ to D, one obtains

$$\begin{aligned} \int_{-\Delta}^{D} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right) dx_2 + (u_2)_D - (u_2)_\Delta &= \frac{\partial V_1}{\partial x_1} - (u_1)_D \frac{\partial D}{\partial x_1} - (u_1)_\Delta \frac{\partial \Delta}{\partial x_1} + \frac{\partial V_3}{\partial x_3} \\ - (u_3)_D \frac{\partial D}{\partial x_3} - (u_3)_\Delta \frac{\partial \Delta}{\partial x_3} + (u_2)_D - (u_2)_\Delta &= \frac{\partial V_1}{\partial x_1} + \frac{\partial V_3}{\partial x_3} = 0 \end{aligned} \tag{40}$$

$$\begin{aligned} &\int_{-\Delta}^{D} \left\{ \frac{\partial a}{\partial t} + \frac{\partial}{\partial x_1}(u_1 a) + \frac{\partial}{\partial x_3}(u_3 a) \right\} dx_2 + (u_2 a)_D - (u_2 a)_\Delta - \int_{-\Delta}^{D} \left(\frac{\partial}{\partial x_3} \lambda \frac{\partial a}{\partial x_3} \right) dx_2 \\ &= \frac{\partial A}{\partial t} + \frac{\partial}{\partial x_1} \int_{-\Delta}^{D} u_1 a \, dx_2 - (u_1 a)_D \frac{\partial D}{\partial x_1} - (u_1 a)_\Delta \frac{\partial \Delta}{\partial x_1} + \frac{\partial}{\partial x_3} \int_{-\Delta}^{D} u_3 a \, dx_2 \\ &\quad - (u_3 a)_D \frac{\partial D}{\partial x_3} - (u_3 a)_\Delta \frac{\partial \Delta}{\partial x_3} + (u_2 a)_D - (u_2 a)_\Delta - \frac{\partial}{\partial x_3} \left(\lambda \frac{\partial A}{\partial x_3} \right) + \frac{\partial}{\partial x_3} \left[\lambda \left(a_D \frac{\partial D}{\partial x_3} + a_\Delta \frac{\partial \Delta}{\partial x_3} \right) \right] \\ &\quad + \left(\lambda \frac{\partial a}{\partial x_3} \right)_D \frac{\partial D}{\partial x_3} + \left(\lambda \frac{\partial a}{\partial x_3} \right)_\Delta \frac{\partial \Delta}{\partial x_3} = \frac{\partial A}{\partial t} + \frac{\partial}{\partial x_1} (L^{-1} V_1 A) + \frac{\partial}{\partial x_1} \int_{-\Delta}^{D} \tilde{u}_1 \tilde{a} \, dx_2 \\ &\quad + \frac{\partial}{\partial x_3} (L^{-1} V_3 A) + \frac{\partial}{\partial x_3} \int_{-\Delta}^{D} \tilde{u}_3 \tilde{a} \, dx_3 - \frac{\partial}{\partial x_3} \left(\lambda \frac{\partial A}{\partial x_3} \right) = 0 \end{aligned} \tag{41}$$

underlined terms cancelling as a result of (31) and (32) and the last four terms of (41) being negligible as compared to $\frac{\partial}{\partial x_3}(\lambda \frac{\partial A}{\partial x_3})$. Indeed :

$$\frac{\partial}{\partial x_3}\left(\lambda \frac{\partial A}{\partial x_3}\right) \sim 0\left(\frac{\lambda a L}{l_3^{2}}\right) \sim 10^{-2}$$

While, for instance

$$\left(\lambda \frac{\partial a}{\partial x_3}\right)_D \frac{\partial D}{\partial x_3} \sim 0\left(\frac{\lambda a}{l_3} \frac{\partial D}{\partial x_3}\right) \sim 10^{-4}$$

assuming a maximum of 10 meters of width variation over depth.

Similarly, integrating eq. (21), one obtains

$$\frac{\partial V_1}{\partial t} + \frac{\partial}{\partial x_1}(L^{-1} V_1 V_1) + \frac{\partial}{\partial x_3}(L^{-1} V_1 V_3) + \frac{\partial}{\partial x_1}\int_{-\Delta}^{D} \tilde{u}_1 \tilde{u}_1 \, dx_2 + \frac{\partial}{\partial x_3}\int_{-\Delta}^{D} \tilde{u}_1 \tilde{u}_3 \, dx_2 = -L \frac{\partial \bar{q}}{\partial x_1} + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial V_1}{\partial x_3}\right) \tag{42}$$

q_v being negligible, the approximation $q \sim \bar{q}$ has been made. This simplification results from eq. (22) which plays no other role in the integrated model.

Integrating eq. (19), one finds, with the same approximation

$$A = -L \frac{\partial \bar{q}}{\partial x_3} \tag{43}$$

One can see that in eqs (41) and (42), integrals of products of deviations around the mean appear. Observations reveal that these terms (the structure of which is reminiscent of the Reynolds stresses) are responsible of longitudinal and vertical dispersion similar to turbulent dispersion but considerably more efficient.

On the model of turbulent diffusion, one sets

$$\frac{\partial}{\partial x_1}\int_{-\Delta}^{D} (-\tilde{u}_1 \tilde{a}) \, dx_2 = \frac{\partial}{\partial x_1}\left(\Lambda_1 \frac{\partial A}{\partial x_1}\right) \tag{44}$$

$$\frac{\partial}{\partial x_3}\int_{-\Delta}^{D} (-\tilde{u}_3 \tilde{a}) \, dx_2 + \frac{\partial}{\partial x_3}\left(\lambda \frac{\partial A}{\partial x_3}\right) = \frac{\partial}{\partial x_3}\left(\Lambda_3 \frac{\partial A}{\partial x_3}\right) \tag{45}$$

$$\frac{\partial}{\partial x_1}\int_{-\Delta}^{D} (-\tilde{u}_1 \tilde{u}_1) \, dx_2 = \frac{\partial}{\partial x_1}\left(N_1 \frac{\partial V_1}{\partial x_1}\right) \tag{46}$$

$$\frac{\partial}{\partial x_3}\int_{-\Delta}^{D} (-\tilde{u}_1 \tilde{u}_3) \, dx_2 + \frac{\partial}{\partial x_3}\left(\nu \frac{\partial V_1}{\partial x_3}\right) = \frac{\partial}{\partial x_3}\left(N_3 \frac{\partial V_1}{\partial x_3}\right) \tag{47}$$

where N_1, N_3, Λ_1, Λ_3 are new diffusivity coefficients.

According to observations, (e.g. Ronday 1975)[1] :

$N_1 \sim 10^2$ à 10^3 m^2/s; $\Lambda_1 \sim 10^2$ m^2/s

$N_3 \sim 10^{-2}$ à 10^{-3} m^2/s; $\Lambda_3 \sim 10^{-2}$ m^2/s

Eq. (40) suggests the definition of a stream function such that

$$V_1 = \frac{\partial \psi}{\partial x_3} \tag{48}$$

$$V_3 = -\frac{\partial \psi}{\partial x_1} \tag{49}$$

Eq. (40) is then identically verified. Eqs. (41), (42) and (43) can be written

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_1}\left(L^{-1}\frac{\partial \psi}{\partial x_3} A\right) - \frac{\partial}{\partial x_3}\left(L^{-1}\frac{\partial \psi}{\partial x_1} A\right) = \frac{\partial}{\partial x_1}\left(\Lambda_1 \frac{\partial A}{\partial x_1}\right) + \frac{\partial}{\partial x_3}\left(\Lambda_3 \frac{\partial A}{\partial x_3}\right) \tag{50}$$

$$\frac{\partial^2 \psi}{\partial t\, \partial x_3} + \frac{\partial}{\partial x_1}\left(L^{-1}\frac{\partial \psi}{\partial x_3}\frac{\partial \psi}{\partial x_3}\right) - \frac{\partial}{\partial x_3}\left(L^{-1}\frac{\partial \psi}{\partial x_3}\frac{\partial \psi}{\partial x_1}\right) = -L\frac{\partial \bar{q}}{\partial x_1} + \frac{\partial}{\partial x_1}\left(N_1 \frac{\partial^2 \psi}{\partial x_1\, \partial x_3}\right) + \frac{\partial}{\partial x_3}\left(N_3 \frac{\partial^2 \psi}{\partial x_3^2}\right) \tag{51}$$

$$A = -L\frac{\partial \bar{q}}{\partial x_3} \tag{52}$$

They form a complete system for the three variables ψ , A and $\bar{q}$.

II.6. Depth-integrated models

Depth-integrated models have been extensively used in shallow continental seas hydrodynamics (e.g. Nihoul 1975). In the case of a partially stratified estuary like the Scheldt Estuary (in zone 2, at least), their utility is less evident. On the one hand, they tend to mask the effects of vertical stratification which, in the gravity field, often play an important role. On the other hand, they are often more difficult to handle than width-integrated models. For instance, although both types of models are two-dimensional as a result of the integration over one space coordinate, in depth-integrated models, one cannot introduce a stream function and subsequently reduce the number of variables if the system is not in a steady state.

---

[1] N_1, N_3, Λ_1 and Λ_3 are in general not constant along the estuary. They must be regarded as control parameters to be determined by theoretical reflexion and simulation exercises based on the data bank.

Indeed, if $x_3 = \zeta$ and $x_3 = -h$ are the equations of the surface and the bottom, respectively, one must have

$$\frac{\partial \zeta}{\partial t} + u_1 \frac{\partial \zeta}{\partial x_1} + u_2 \frac{\partial \zeta}{\partial x_2} = u_3 \quad \text{in} \quad x_3 = \zeta \tag{53}$$

$$u_1 \frac{\partial h}{\partial x_1} + u_2 \frac{\partial h}{\partial x_2} = -u_3 \quad \text{in} \quad x_3 = -h \tag{54}$$

Defining

$$U_1 = \int_{-h}^{\zeta} u_1 \, dx_3 \tag{55}$$

$$U_2 = \int_{-h}^{\zeta} u_2 \, dx_3 \tag{56}$$

and integrating (18) from $-h$ to ζ, one obtains

$$\int_{-h}^{\zeta} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right) dx_3 + (u_3)_\zeta - (u_3)_{-h} = \frac{\partial U_1}{\partial x_1} - (u_1)_\zeta \frac{\partial \zeta}{\partial x_1} - (u_1)_{-h} \frac{\partial h}{\partial x_1} + (u_3)_\zeta$$
$$+ \frac{\partial U_2}{\partial x_2} - (u_2)_\zeta \frac{\partial \zeta}{\partial x_2} - (u_2)_{-h} \frac{\partial h}{\partial x_2} - (u_3)_{-h} = \frac{\partial \zeta}{\partial t} + \frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial x_2} = 0 \tag{57}$$

using (53) and (54).

In wide, well-mixed estuaries with flood - and ebb - channels, like zone 3 of the Scheldt, it may be interesting to combine, to the width-integrated model, a depth-integrated model to have additional information on the lateral distribution of velocity.

In partially mixed estuaries, depth-integration is only advantageous if the vertical distribution of buoyancy is known from observations. Otherwise, the integration over depth, transforming partial differential equations into integral equations*, does not lead to any real simplification, even if it appears to reduce the number of variables.

II.7. Cross-section integrated models

In a first approach, when one is mainly interested in longitudinal variations, one may use a cross-section integrated model.

The operation of integration over the cross-section is similar, mathematically, to the integration over the width and the details will not be reproduced here.

---

* because
$$q = \frac{p_a}{\rho_m} + g\zeta + \int_{x_3}^{\zeta} a \, d x_3$$

The final one-dimensional equations appropriate to the Scheldt Estuary are given in (Nihoul and Ronday 1976).

III. RESIDUAL CIRCULATION AND SILT DEPOSITION IN THE SCHELDT ESTUARY

III. 1. Flocculation and variations of the suspended load in the Scheldt Estuary

Critical salinity values of 1 to 5 °/₀₀ are typically found in the central reach of the estuary (fig. 4). At such values of the salinity, intense flocculation and sedimentation take place and the suspended load decreases abruptly. (There is about one order of magnitude difference between the values of turbidity and COD upstream and downstream of the critical region).

Variations of the river flow rate Q and the subsequent displacement of the region of critical salinity reflect on the sedimentation and tend to spread the deposition of suspended matter over most of zone 2 (fig. 8).

It is found however that bottom sediments tend to accumulate in a narrower region than one might expect and this is explained in the following with the help of a two-dimensional width-integrated residual circulation model.

III. 2. Model of residual circulation

The time-dependent width-integrated hydrodynamic equations have been derived in section II.5. If V_1, V_2 and A represent respectively the integrals of u_1, u_2 and a over the width, these equations can be written

$$\frac{\partial V_1}{\partial x_1} + \frac{\partial V_3}{\partial x_3} = 0 \qquad (58)$$

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_1}(L^{-1} V_1 A) + \frac{\partial}{\partial x_3}(L^{-1} V_3 A) = \frac{\partial}{\partial x_1}\left(\Lambda_1 \frac{\partial A}{\partial x_1}\right) + \frac{\partial}{\partial x_3}\left(\Lambda_3 \frac{\partial A}{\partial x_3}\right) \qquad (59)$$

$$A = - L \frac{\partial \bar{q}}{\partial x_3} \qquad (60)$$

$$\frac{\partial V_1}{\partial t} + \frac{\partial}{\partial x_1}(L^{-1} V_1 V_1) + \frac{\partial}{\partial x_3}(L^{-1} V_1 V_3) = - L \frac{\partial \bar{q}}{\partial x_1} + \frac{\partial}{\partial x_1}\left(N_1 \frac{\partial V_1}{\partial x_1}\right) + \frac{\partial}{\partial x_3}\left(N_3 \frac{\partial V_1}{\partial x_3}\right) \qquad (61)$$

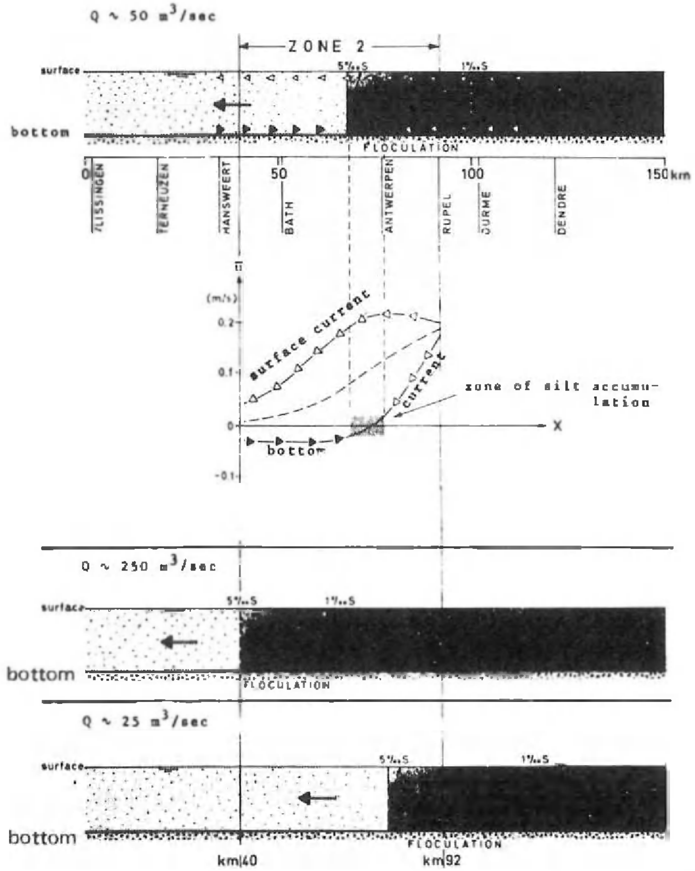


Fig. 8. : Flocculation in the Scheldt Estuary for different river flows.

The residual circulation can be defined as the mean circulation over a period of time sufficiently large to eliminate tidal oscillations and transitory wind induced currents.

The residual circulation is essentially related to the river flow rate and to gravity currents associated with the stratification. A width-integrated model is thus appropriate. The residual width-integrated equations are derived from eqs. (58) to (61) by further averaging over time. In this process, time derivatives are replaced by finite differences which can be made as small as desired by increasing the period of time over which the averages are taken. Thus if the residual currents are defined over a sufficiently long period of time, they can be described by steady state equations.

If < >indicates a time average and if

$$U_i = \langle V_i \rangle \qquad i = 1,2 \tag{62}$$

$$V_i = U_i + W_i \qquad i = 1,2 \tag{63}$$

$$B = \langle A \rangle \tag{64}$$

$$A = B + C \tag{65}$$

$$\pi = \langle \bar{q} \rangle \tag{66}$$

one gets, integrating eq. (58) to (61) over time :

$$\frac{\partial U_1}{\partial x_1} + \frac{\partial U_3}{\partial x_3} = 0 \tag{67}$$

$$\frac{\partial}{\partial x_1}(L^{-1} U_1 B) + \frac{\partial}{\partial x_3}(L^{-1} U_3 B) = \frac{\partial}{\partial x_1}\left\{ \Lambda_1 \frac{\partial B}{\partial x_1} + L^{-1} \langle - W_1 C \rangle \right\} + \frac{\partial}{\partial x_3}\left\{ \Lambda_3 \frac{\partial B}{\partial x_3} + L^{-1} \langle - W_3 C \rangle \right\} \tag{68}$$

$$\frac{\partial}{\partial x_1}(L^{-1} U_1 U_1) + \frac{\partial}{\partial x_3}(L^{-1} U_1 U_3) = - L \frac{\partial \pi}{\partial x_1} + \frac{\partial}{\partial x_1}\left\{ N_1 \frac{\partial U_1}{\partial x_1} + L^{-1} \langle - W_1 W_1 \rangle \right\} + \frac{\partial}{\partial x_3}\left\{ N_3 \frac{\partial U_1}{\partial x_3} + L^{-1} \langle - W_3 W_1 \rangle \right\} \tag{69}$$

$$B = - L \frac{\partial \pi}{\partial x_3} \tag{70}$$

One can see that the average of the quadratic terms gives again two contributions ; the first of which containing the product of the means and the second the mean product of the fluctuations.

The latter contributes to the dispersion and combines to turbulent mixing and shear effect. In a first approach, one can parameterize the general effect of the dispersion processes, on the model of turbulent dispersion, introducing new dispersion coefficients K_1, K_3, M_1, M_3 such that

$$\Lambda_1 \frac{\partial B}{\partial x_1} + L^{-1} \langle - W_1 C \rangle = K_1 \frac{\partial B}{\partial x_1} \qquad (71)$$

$$\Lambda_3 \frac{\partial B}{\partial x_3} + L^{-1} \langle - W_3 C \rangle = K_3 \frac{\partial B}{\partial x_3} \qquad (72)$$

$$N_1 \frac{\partial U_1}{\partial x_1} + L^{-1} \langle - W_1 W_1 \rangle = M_1 \frac{\partial U_1}{\partial x_1} \qquad (73)$$

$$N_3 \frac{\partial U_1}{\partial x_3} + L^{-1} \langle - W_3 W_1 \rangle = M_3 \frac{\partial U_1}{\partial x_3} \qquad (74)$$

with, according to observations (MKS) (e.g.Ronday 1975) :

$$K_1 \sim 10^2;\ K_3 \sim 10^{-2};\ M_1 \sim 10^3;\ M_3 \sim 10^{-1}$$

K_1, K_3, M_1 and M_3 must be regarded as control parameters to be determined by inspection of the data bank. In general, they are functions of x_1 even if, in first approximation, they can be assumed independent of x_3.

If l_1 and l_3 are characteristic length scales of respectively horizontal and vertical variations of the velocity field, eq. (67) gives

$$\frac{U_1}{l_1} \sim \frac{U_3}{l_3}$$

In eq. (69), the terms in the left-hand side are of the order

$$\frac{L^{-1} U_1 U_1}{l_1}$$

The second term of the right-hand side is of the order

$$\frac{M_1 U_1}{l_1^2}$$

while the last term of the right hand side is of the order

$$\frac{M_3 U_1}{l_3^2}$$

Taking (Ronday 1975),

$$l_3 \sim 10;\ l_1 \sim 10^4;\ L^{-1} U_1 \sim 10^{-2}$$

one can see that vertical dispersion is several orders of magnitude larger than advection and horizontal dispersion and must accordingly be balanced by $- L \frac{\partial \pi}{\partial x_1}$.

A similar simplification is not, in general, possible for eq.(68) which does not contain a term analogous to the pressure gradient and where vertical dispersion is comparatively less important ; vertical salinity gradients being smaller than vertical velocity gradients in many cases.

Eq. (67) suggests the introduction of a residual stream function ψ_o such that :

$$U_1 = \frac{\partial \psi_o}{\partial x_3} \qquad (75)$$

$$U_3 = - \frac{\partial \psi_o}{\partial x_1} \qquad (76)$$

Eliminating U_1, U_3 and B, one obtains

$$\frac{\partial \psi_o}{\partial x_3} \frac{\partial^2 \pi}{\partial x_1 \partial x_3} - \frac{\partial \psi_o}{\partial x_1} \frac{\partial^2 \pi}{\partial x_3{}^2} = \frac{\partial}{\partial x_1} \left\{ K_1 \frac{\partial}{\partial x_1} \left(L \frac{\partial \pi}{\partial x_3} \right) \right\} + \frac{\partial}{\partial x_3} \left\{ K_3 \frac{\partial}{\partial x_3} \left(L \frac{\partial \pi}{\partial x_3} \right) \right\} \qquad (77)$$

$$\frac{\partial}{\partial x_3} \left(M_3 \frac{\partial^2 \psi_o}{\partial x_3{}^2} \right) = L \frac{\partial \pi}{\partial x_1} \qquad (78)$$

Eqs. (77) and (78) form a complete system for the two variables ψ_o and π.

III. 3. Application to the deposition of silt in the Scheldt Estuary

The residual circulation model has been applied to a section of the Scheldt Estuary from the Rupel to the sea (zones 2 and 3), taking into account the geometry of the basin (for instance, the function $L(x_1, x_3)$) and the appropriate boundary conditions.

To determine the dispersion coefficients, observed salinity and SiO_2* distributions were used. The model was run in preliminary hindcasting exercises until by successive improvement of K_1, K_3 and M_3 a satisfactory agreement was found between predictions and observations.

* The buoyancy equation (68) is easily adapted to describe the distribution of SiO_2 . In the right-hand side two additional terms appear ; the first one representing the consumption by diatoms, the second one the lateral inputs.

Fig. (9) represents the lines of equal width-averaged residual horizontal velocity calculated by the model, in its final, calibrated form, for a flow rate equal to four times its low water value.

Currents are expressed in m/sec. The curve in dot and dashes (—·—·—·) noted n=4 indicates the line of zero residual current. It is readily seen on the salinity scale printed on the top of the diagram (and corresponding to the same flow rate) that the residual current goes to zero, at the bottom, at the very place where critical salinity values and flocculation are observed.

Mud sedimentation and accumulation thus occurs in a region where the average sediment transport on and near the bottom is negligible.

For reference, two curves in dashes (— — —) noted n=1 and n=10 respectively, indicate the lines of zero residual current for flow rates equal to the low water value and 10 times that value. The salinity scale is, of course, shifted accordingly and the zone of precipitation appears to coincide in most cases with the region of zero bottom current.

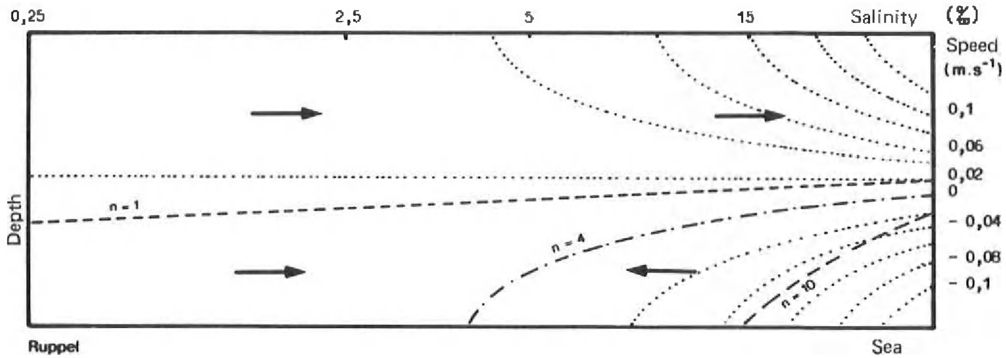


Fig. 9. : Lines of equal width-averaged residual horizontal velocity calculated by the model.

However, when the flow rate increases or decreases and comes back to typical values ($n \sim 4$), a net motion near the bottom appears in the regions where it was zero for lower and higher flow rates and directions are such that, as indicated by the arrows in fig. 9, they tend to carry back the freshly deposited sediments to the median zone.

IV. ACKNOWLEDGEMENT

This work was conducted in the scope of the Belgian National Environment Program, sponsored by the Science Policy Administration, Office of the Prime Minister.

V. REFERENCES

Bowden K.F. (1965), J. Fluid Mech. 21, 83.

Coen I. (1974), *Debieten van het Scheldebekken*, Antwerpse Zeediensten, Ministerie van Openbare Werken, Belgium.

De Pauw N and Peters J.J. (1973), *Contribution to the Study of the Salinity Distribution and Circulation in the Western Scheldt Estuary*, Belgian National Environment Program, Sea Project, Ministry for Science Policy, Belgium.

Fisher H.B. (1972), J. Fluid Mech., 53, 671.

Jäger Ph. (1973), *Dynamical parameters of the Scheldt Estuary*, Belgian National Environment Program, Sea Project (compiled from "Stormvloeden of de Schelde", 1966, Ministerie van Openbare Werken, vol. II, IV, V), Ministry for Science Policy, Belgium.

Munk W.H. and Anderson E.R. (1948), J. Mar. Res. 7, 276.

Peters J.J. (1974), *Model voor de studie van de verontreiniging van het Scheldeestuarium*, Symposium The Golden Delta, 2, Pudoc.

Nihoul J.C.J. (1975), *Modelling of Marine Systems*, Elsevier Publ., Amsterdam.

Nihoul J.C.J. and Ronday F.C. (1976), *Modèles d'un estuaire partiellement stratifié*, in Recherche et Technique au Service de l'Environnement, edited by Le Conseil Scientifique de l'Environnement, Liège, 315-338.

Pritchard D.W. (1967), *Estuaries*, AAAS Publ., USA.

Ronday F.C. (1975), Annales des Travaux Publics de Belgique, 4, 1.

Theuns J. and Coen I. (1973), Annales des Travaux Publics de Belgique, 3, 139.