The James Forrest Lecture was then delivered.

JAMES FORREST LECTURE, 1947

The President said that the James Forest Lecture was founded in 1891 in honour of the late James Forrest, who was the Secretary of the Institution from 1859 to 1896 and Honorary Secretary from 1896 until his death in 1917. The Lecture to be delivered was the fifty-third Lecture of the series.

Mr. Forrest had bequeathed to The Institution some pieces of silver plate which had been presented to him during the course of his life, and normally they were displayed on the occasion of each James Forrest Lecture, but the practice had been dropped on that evening owing to the fact that a public exhibition was being held in the Institution building.

The Lecturer, Professor Jack Allen, M.I.C.E., was Professor of Engineering at the University of Aberdeen. For many years he assisted Professor A. H. Gibson, M.I.C.E., at Manchester University, in the design, construction, and operation of hydraulic models. He acted as Consultant for hydraulic investigation to the Great Ouse Catchment Board, the National Physical Laboratory, and the Dundee Harbour Board, and was a member of the new Hydraulic Research Board which was recently set up by the Department of Scientific and Industrial Research.

In that connexion, the President desired to announce that the first Director appointed for the new Laboratory was Sir Claude Inglis, M.I.C.E.

"Model Experiments in relation to Harbours and Waterways."

By Professor Jack Allen, D.Sc., M.I.C.E.

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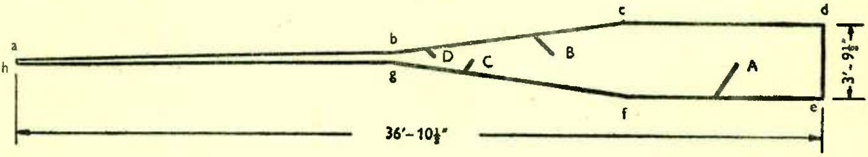
INTRODUCTION.

It is appropriate that the first name we should connect with this subject, and with this Lecture, is that of Osborne Reynolds. Not only was he a great pioneer in this particular field, but also his unique combination of mathematical insight, mechanical ingenuity, and experimenter's art exemplified most vividly the interdependence of abstract science and engineering which is the underlying theme of the James Forrest Lectures.

In 1885, Reynolds was interested in the estuary of the Mersey, primarily in the motion of the water in the estuary and the way in which the great eddies of circulation became superimposed upon the general movement of the tide.[1] Accordingly he constructed a model of the Mersey, with a horizontal scale of 1 : 31,800 and a vertical scale of 1 : 960. Almost immediately he realized that the corresponding time-scale should be $\sqrt{960}$: 31,800, and he was impressed by the way in which the bed of his model, fortunately moulded in sand, was formed into banks and channels by his mechanically operated tides. The general *pattern* of these bore as close a resemblance to any of the charts of the estuary as those charts did to one another. This encouraged him to undertake a prolonged study of the possibilities of such models and of the desirable relationship between the horizontal and vertical scales, at the end of which he declared: "this method of experimenting seems to afford a ready means of investigating and determining beforehand the effects of any proposed estuary and harbour works; a means which, after what I have seen, I should feel it madness to neglect before entering upon any costly undertaking." *Fig. 1* illustrates one of his experiments.

Fig. 1.



a b c d e f g h denotes model estuary (de is seaward end). A, B, C, D denote groynes or spur dikes, introduced to render the model asymmetrical. (One of a series of experiments on models of hypothetical, "geometrical" shape and of different sizes, to investigate the general possibility of experiments on a small scale.)

LAY-OUT OF ONE OF OSBORNE REYNOLDS'S TIDAL MODELS.

Mr. L. F. Vernon-Harcourt, M.I.C.E., was quick to follow Reynolds's example by making a model of the Seine estuary[2] with a horizontal scale of 1 : 40,000 and a vertical scale of 1 : 400. Vernon-Harcourt was greatly impressed by the revelations of this model, even though it was simple in construction and—we should say—primitive in its mode of operation. He extended Reynolds's work by trying different bed-materials, including powdered coke, pumice, coffee grains, and flowers of sulphur. Finally, he adopted a sand almost identical with that which Reynolds apparently had used throughout. Its mean diameter, as viewed under the microscope, is believed to have been about 0·0065 inch.

For the next thirty years, only three or four tidal models appear to have

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[1] The references are to the Bibliography on p. 410, *post*.

been made in Great Britain. One of these, which has been described by by Mr. G. E. W. Cruttwell, M.I.C.E.,[3] was a model of the Thames with a horizontal scale of 1 : 10,560 and a vertical scale of 1 : 384. It failed to show any bed-movement with the sand used; yet Mr. Cruttwell affirmed that the observation of the movement of the water alone was extremely instructive.

The great revival of interest among British engineers began, however, in 1926, when Professor A. H. Gibson, M.I.C.E., started his long investigation of the proposed Severn barrage.[4] In the course of this, he devoted many experiments to fundamental matters of laboratory technique, from the viewpoints of both water-movement and of sand and silt. For the first time, the rate of deposit of very fine mud was tackled, taking into account the electrolytic effect of the salts of the sea as well as the effect of the vertical exaggeration of scale. A weak solution of potash alum was employed in the model to allow for both of these effects.

Later (1929), Professor Gibson was asked to build a model of the Mersey and Liverpool Bay to study the problem of extending the training walls, and subsequently no fewer than nine major laboratory investigations of tidal river problems have been completed or initiated in Great Britain. Several other kinds of model experiments have also been made, including non-tidal rivers, the discharge of weirs, and the protection of the toes of spillways against scour. Only four years ago, however, was any British *national* research establishment concerned in this work, the Engineering Division of the National Physical Laboratory then beginning the construction of its first tidal model.

In November 1944, Mr. Wentworth-Sheilds, O.B.E., devoted a part of his Presidential Address to The Institution[5] to a plea for the establishment in Great Britain of a national hydraulics station comparable with those foreign laboratories, such as the U.S. Waterways Experiment Station at Vicksburg, Miss., which have been created specially for the study of maritime engineering problems; and the Department of Scientific and Industrial Research has recently resolved that such a station shall be built in this country. Its principal interest will be in "loose-boundary hydraulics," that is, the problems which involve accretion and scour. *Fig. 2* shows the first Vicksburg model (1930), and *Fig. 3* is a recent photograph of the Vicksburg Station.

I propose now to describe briefly a few examples of river and harbour models which have played an important part in the design of actual works.

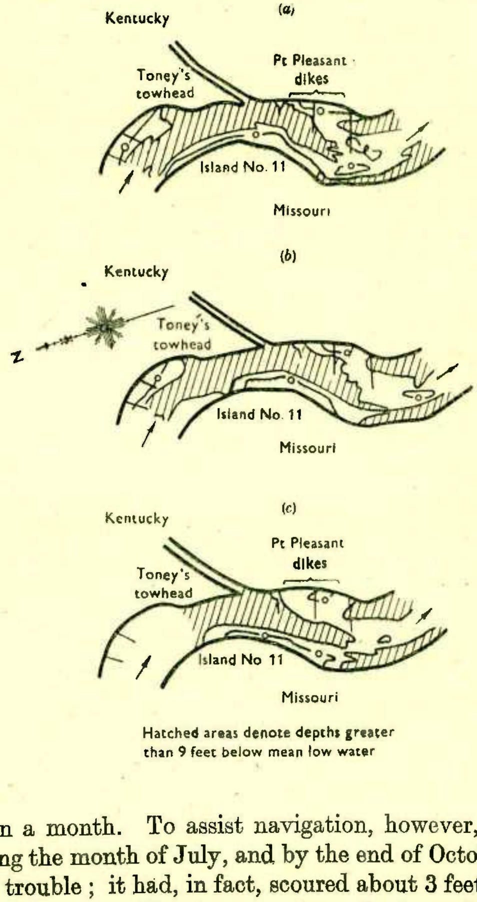
EXAMPLES OF MODELS.

(1) Three groynes, more than 8,000 feet in total length, had been completed during the year 1932 in the Point Pleasant reach of the Mississippi. (*Figs. 4* (*a*).) Observations made in August of that year revealed no improvement in the channel and the costly process of removing the groynes

was seriously contemplated. Model experiments made at Vicksburg,[6] however, indicated that the groynes would eventually encourage navigable depths. (*Figs. 4* (*b*).)

By July 1933, there was a depth of 10 feet over almost the whole of the critical stretch (*Figs. 4* (*c*)) and every prospect of a 10-foot continuous channel within a month. To assist navigation, however, some dredging was done during the month of July, and by the end of October the channel was giving no trouble; it had, in fact, scoured about 3 feet deeper.

Figs. 4.



Hatched areas denote depths greater than 9 feet below mean low water

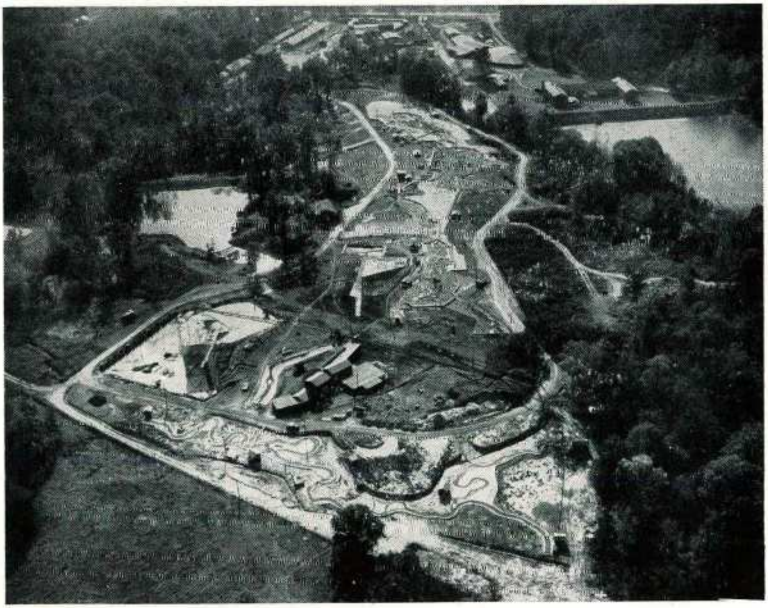
The model in this case (*Fig. 5*) was made to a horizontal scale of 1 : 1,000 and a vertical scale of 1 : 125, but an additional distortion of *longitudinal* slope was provided in order to ensure proper bed-movement. During a run of 8 hours, a cycle of stages of water-level, changing by 10-foot intervals from 10 feet to 40 feet and back, was simulated in proportion to the periods occupied by these changes in Nature as based upon 5 years of observations.

(2) Professor Gibson's tidal model of the Mersey and Liverpool Bay

Fig. 2.

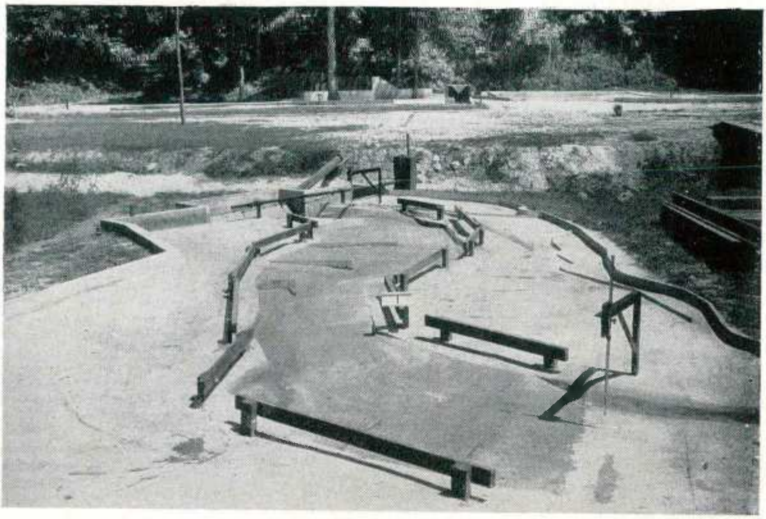
THE FIRST VICKSBURG MODEL, 1930 (THE LOWER REACHES ILLINOIS RIVER).

Fig. 3.



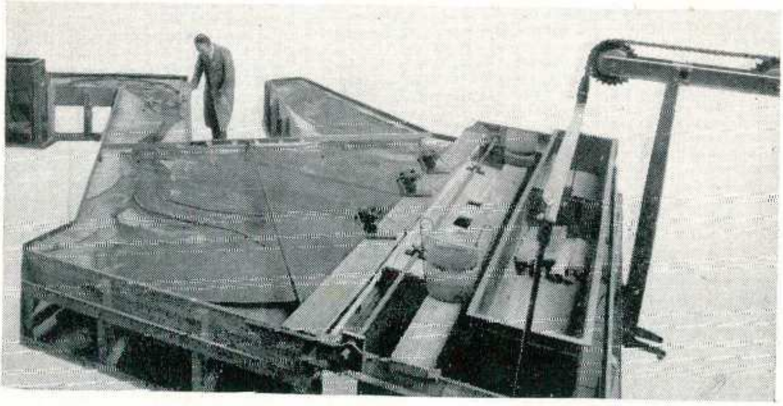
A RECENT PHOTOGRAPH OF THE U.S. WATERWAYS EXPERIMENT STATION, VICKSBURG, MISSISSIPPI.

Fig. 5.



Point Pleasant Model with Point Moulded to Configuration shown by Survey of August 1932.

Fig. 7.

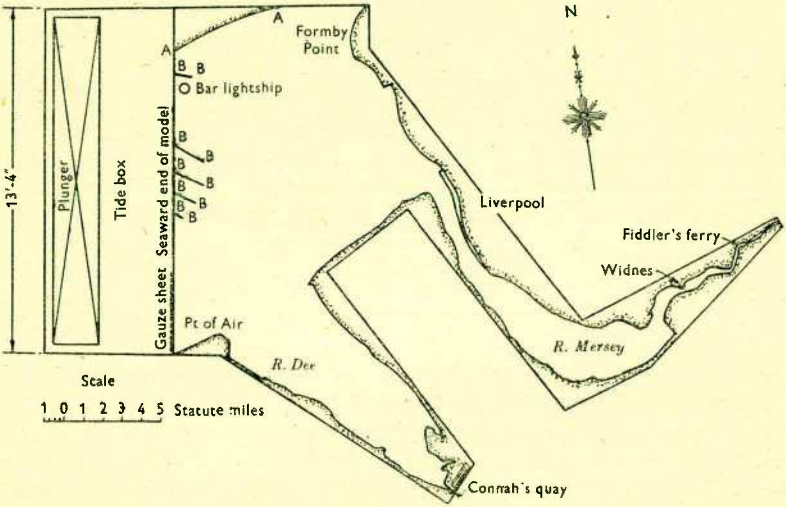


Model of Liverpool Bay and the Mersey and Dee Estuaries.

embraced the area shown in *Fig. 6*, from which it will be seen that the Dee estuary was included as having possibly some secondary effect. The seaward limit of the model was chosen to lie a little west of the Bar lightship and the northern artificial boundary was adopted as a result of float observations in Liverpool Bay itself, which indicated a general tendency for the flood-tide currents north of that line to make northwards towards the Ribble.

The estuary of the Mersey was moulded up to a point about 1 mile below Warrington and a small basin was added to represent the area of the remaining portion of the river subjected to spring tides. Considerations of space led to the canalized portion of the Dee between Connah's Quay and Chester weir being replaced by a zig-zag channel or labyrinth.

Fig. 6.



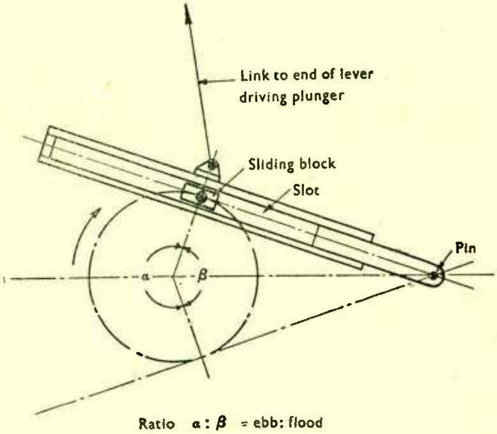
SCOPE OF MERSEY MODEL.

The tides were produced by the vertical motion of a balanced steel plunger of 21 square feet mean sectional area in plan (*Fig. 7*), the stroke of which was varied automatically from springs to neaps by epicyclic gearing. The flood tide was made to occupy a shorter time than the ebb by constraining the main crank-pin of the driving mechanism within a slotted bar pivoted at one end. (*Fig. 8.*)

After much preliminary trial and error, the set of the flood tide from the western boundary into the bay was reproduced with the accuracy demonstrated in *Fig. 9*. This degree of agreement was achieved by the introduction of the curved boundary AA in *Fig. 6* and the five short baffles BB, extending to high-water level. Again, in order to obtain the proper flood current velocity past the Point of Air, a sheet of copper gauze was placed as shown in *Fig. 6,* this giving the local reduction of velocity caused in Nature by the Chester Flats, which lie outside the scope of the model.

The shape and stroke of the plunger were adjusted so that a close agreement was obtained as between the tide curves at the mouths of the Dee and the Mersey and the corresponding natural curves. Thereafter, observations were taken of the rate of rise and fall of the tide at points farther upstream (*Fig. 10*). The only apparent discrepancy of any significance was discovered at Connah's Quay, in the Dee, where a surge was observed at high water of spring tides in excess of any such irregularity for which evidence in Nature has been forthcoming. This peculiarity was also found afterwards in a model of the Dee estuary alone,[7] its magnitude then being dependent largely upon the configuration of the sandbanks and channels between Parkgate, Flint, and Connah's Quay. In any event, this phenomenon could have had no conceivable influence upon the conditions in the channels leading towards Liverpool.

Fig. 8.

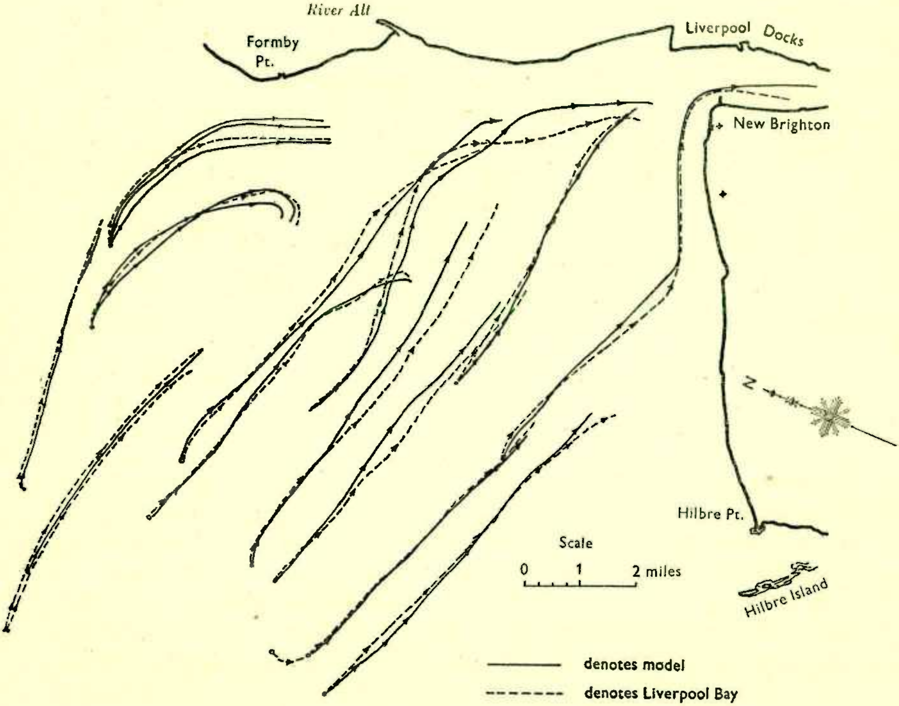


The bed material used in the Mersey model was a sand of specific gravity 2·64 and of a mean diameter (as viewed under the microscope) of 0·0071 inch, which is very nearly three-quarters of the average size of sand found in Liverpool Bay and the more important channels. This relative size ($\frac{3}{4}$) was chosen as a result of the exhaustive tests previously made with fifteen different materials in the Severn model investigation.

In addition, mud from the Mersey itself was supplied in suspension with the little streams of water which were fed to the model at the upper limits of the Mersey and the Dee to represent the run-off of the catchment areas. The quantity of silt so introduced was adjusted so as to maintain an average concentration in mid-river opposite Prince's landing stage of 1 : 11,000 parts by weight or thereabouts; samples from this locality in Nature showed average values of approximately 1 part in 6,000 and 1 in 17,000 at springs and neaps respectively, in comparison with 1 : 12,000 and 1 : 24,000 near the Formby lightship.

Owing to the vertical distortion adopted in such a model, it is necessary that particles of silt should fall more rapidly than in Nature. In any horizontal displacement, they have to descend *proportionately* through a bigger depth. With the scales used in this model (1 : 7,040 horizontally and 1 : 190 vertically) the particles should, in fact, fall $\frac{7040}{190^{\frac{3}{2}}}$, or 2·68 times as quickly as in Nature. There is, moreover, the added complication that the salts of the sea encourage coagulation of the silt particles, so that they settle far more rapidly than in fresh water. In order to cover both of these effects, the model was operated with a weak solution of potash alum

Fig. 9.



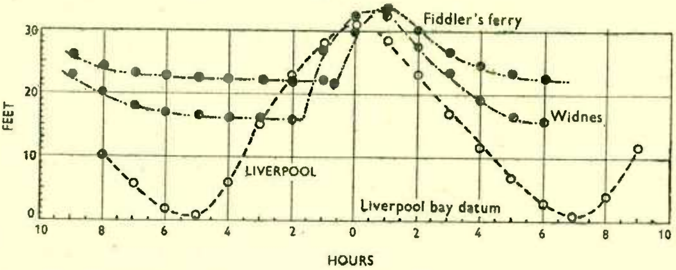
Drift of Floats on Flood Tides (Spring Tides).

introduced by means of siphons floating in tanks above the seaward end of the model. The siphons enabled the alum to be dropped into the model at such a rate as to compensate for the continual dilution caused by the inflow of the fresh-water rivers and the outflow through the drain and over the spillway provided near the tide-producing plunger. The quantity of alum actually needed was rather less than $\frac{1}{10}$ lb. per 24 hours.

Some allowance for wave-action was also made in this model. A battery of fans was mounted over the mouth so as to blow from a direction between west and west-south-west; these fans blew for the equivalent of three months in each year of tides. (One " year " was 17·2 hours). The inclination of the fans to the water-surface was arranged so as to generate waves corresponding with a height of 3 feet in Nature.

In addition, in order to obtain some idea of the likelihood of heavy gales lifting bed material from the back of the training walls and depositing it in the main channels, a wooden board was mounted across the seaward end of the model. This board, which dipped into the water, was fixed to a horizontal shaft, and a lever attached to this shaft was occasionally worked by hand for about four or five tides. The motion of this paddle produced waves equivalent to about 16 feet in height and caused silt to be carried into suspension and some stretches of the foreshore and sandbanks to be eroded by as much as 3 feet.

Fig. 10.



The effect of the *fan-produced waves* was, however, confined to the higher sandbanks and to the more exposed foreshores; it should be understood that this wave-action, however produced in such a model, can give only *qualitative* results—primarily for the reason that the length of the waves is not in proportion to their height (because of the vertical distortion of scale). But, apart from such general indications as the waves may serve to show, they have a pronounced beneficial effect in keeping the sand free for transportation by the ordinary currents.

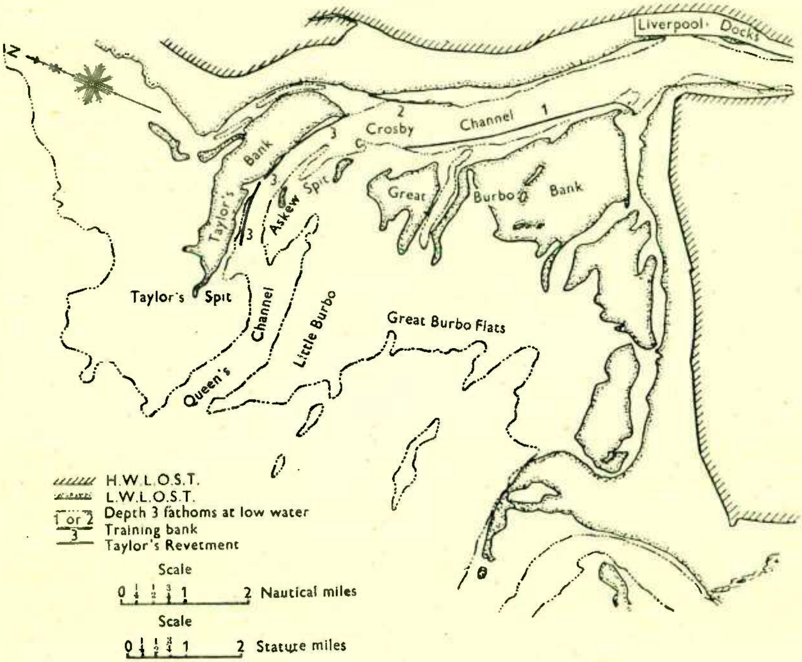
In this model, many widely differing schemes of training walls were tried. These were made of sheet lead or similar material, bent to the required shape and pressed into the sand. Attention was not restricted to the resulting bed changes; numerous measurements were made of the effect on tides and currents: these experiments alone were of immense value.

The questions which arise in the design of such training works include:—

(*a*) the most effective position;
(*b*) the best height of the walls.

If the two walls which are intended to embrace the channel are set too far apart, meandering and general instability will occur between them; if they are too low, their influence in guiding the currents is largely lost; if they are too high, they may cause current-velocities high enough to embarrass navigation. By observing the lines of flow in the model, through the agency of dye and of floats, and by measuring velocities, it is possible to demonstrate these phenomena with beautiful effect.

Fig. 11.

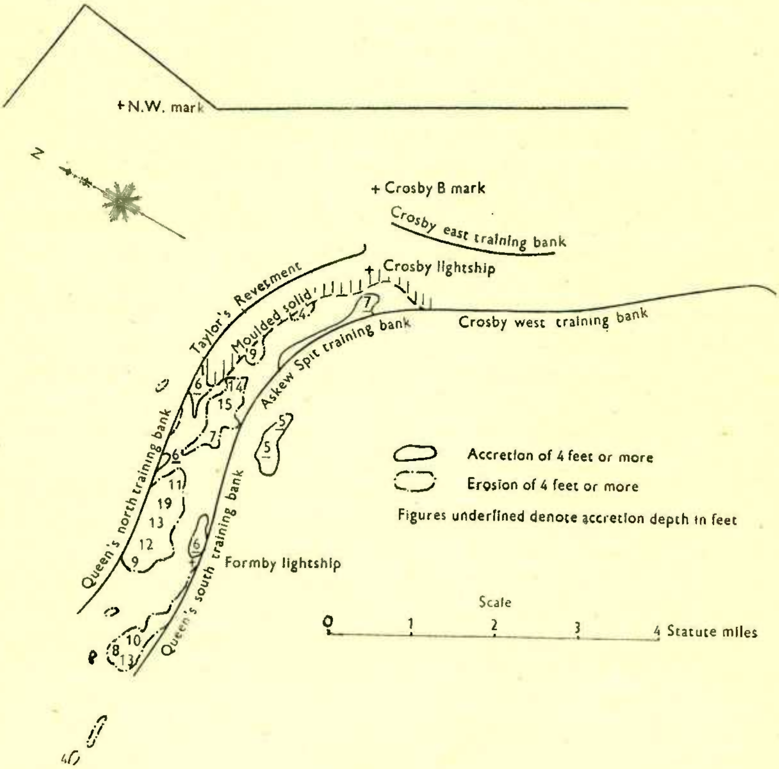


LIVERPOOL BAY, 1929.

In this particular investigation, it was found that the walls need not be built up to anything like the height which had been envisaged; indeed, that such high walls would increase the velocities to an inconvenient if not dangerous degree; low walls would increase the ebb currents, especially near the time of low water when scour is most active, more proportionately than the flood velocities.

In 1932, the model was dismantled in Manchester and re-erected in Liverpool, where the Mersey Docks and Harbour Board continued the investigation, especially of the more westerly portions of the approach channels. *Fig. 11* shows the 1929 plan of the Bay. For the purpose of the final experiments, the channel alongside Taylor's revetment and Askew Spit was moulded as a solid, permanent feature. Owing to the vertical exaggeration of scale, it was impossible for the sand to stay at the angle demanded by this channel, and at Manchester it had been moulded under water as nearly practicable to the soundings shown in the 1929 survey. A few tides were then run to permit initial settlement, and the result was surveyed and adopted as the standard basis of reference, or "model equivalent of 1929 conditions," to be used for the beginning of each comparable pair of tests, with and without proposed works. It was found

Fig. 12.



MODEL RESULTS AFTER 15,300 TIDES FROM 1934 BAY CHART.

that the depth of this channel obtainable in the model was about 45 feet, in comparison with a maximum of about 62 feet in Nature, although this 45 feet did allow some little further deepening under the action of the currents and such deepening was interpreted as an indication of probable scour.

The investigation as a whole suggested that it would be practicable, by extending the " south " wall a distance of approximately 6 miles, and the other wall about 3¼ miles along certain lines, to improve the alignment of the channel and to reduce materially the amount of maintenance dredging afterwards required.

The cause of the excessive curvature in the old channel was the projection of Taylor's Spit, and it was decided to correct this and to bring the channel into its new alignment by the removal of this material. This was to be done by intensive dredging and by maintaining a careful relationship between the rate of dredging and the rate of extension seawards of the training banks. *Fig. 12* shows the scheme finally evolved, and the model indications of scour and accretion.

Work along these lines (as suggested by the model) began in June 1932, but unfortunately had to be abandoned in September 1939, by which time the " south " wall had been carried forward about 4½ miles, and the " north " wall 1½ mile. During the same period, about 95,000,000 tons had been dredged in the channels generally, a third of this quantity being from Taylor's Spit. From July 1939 to July 1945, less than 8,000,000 tons was dredged from the bar, Queen's Channel, and Crosby Channel ; this material was devoted wholly to the local maintenance of the channels existing at that time.

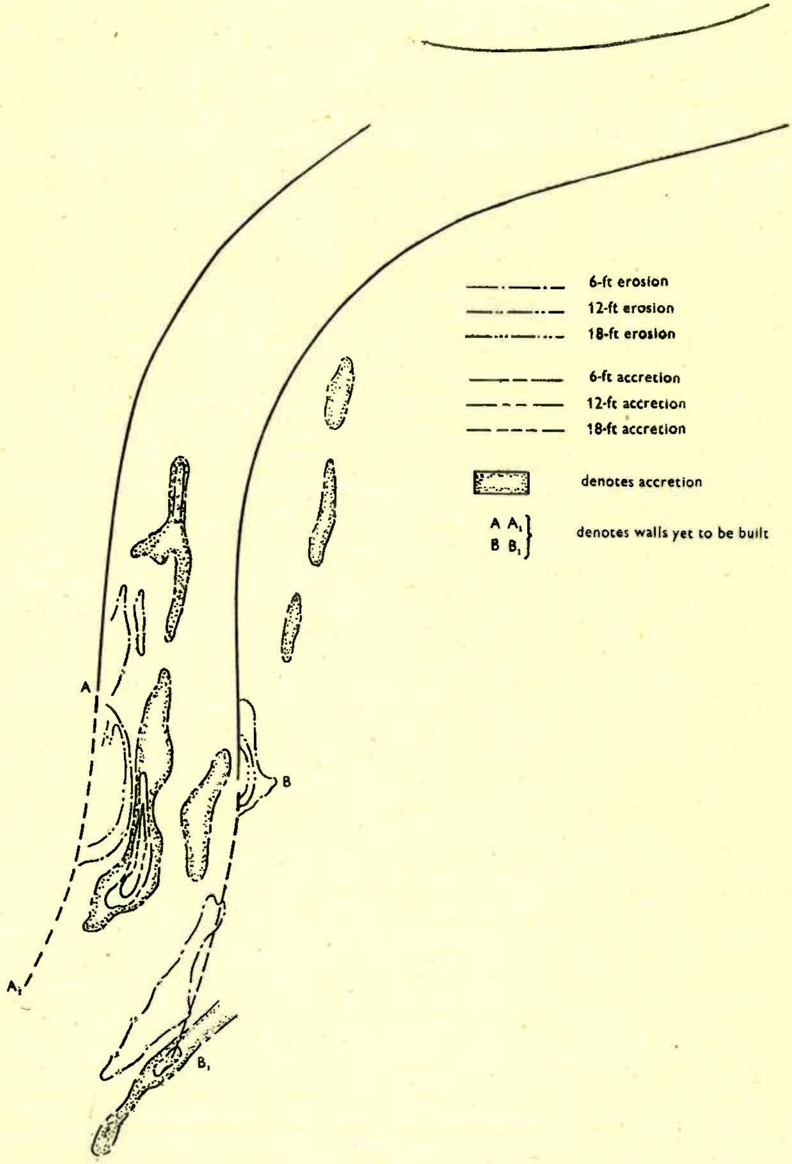
Despite the small amount of dredging done during the war, and the fact that the walls are not yet completed, it is true to say that the channel behaved reasonably well and to infer that this behaviour during those critical years is largely attributable to the construction of the walls up to the stage reached in 1939.

Fig. 13 indicates the scour and accretion in the seaward approaches during the 1939–45 period of virtually no dredging ; it remains only to state that the average rate of dredging from the bar, Queen's Channel, and Crosby Channel during the 40 years previous to the start of the model investigation in 1929 was not less than 10,000,000 tons per annum.

(3) As the third example, I should like to recall the very detailed study made by Messrs. Sir Alexander Gibb and Partners, in 1932–35, of the conditions outside the port of Rangoon. Mr. Oscar Elsden, M.Sc., A.M.I.C.E., has described the tidal model used for this purpose[8] (*Fig. 14*). The horizontal scale was 1 : 8060 and the vertical scale 1 : 192. Two tide-producing plungers were employed, one running at twice the speed of the other ; the smaller (and slower) plunger was added in order to reproduce the pronounced diurnal variation of tides experienced at Rangoon. I have time

only to mention some points of special interest in this magnificent series of experiments :

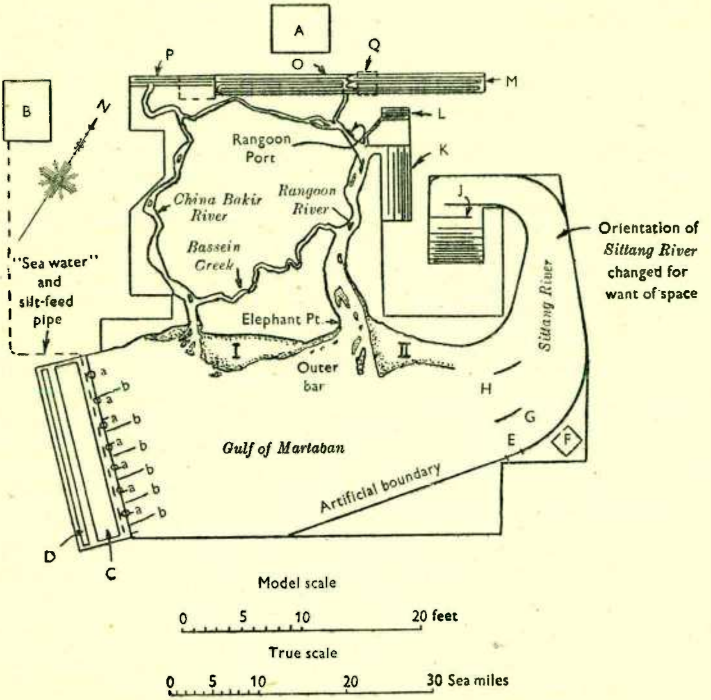
Fig. 13.



Erosion and Accretion in Liverpool Bay, 1939–45.

(*a*) The principal bed-material was a sand having a size of about three-quarters of that of the natural Rangoon sand. This ratio was again based upon the experiments made in the Severn models. As, however, the Port Commissioners' charts showed that large parts of the bed were covered, by the year 1932, with silty material, a bed of similar composition was moulded in the model over the corresponding areas.

Fig. 14.



I: China Bakir Flats
A: Silt tank for river supply
C: Main tide-producing plunger
E: Weir for maintaining mean tide level
G, H: Baffles to correct centrifugal effects
a: Fans for Monsoon winds
K: Labyrinth for Pegu River
M: Labyrinth for Hlaing and Rangoon Rivers
P: Labyrinth for China Bakir River
II: Eastern Grove flats
B: Alum and silt tank for sea supply
D: Diurnal tide-producing plunger
F: Weir-control motor
J: Labyrinth for Sittang River
b: Adjustable baffles controlling tidal stream
L: Labyrinth for Pazundaung Creek
O: Labyrinth for Panhlaing
Q: River-control motor

Layout of Rangoon Tidal Model.

(*b*) A study of silt samples taken in the Gulf of Martaban indicated that a larger mass of silt enters the Gulf on the flood tide, along the line chosen for the western limit of the model, than leaves it on the ebb tide along the same line. This "excess" of "tidal silt" was therefore introduced in suspension with alum-charged water from a perforated trough placed across the mouth of the model near the plungers. The perforations were so designed as to discharge the greater part of this silt inshore.

(*c*) Silt was also supplied in suspension to the various tributary rivers, and monsoon conditions were automatically reproduced through a timing device which increased the supply of water and silt to the rivers at the appropriate times. Finely divided clay was used for silt.

(*d*) In certain tests, two different clays were employed for tidal and river silt. They contained different proportions of titanium dioxide, and this feature was used as a means of identification for the silt mixture extracted from the model after a period of running: the results established that: "The major part of the Outer bar deposits had come from the China Bakir river or from further west, and that only a small proportion of the silt leaving the Rangoon river found its way on to the bar."

(*e*) South-westerly monsoon winds were reproduced by a battery of seven fans, brought into action automatically at the proper times through the seasonal control-gear. They created waves of about the correct height, but of incorrect length (because of the distortion of scale).

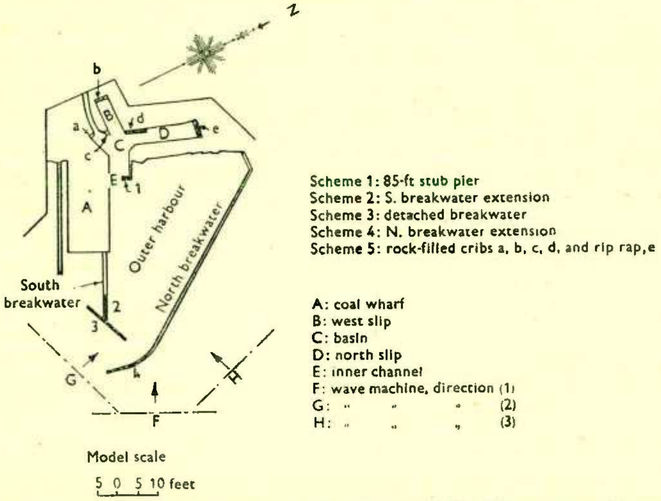
(*f*) An attempt was made to simulate the rate of erosion for certain stretches of coastline or river bank. Mixtures of sand and clay and cement were tried, but finally the effort had to be abandoned. Instead, the eroding coastline was moulded in a heavy puddle-clay, which was cut back by hand at frequent and regular intervals to agree with the erosion shown on charts of the river. Simultaneously, the appropriate quantities of sand and silt were added daily along the length of river bounded by the eroding banks. The mixture added in this way consisted of 5 per cent. of sand and 95 per cent. of silt: these proportions were based upon analysis of the material in the actual river.

As a result of this investigation, Messrs. Sir Alexander Gibb and Partners advised the Port Commissioners of Rangoon that the conditions at the bar would probably *improve* and that the proposed training (which was to have been a costly undertaking) would be worse than useless.

No important prediction ever made as the result of laboratory investigation has been so clearly—even dramatically—fulfilled. In 1860, the ruling depth over the bar was 24 feet at low water of ordinary spring tides; it continually decreased until in 1931 it was 12 feet, and it was *still* 12 feet when the model-experiments finished and the Report was issued. During the next four years, the depth improved by as much as 8 feet.

(4) From river-models in which the water-currents are a dominating feature, let us turn attention next to examples where *wave-action* is the vital factor. The design of breakwaters is an obvious case, and many scale-model experiments have been made to help in the solution of this different kind of problem.

Fig. 15.



1 : 50 Model of Port Washington (Based on U.S. Waterways Experiment Station Bulletin No. 10, Fig. 2).

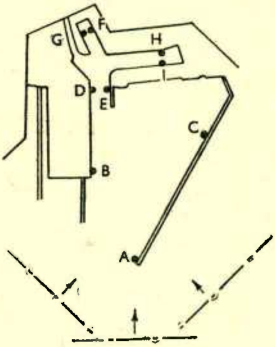
For example, the U.S. Waterways Experiment Station at Vicksburg has undertaken several wave-action studies using scales of 1 : 50 or 1 : 100, with no vertical distortion, and a wave-generator consisting of a triangular-shaped plunger, to which a vertical motion is imparted through gearing from a variable-speed motor; the travel of the plunger and the speed of the motor may be adjusted to produce the waves required.

Fig. 15 illustrates the general lay-out of a model of the harbour of Port Washington, on the western shore of Lake Michigan.[9] During the winter of 1934–35 storms inflicted damage, estimated at $80,000, to docks and small craft within the harbour, despite the construction of the new north breakwater. The model-experiments revealed that further suggested extensions would not materially reduce the height of the waves in the inner basin due to storm disturbances moving approximately along the axis of the channel. Finally, it was recommended that wave-absorbers in the form of sloped rip-rap and rock-filled cribs should be constructed. To quote the Vicksburg Report, however ;

> "Because of insistent demands from local fishing interests for protection from easterly to southerly storms, the south breakwater extension was built. This produces benefits from southerly storms, but a series of wave-height observations at various points in the harbour, before and after construction of the extension, confirms the results of the model tests, that the structure would be of little benefit for storms entering the harbour about along its axis. Although the south breakwater extension has aided in reducing the wave heights in the harbour to some extent, there is still too much disturbance for safe mooring of small boats in the slips or outer basin or for the unloading of large coal boats during major storms running directly into the harbour. Wave absorbers have not been constructed to date. A direct comparison of model and field data shows a marked similarity of wave heights. All this sums up to prove that the model has correctly evaluated the effect to be obtained from the extension of the south breakwater."

Fig. 16 demonstrates the standard of agreement between the wave-heights as measured in the model and in the harbour itself: this agreement is remarkably good in view of the difficulty of measurement and of ensuring comparable conditions.

Fig. 16.

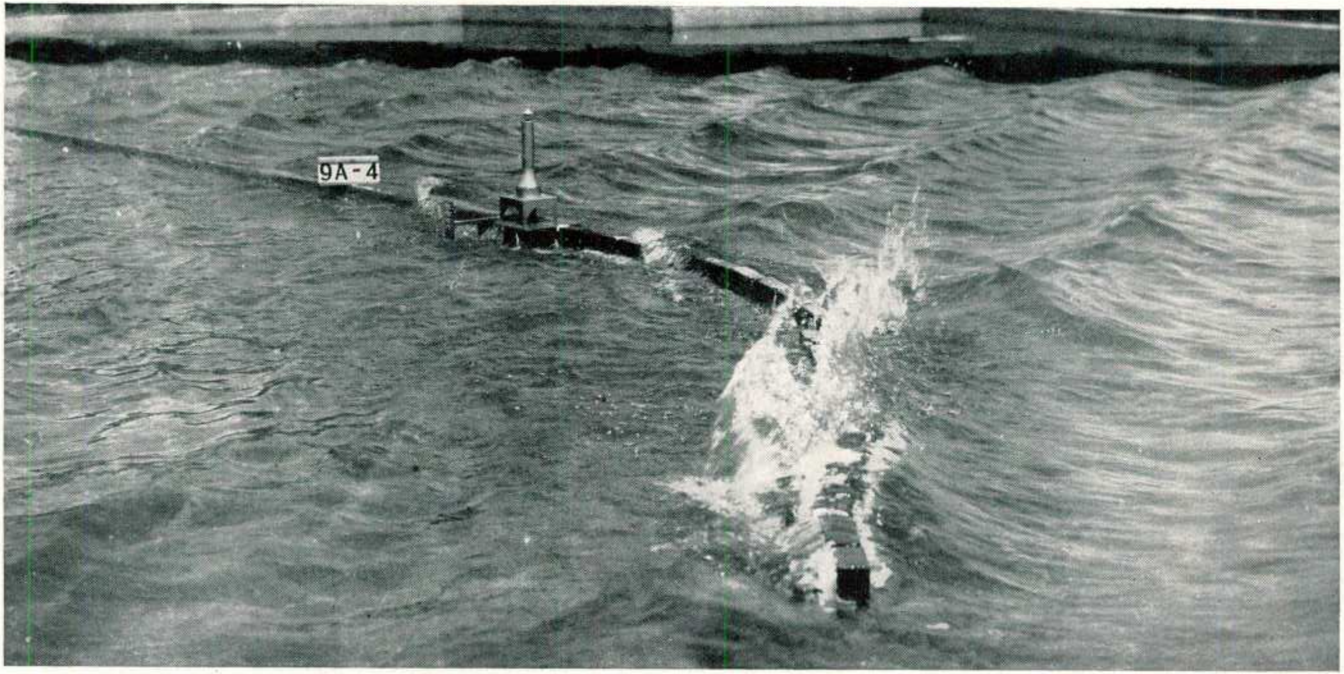


Staff gauge	Wave height: ft	
	Prototype	Model
A	10·0	9·0
B	2·0	3·0
C	6·0	5·0
D	4·5	4·0
E	4·5	3·0
F	4·0	4·0
G	4·0	4·0
H	3·5	3·0
I	3·0	2·0

PORT WASHINGTON—WAVE HEIGHTS IN PROTOTYPE AND MODEL (BASED ON U.S. WATERWAYS EXPERIMENT STATION HYDRAULICS BULLETIN NO. 10, FIG. 8).

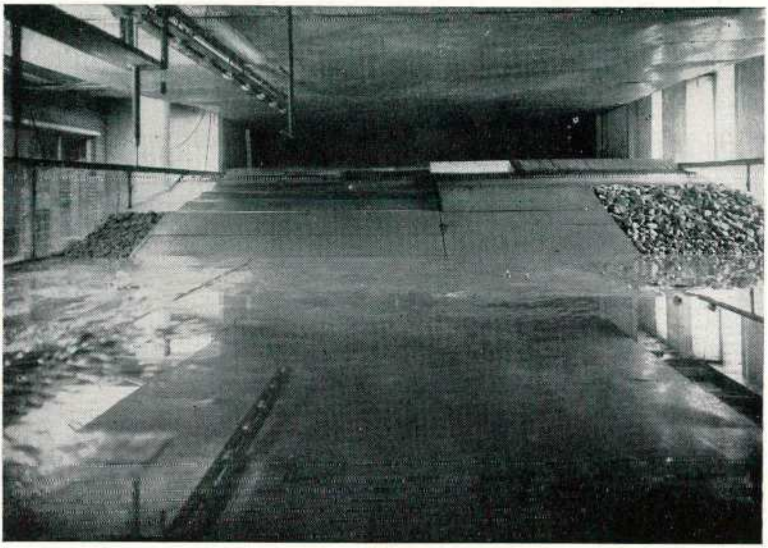
The plunger used in this model was 50 feet long and could be moved to

Fig. 17.



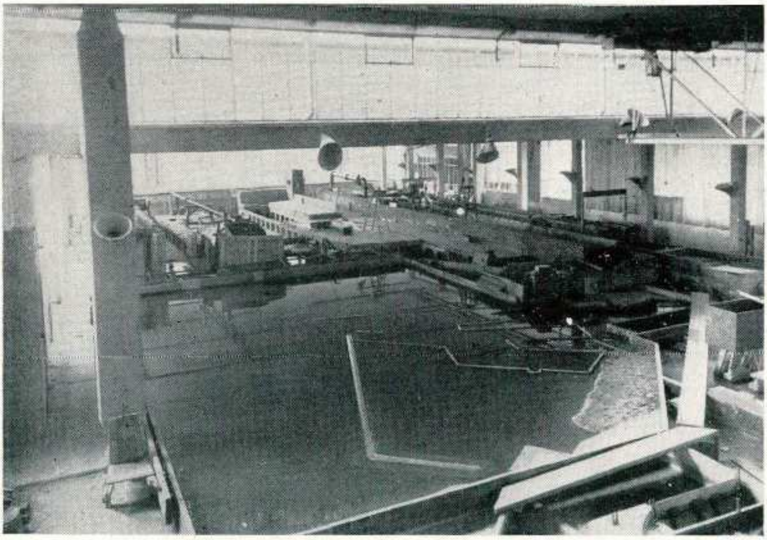
Model of Port Washington, Lake Michigan.

Fig. 19.



FLUME USED FOR LEITH BREAKWATER EXPERIMENTS AT DELFT.

Fig. 20.



MODEL OF LEITH HARBOUR AT DELFT.

any one of three positions so as to make waves from different directions. In order to cover the range of storm waves in the lake, waves of height equivalent to 7 feet, 10 feet, 15 feet and 20 feet were tried in each direction. *Fig. 17* shows typical conditions in the model.

(5) My next example—experiments made in the Netherlands Government Laboratory at Delft, as a prelude to the design of two new breakwaters at Leith—is derived from the Inaugural Address delivered by Mr. J. Dalgleish Easton, M.I.C.E., to the Edinburgh and District Association of The Institution in October 1944.†

These experiments were divided into two main series :

(*a*) To find a satisfactory cross section for the breakwaters so as to withstand the heaviest storms.

(*b*) To find the most suitable position of the seaward ends of the two breakwaters so as to form a new entrance to the port and to minimize the disturbance in the harbour during storms.

The first series was made, to a scale of 1 : 25, in a *covered* flume 13·2 feet wide and 88 feet long. The depth of water could be varied, waves produced at one end, and a current of air blown over the water surface. Two models, placed side by side and having different profiles, were tried at one and the same time. These were constructed of wood and about 13¾ inches width of each was completed with a layer of rubble (represented by fine stone chips of various dimensions) tipped stone (pieces of whinstone of various sizes and weights represented to scale ; for example, 1 ton by 64 grams), and pitching (marble blocks and, for the smaller pieces, cement blocks to which filings were added according to the specific gravity required.)

The sides of the flume near the models were made of glass to facilitate observations. Ten different water-levels were used, corresponding with various stages of the tide, and, with each, the wave-height was stepped up by stages to its maximum. It was found impossible to imitate properly the stone pitching, the resistance of which is greatly affected by the degree of interlocking. To be on the safe side, therefore, the stones were arranged hardly supporting one another—a very loose pitching. In order to overcome the " personal error " in interpreting the behaviour of the models by visual observation, it was decided to have all the tests made by the same observer and the placing of the pitching stones done by one workman.

Fig. 18 shows the design evolved from the behaviour of the models and from observing the water movements over them, such as the backwash of the waves. *Fig. 19* is a photograph of the flume.

The second series of experiments was made in a model of the harbour built to a scale of 1 : 180. Two wave-generators were used, acting from different directions and consisting essentially of swinging boards with an adjustable movement. In order to prevent reflexion of the waves from the

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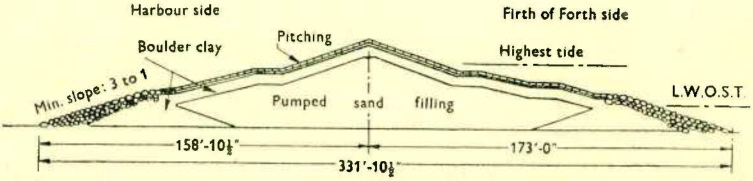
† J. Instn Civ. Engrs, vol. 23 (1944–45), Notices Section, p. [10] (Dec. 1944).

artificial boundary of the tank, a slope of sand was placed there on which the waves could break. The biggest wave-height outside the harbour was 0·9 inch, representing 13½ feet. A point worthy of notice is that at the two places where the wave-makers were situated, the depth of water, according to a strict application of the scale of 1 : 180, would have been only 2·67 inches (representing 40 feet). This depth would have been too small for the production of regular waves. Accordingly, the depth in these regions was arbitrarily increased to 7 inches.

Eleven arrangements for the new harbour entrance were tested. One has actually been constructed "to the satisfaction of the Pilots and Shipmasters using the Port." 10 *Fig. 20* is a photograph of the harbour model.

Fig. 18.



Cross-section of Breakwater at Leith.

THE TECHNIQUE OF MODEL EXPERIMENTS :

Having discussed very inadequately these few specific examples of river and harbour models, I propose to devote the remainder of this Lecture to some of the underlying theory.

(1) There are two manners of motion of a fluid—streamline and turbulent—for which the laws of resistance are quite distinctive. In the one, the resistance is proportional to the velocity and in the other (turbulent motion) more nearly to the square of the velocity. If the motion in the prototype is turbulent, so also should it be in the model.

Experiment 11 has shown that streamline motion generally becomes turbulent, in a *straight* open rectangular channel, if the Reynolds number $\frac{vm}{\nu}$ exceeds 1,400. Here v denotes the mean velocity, m the hydraulic mean depth and ν the kinematic viscosity of the fluid. This means that even in such an "ideal" channel 100 feet wide and 3 feet deep the flow of water at ordinary temperatures will almost certainly be turbulent if the mean velocity exceeds 0·07 inch per second, whilst in a channel 1 foot wide by $\frac{3}{100}$ foot deep, the critical velocity would become approximately 7·3 inches per second.

In a very rough or tortuous channel,11a or one having abrupt changes of width or depth, the motion will be turbulent at velocities appreciably

lower than $v = \frac{1400\nu}{m}$, and indeed this may be taken as a very safe criterion in dealing with river models.

In a tidal model, however, the velocities are changing all the time, as the tide ebbs and flows. The maximum current-velocity, v knots, during the flood tide near the mouth of the natural estuary will occur usually at about half-tide, when the mean depth of the water in the seaward channels is, say, H feet. If the vertical scale of the model is $1 : y$, the corresponding depth will be $\frac{H}{y}$ feet and the velocity will be $\frac{v}{\sqrt{y}}$ knots, or $\frac{1{\cdot}69v}{\sqrt{y}}$ feet per second.

It is true that the *hydraulic mean depth* will differ appreciably from $\frac{H}{y}$, especially in a model with exaggerated depth-scale; nevertheless, owing to the fact that the channels are far from straight or regular in section, turbulent motion may be confidently expected in the model if

$$\frac{1{\cdot}69v}{\sqrt{y}} \cdot \frac{H}{y} \not< 1{,}400\nu,$$

or, with water at "ordinary" temperatures, if

$$\frac{1{\cdot}69vH}{y^{\frac{3}{2}}} \not< 0{\cdot}017.$$

In "round numbers" this reduces to the condition $y^{\frac{3}{2}} \not> 100vH$, and this is likely to be a safe limiting criterion for the vertical scale (from this point of view), especially if the tide under consideration is a neap. It may also be safely assumed that the turbulence generated in the deep channels near the mouth will not completely decay during slack water; moreover, it will be transmitted to the shallower channels upstream, where sudden changes of section or shape will tend to contribute additional eddies.

(2) We have noticed the use of a "*vertical exaggeration*" in our examples of river models. How far is this desirable or justifiable?

Let us consider first the case of uniform flow along a straight open channel. The resistance depends not only upon the textural roughness of the sides and the bed, but also upon the depth of water: the channel is proportionately rougher when it is shallow than when it is deep.

Thus, the value of Chézy's coefficient C in $v = C\sqrt{mi}$ is, for a great variety of open channels (and rough pipes, too) connected with the hydraulic mean depth m by the equation:

$$C = 37{\cdot}6 + 32{\cdot}1 \log_{10}(m/k) \quad \text{ft.}^{\frac{1}{2}}\,\text{sec.}^{-1} \quad . \quad . \quad . \quad . \quad (1)$$

where k denotes the height of the roughening excrescences.[12]

If the channel is 100 feet wide by 5 feet deep and is, in effect, roughened by many boulders which project 6 inches into the stream, then $m = 4{\cdot}55$ feet, $m/k = 9{\cdot}10$, and $C = 68{\cdot}4$. With twice the depth of water, m

300
1:6

becomes 8·33 feet, $m/k = 16{\cdot}66$, and $C = 76{\cdot}8$—a very appreciably smoother channel than before. *Fig. 21* shows the general relationship between C and m/k.

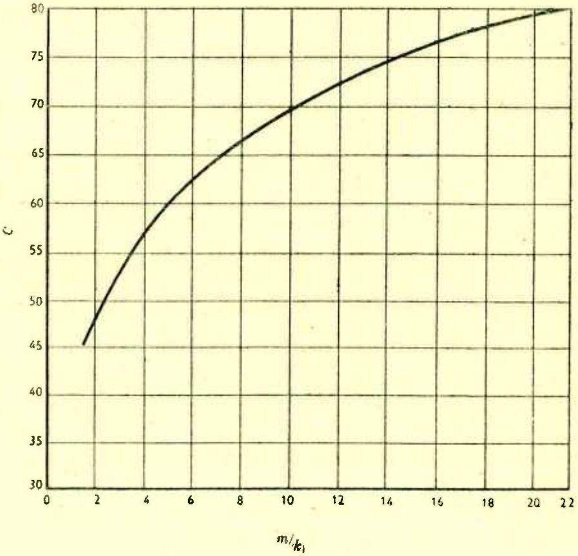
Let us now consider a model channel with a horizontal scale of $1 : x$ and a vertical scale of $1 : y$. The suffix (1) will identify the large channel and the suffix (2) the model.

$$\frac{i_2}{i_1} \text{ should equal } \frac{x}{y};$$

But
$$i = \frac{v^2}{C^2 m}$$

Therefore
$$\frac{x}{y} = \left(\frac{v_2}{v_1}\right)^2 \left(\frac{m_1}{m_2}\right) \left(\frac{C_1}{C_2}\right)^2 \quad \ldots\ldots \quad (2)$$

Fig. 21.



For *given* values of x, y, C_1 and C_2, this equation may be satisfied by choosing the velocity scale so as to make

$$\left(\frac{v_2}{v_1}\right)^2 = \left(\frac{x}{y}\right) \left(\frac{m_2}{m_1}\right) \left(\frac{C_2}{C_1}\right)^2; \quad \ldots\ldots \quad (3)$$

and this solution has, in effect, been applied in many river model studies: the rate of flow has been adjusted to give (nearly enough) the appropriate water-surface gradient.

Since, however, for turbulent flow in open channels, the loss of head is proportional nearly to v^2, it is natural to aim at a velocity scale given by

$$\frac{v_2}{v_1} = \frac{1}{\sqrt{y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Indeed, in the particular case of tidal models, this velocity-scale is demanded by the fact that the velocity of propagation of the tidal wave is proportional to the square root of the depth of water in which it travels. And again, if there are other incidental considerations involved, such as the discharge of sluice-gates or weirs, the appropriate velocity-scale for these is given by equation (4). So the really fundamental method of attack, if practicable, is to make $\frac{v_2}{v_1} = \frac{1}{\sqrt{y}}$, and the corresponding scale of rates of discharge

$$\frac{q_2}{q_1} = \frac{1}{xy} \cdot \frac{1}{\sqrt{y}} = \frac{1}{xy^{\frac{3}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equation (2) then becomes

$$\frac{x}{y} = \frac{1}{y} \cdot \frac{m_1}{m_2} \left(\frac{C_1}{C_2}\right)^2 ; \quad \text{or} \quad x = \frac{m_1}{m_2} \left(\frac{C_1}{C_2}\right)^2 \quad . \quad . \quad . \quad . \quad (6)$$

But if there is no vertical exaggeration of scale, $y = x$ and $\frac{m_1}{m_2} = x = y$. In that event, equation (6) shows that C_2 must equal C_1, and since the depth in the model is only $\frac{1}{x}$ of that in the prototype channel, this can be satisfied only by making the textural roughness of the model surfaces less than that of the actual full-size channel. If the model is still operating within the range to which equation (1) applies, this further means that the height of the roughening excrescences in the model should be $\frac{1}{x}$ of those in the prototype: in small models, or if the full-size channel is itself comparatively smooth, this may well mean an impracticable degree of smoothness in the model.

This problem becomes much easier to solve if the depths are purposely exaggerated in the model, relative to the horizontal dimensions. Then it may even be practicable to make the surface of the model of the same texture as the full-size channel.

To clarify the situation, consider Bazin's formula

$$C = \frac{157 \cdot 6}{1 + \frac{N}{\sqrt{m}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which N depends only upon the character of the channel sides and bed. Combining equations (6) and (7) :—

$$\frac{\sqrt{m_1} + N_1}{\sqrt{m_2} + N_2} = \frac{m_1}{m_2} \cdot \frac{1}{\sqrt{x}} \quad \ldots\ldots \quad (8)$$

Suppose the full-size channel to be of rough earth. N_1 may then be equal to 3·0 or thereabouts. A model built to equal horizontal and vertical scales of 1 : 200 would require to have its $N_2 = \frac{N_1}{\sqrt{x}}$ or $\frac{3}{\sqrt{200}}$; that is, 0·21, which *is* a possible value for a channel of fairly smooth wood or cement.

But if the scales are both 1 : 2,000, ($x = y = 2{,}000$), N_2 must equal $\frac{3}{\sqrt{2000}} = 0{\cdot}067$, meaning a surface smoother than *smoothed* cement or *planed* wood. One way of overcoming this difficulty would be to set the model to an exaggerated longitudinal slope (tilting the model in the direction of its length), or again, as has previously been implied, by determining the velocity scale experimentally instead of adopting the fundamental $v_2 = v_1 \cdot \frac{1}{\sqrt{y}}$. The further complication then arises, however, that the angle of tilt, or the scale of velocities thus empirically determined, may not apply to other stages of flow.

Now let the vertical dimensions of the model be exaggerated. Suppose the natural channel to be 400 feet wide by 20 feet deep, and $N_1 = 3{\cdot}0$, as before. First, let the model scales be 1 : 200 (horizontally) and 1 : 20 (vertically)—a distortion of 10 : 1.

$$m_1 = 18{\cdot}2 \text{ feet.}; \quad m_2 = 0{\cdot}5 \text{ foot}; \quad x = 200.$$

Equation (8) then requires $N_2 = 2{\cdot}1$, instead of 0·21, when $x = y = 200$.

Again, if $x = 2{,}000$ and $y = 200$, still involving a vertical distortion of 10 : 1, N_2 must equal 0·67, instead of 0·067 for the undistorted scale of 1 : 2,000. This is a real advantage, because $N_2 = 0{\cdot}67$ corresponds with a reasonably practicable roughness, whereas $N_2 = 0{\cdot}067$ means an unattainable smoothness.

The advantage in the first case ($x = 200$; $y = 20$) is not so obvious, although it is there, and it seems that, as regards the problem of attaining the proper friction losses, there is a great deal to be said in favour of a vertical distortion of scale—but more especially when x is large ; that is, in the smaller-scale models.

This argument is strongly reinforced by the fact that, for a chosen horizontal scale, the velocity and the hydraulic mean depth (and consequently the Reynolds number) increase as the degree of vertical distortion increases : the possibility of achieving turbulent flow in the model is enhanced by the use of a vertical exaggeration of scale. Here again, this is more significant in the case of the smaller models, which otherwise tend (because of their small dimensions) towards non-turbulent regime.

Moreover, if the problem of bed movement be considered, it will appear that vertical exaggeration is desirable—if not, indeed, essential. Experiments on flat beds of granular material of a given density, shape, uniformity, and size, indicate that the mean velocity in a rectangular channel, to initiate movement of the bed, is proportional to $h^{0{\cdot}06}\, m^{0{\cdot}22}$, where h denotes the actual depth and m the hydraulic mean depth.[13]

To appreciate the general significance of this, suppose the channel to be broad and h sensibly equal to m. The mean velocity to generate bed-movement is then proportional to $h^{0{\cdot}28}$. In such a case, to double the vertical distortion would mean that the velocity needed for disturbing a bed of a certain granular material would be increased by about 22 per cent.; the actual stream-velocity would, by the same change of distortion, be raised by $\sqrt{2} : 1$, or roughly 40 per cent.—that is, a considerably brighter prospect of transporting the grains would have resulted from the increased exaggeration of scale.

An interesting numerical illustration of some of these remarks is provided by Professor Gibson's model-investigation of the Severn.[14] The effect of neap tides is not felt above Framilode, and accordingly the 12 miles or so between there and Gloucester is subject to the rules of unidirectional flow at neap tides.

The mean width of the river is 265 feet, and when the discharge is 4,100 cusecs the average hydraulic mean depth is 5·92 feet and the mean velocity is 2·5 feet per second, whilst the drop in the water surface between Gloucester and Framilode is 3·7 feet. These data make Chézy's $C = 134$ and Bazin's $N = 0{\cdot}43$, this absorbing the effect of the resistance of the bends as well as the channel roughness.

In the first Severn model, the horizontal scale was 1 : 8,500 and the vertical scale 1 : 100; that is, $x = 8{,}500$ and $y = 100$. The scale of river discharges was taken to be $1 : xy^{\frac{3}{2}}$, and the scale of velocities as $1 : \sqrt{y}$. At a flow equivalent to 4,100 cusecs, the value of m in the model was 0·0124 foot and v was 0·25 foot per second, the fall being 0·037 foot. These values make $C = 31{\cdot}9$, whereas, if N had been 0·43, as in the river itself, C would have been 32·4.

In the second Severn model, the value of x was 8,500 and that of y was 200. The mean velocity then obtained with a discharge of $\dfrac{4100}{8500 \times 200^{\frac{3}{2}}}$ was 0·17 foot per second; the value of m was 0·0104 foot; the fall between Gloucester and Framilode was 0·0185 foot. These data make $C = 33{\cdot}5$, in comparison with 30·2 by substituting $N - 0{\cdot}43$ in $C = \dfrac{157{\cdot}6}{1 + \dfrac{N}{\sqrt{m}}}$.

Here, then, are cases in which the roughness of the model channels was only a little less than that of the river itself (a very reasonable conclusion in view of the facts that the bed was composed of material differing little from that of the natural river and that the model channel contained the same number of bends), and the large vertical exaggerations employed served to give the correct water-gradient.

Incidentally, these numerical examples will have shown that the scale-ratio of hydraulic mean depth in such models is much more complicated than are the simple geometrical scale-ratios. This would be true even of channels of rectangular cross section, let alone irregular shapes, when the depths are not converted in the same ratio as the widths.

We have now reached the stage where, on theoretical grounds, there is a clear case for the distortion of scale in river-models. This distortion eases the problems of friction, bed-movement, and turbulence. The counter argument is that the vertical exaggeration places a restriction upon the slopes which can be simulated in the model, the maximum slope being that of the angle of repose of the mobile material. This complication is aggravated in the case of scour-holes around the projecting ends of spur dikes or groynes or portions of coastline; these scour-holes are often very steep in actual rivers and would require to be impossibly steep in a model with exaggerated depths.

The choice of scale must take into account the relative importance of such steep slopes. Often they are of a very local character and comparatively unimportant, but if, in particular, a scheme involving the construction of groynes is under consideration, it is essential to have a small vertical exaggeration of scale, with correspondingly large horizontal dimensions.

Osborne Reynolds wrote of one of his Mersey models in the following terms: [15]

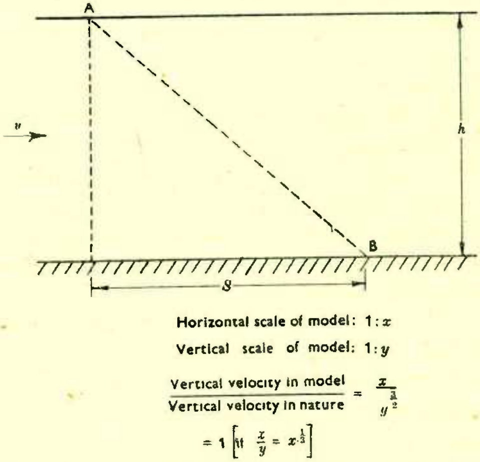
> "In one respect the great difference between the model and the estuary calls for remark: this is the much greater depth of the model as compared with its length and breadth. The vertical scale being 33 feet to an inch, and the horizontal scale 880 feet to an inch, so that the vertical heights are nearly twenty-seven times greater than the horizontal distances, such a difference is necessary to get any results at all with such small-scale models; and it is only natural to suppose that it would materially affect the action. As a matter of fact, however, it does not seem to do so. And, further, it would seem that, notwithstanding the general resemblance on the regime of the beds of large and small streams running over sand, there is in these a similar difference in vertical scale, the smaller streams not only having a greater slope, but also having a greater depth as compared with their breadth and steeper banks. . . . In the model it certainly seems that the general regime is determimed by the momentum effects, and from the almost exact resemblance which this regime bears to that of the estuary, it would

seem that, although the momentum effects may be diminished by the greater resistance on the bottom, they are still the prevailing influence in determining the configuration of the banks. Further investigation will doubtless explain this, and also determine the best proportional depths. From my present experience in constructing another model, I should adopt a somewhat greater exaggeration of the vertical scale."

Professor Gibson has developed one of Reynolds's arguments in the following way :[16]

> "It is perhaps worth noticing in passing that what is in effect a distortion of scale is usual in Nature, since small streams flowing through alluvial ground have much steeper side slopes and gradients than large rivers of similar regime in similar ground. In a very large river such as the Mississippi, the Ganges or the Irrawaddy, the maximum depth will rarely exceed 1 : 50 of the maximum width, while in a small stream in similar ground this ratio will seldom be less than 1 : 5."

Fig. 22.



There is still another way of looking at this question of vertical exaggeration. Referring to *Fig. 22*, suppose a particle to travel along some path through the depth of the stream from a point A to a point B. It moves horizontally a distance S in a time proportional to $\frac{S}{v}$ and it descends through a depth h. In the model, the corresponding horizontal movement should be $\frac{S}{x}$ in a time $\frac{S\sqrt{y}}{vx}$, the depth of fall being $\frac{h}{y}$.

It would then appear that the vertical velocity in the model is proportional to $\frac{hxv}{yS\sqrt{y}}$ and in the prototype, to $\frac{hv}{S}$.

The ratio of these is $\frac{x}{y^{\frac{3}{2}}}$.

If this is made equal to unity, that is, if it be assumed to be necessary to have particles falling in the model at the same speed as in Nature, which would be true of the terminal velocities of identical grains falling through identical media, then

$$y^{\frac{3}{2}} = x.$$

This agrees with a theory of Mr. Gerald Lacey, M.I.C.E., which has frequently been used in proportioning the vertical scale of river models to the horizontal scale.[17] Further experimental evidence is needed to establish whether this really is the ideal relationship. It certainly conveys the correct trend, because it implies that the vertical distortion, $\frac{x}{y}$, should be equal to $x^{\frac{1}{3}}$. If $x = 1$, that is if the "model" is full-size, then $y = 1$—as it should be ; if x is very large (the model then having small horizontal dimensions), the vertical exaggeration $\frac{x}{y}$ should be very great.

The general practice in continental and American laboratories is to adopt vertical distortions (for river models) of the order of 5 : 1 or 10 : 1. But we have seen that much higher exaggerations have been successfully employed by British experimenters.

(3) A great deal could be said about the choice of the mobile material. I have already specified the materials which have been used in certain investigations. It is quite remarkable that the sand used by Reynolds in his tidal models was very nearly the same as that finally considered to be best in the Severn investigations made forty years later.

One feature commonly encountered in river-models is the formation of sand ripples. Whilst there is every reason to suppose that ripples, often of great dimensions, exist in natural rivers and estuaries, yet it is believed that in the models they have sometimes reached disproportionate sizes, especially as regards their length from crest to crest.

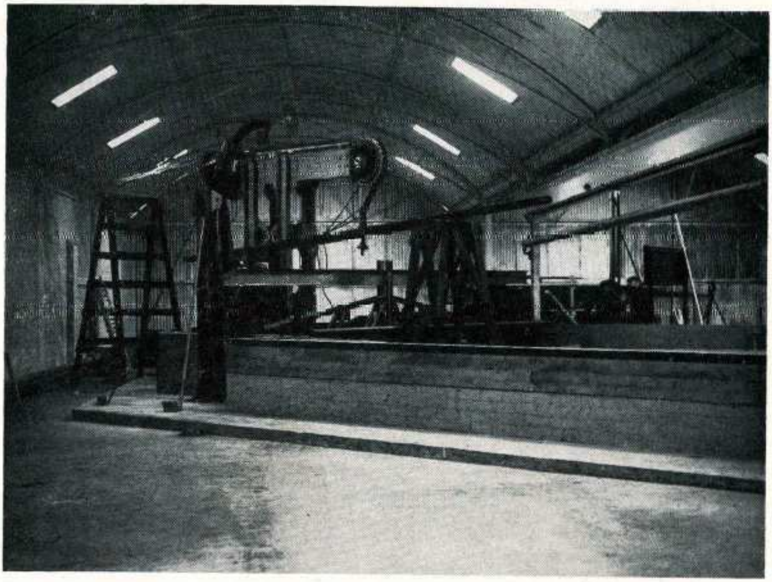
One experimenter, Captain Kramer, [18] has devised the technique of first testing the sand to be used in the model in a straight channel, determining therein the critical tractive stress of the water over the sand (*a*) when the grains are all in motion, and (*b*) when ripples of a height equal to 8 per cent. of the depth of water are formed during a run of 1 hour and such that subsequent lower stages of flow fail to remove the ripples entirely. He has then chosen his model scales so that the operating conditions will lie between these lower and upper limits of tractive stress. Whilst the figure of 8 per cent. is arbitrary, nevertheless the method is ingenious and deserving of further study, as also, indeed, are the ripples and tidal ridges to be found in Nature. Some of these have been described in books by Dr. Vaughan Cornish [19] which, I think, are among the delights of scientific literature. *Fig. 23* shows some wonderful sand ridges in the Cheshire Dee.

Some experimenters have adopted the device of tilting their river models so as to give them extra longitudinal slope, thereby meeting the

Fig. 23.

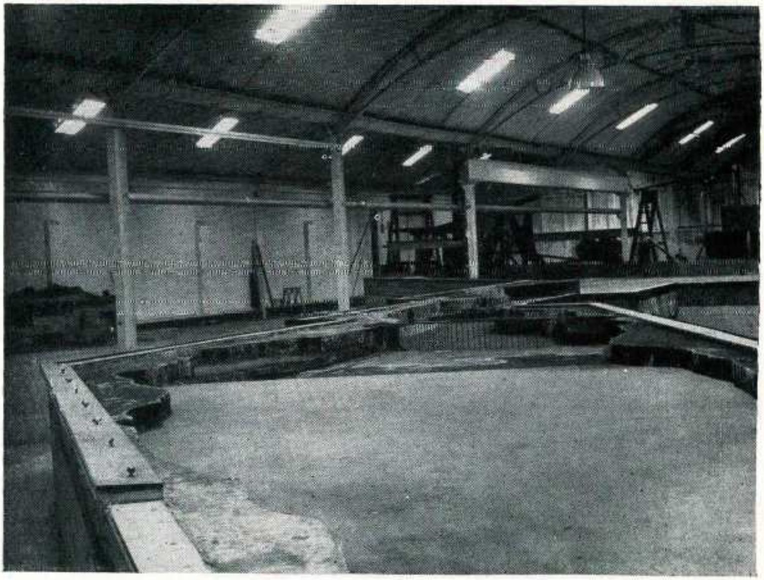
Sand Ridges in the Estuary of the Cheshire Dee.

Fig. 25.



Firth of Tay Model under Construction.

Fig. 26.



Firth of Tay Model under Construction.

friction problem half-way, as it were, and also assisting bed-movement. In tidal models, this is impracticable (because of the alternating flow).

One criticism levelled against the use of models, by sceptics in the past, has been that each grain of sand represents, to scale, some very large boulder. This overlooks the fact that we are concerned—in this type of model—with the movement of banks or shoals of appreciable size in which each grain is comparatively minute. There must be half-a-million grains or so of sand, as commonly used in models, in one sandbank only 1 cubic inch in volume.

Experience has now given us a fairly clear notion as to the sort of material to use in any particular model. If it is possible also to check our choice by comparing changes in the model with changes—under similar conditions—in Nature, so much the better. In any case, I believe in the technique of operating the model from some standard initial state under identical conditions of tides, river-flow, and so forth, with and without the scheme of works under investigation.

I think also that when time, space, and expense permit, there is every argument in favour of models made to different scales for one and the same investigation. The presence of scale-effect may often be judged in this way; there is also the advantage that schemes may be studied rapidly in a small model and only the more promising ones subjected to prolonged scrutiny in the large model.

A guide as to the prospects of adequate sand-movements being found in a tidal model may be based upon the observed behaviour of mounds of granular material in open rectangular channels: [20]

(*a*) Let L be defined as the length and B as the breadth (in inches) of the average sand-grain as viewed under the microscope. Assume a mean grain-size of $\frac{L+B}{2}$ for use in the model—say about three-quarters of the $\left(\frac{L+B}{2}\right)$-value of the sand found in the natural estuary.

(*b*) Calculate the depth h (inches) equivalent to the mean-tide depth of water in the proposed model at a few important parts of the channels.

(*c*) Calculate the value of $1{\cdot}50 \left(\frac{L}{B}\right)^{0\cdot27} h^{0\cdot23}$.

If this proves to be not greater than the maximum flood or ebb velocity expected in the model, in feet per second, on spring tides at the places under consideration, then there is a very good chance indeed that the proposed vertical scale of the model will be satisfactory from the aspect of bed-movement. I have here assumed the sand to have a specific gravity of 2·64.

Thus, if the $\frac{L+B}{2}$ value of the model sand is 0·01 inch, and if $h = 4$ inches, the model-velocity would have to be not less than

$$1{\cdot}50 \times 0{\cdot}01^{0{\cdot}27} \times 4^{0{\cdot}23} \text{ foot per second,}$$

or 0·59 foot per second.

With a vertical scale of 1 : 144, this velocity would represent 0·59 × 12 feet per second, or 4·2 knots, whilst the depth of 4 inches would represent a channel 48 feet deep in Nature. So, if the surface velocities in this channel of the actual estuary are of the order of 4·2 knots, on spring tides, it may confidently be expected that the sand specified will move sufficiently in the model, the vertical scale of which is 1 : 144.

But a new field may well have been opened, so far as mobile materials are concerned, by the use of very light " sands " such as powdered Perspex, which Brigadier R. A. Bagnold has tried in his wave experiments [21] and which, he points out, has the virtue of producing no apparent biological contamination of the water. Therefore it may retain freedom of movement for a long time, whereas the ordinary fine sands often lose their mobility by the chemical or biological contamination of their surface of contact with the water.

A PARADOX.

Models for the study of relatively small structures often require to be of larger size, proportionately, than models concerned with works of greater magnitude. For example, if it is proposed to study the formation of a localized shoal outside a dock-entrance, a large model is imperative in order that this shoal may be of reasonable size when scaled down. But that is only the beginning : for suppressing the unnatural conditions in the vicinity of the artificial limits of the model, these limits must be placed remote from the spot under investigation. Altogether, then, considerable floor-space may be demanded.

Let me illustrate this by reference to a model now under construction by the Dundee Harbour Trust. One object is to determine the probable effect of proposed extensions to the harbour, involving the construction of new docks projecting about 1,100 feet into the estuary from the present frontage. Not only is it desired to discover the ultimate result of this structure, but also its effect at different stages of its construction. Therefore, it is essential that the model should be of reasonably large size.

A horizontal scale of 3 feet to 1 mile has been chosen, and a vertical scale of 1 inch to 12 feet. Therefore a spring tide of 15 feet will be represented by 1·25 inch in the model, and the tidal period will be $\frac{12{\cdot}4 \times \sqrt{144}}{1760}$ hours, or 5 minutes 4 seconds. Incidentally, the scales for this model agree with the Lacey formula, or with the conception of vertical velocities being equal to those in Nature : thus, $144^{\frac{3}{2}} = 1{,}760$ approximately.

Fig. 24 shows the scope of the Tay model: not only is it necessary to have large horizontal dimensions in order to investigate the effects which have been mentioned, but also it is essential that the boundaries of the model should be remote from the works under consideration so that it may be safely assumed that conditions at the entrance to the model will be sensibly independent of the proposed works.

This has meant choosing the seaward end not nearer than the mouth of the estuary itself and, since it is hoped to gain some information concerning the outer bar, which lies south-east of Buddon Ness, an artificial boundary well beyond Buddon has been adopted.

Again, the flood tide makes along the general directions indicated by the arrows in *Fig 24*: this accounts for the position of the tide-box in which the tides are to be generated and for the direction of the easternmost boundary of the model. Experiment will show whether it is necessary (as in the model of Liverpool Bay) to guide the flood-tide currents out of the tide-box along the required lines by means of baffles.

The southernmost limit of the model has been placed so as to minimize the risk of reflexions of the flood-tide wave into the most important area and also to permit the ebb tide, at certain stages, to take an approximately south-easterly direction from the mouth of the estuary. *Figs. 25* and *26* illustrate the model under construction.

WAVE-ACTION.

(1) As a guide in fixing the best position of breakwaters for the protection of harbours, a model of the fixed-bed type is of great value. Waves of many kinds may be involved, as, for example:

(*a*) Deep-water waves of oscillation, the velocity c of which is related to the wave-length λ by the expression [23]

$$c^2 = \frac{g\lambda}{2\pi}$$

(*b*) Shallower-water waves, in which particles of water situated near the surface describe an ellipse of horizontal axis a and vertical axis b: then

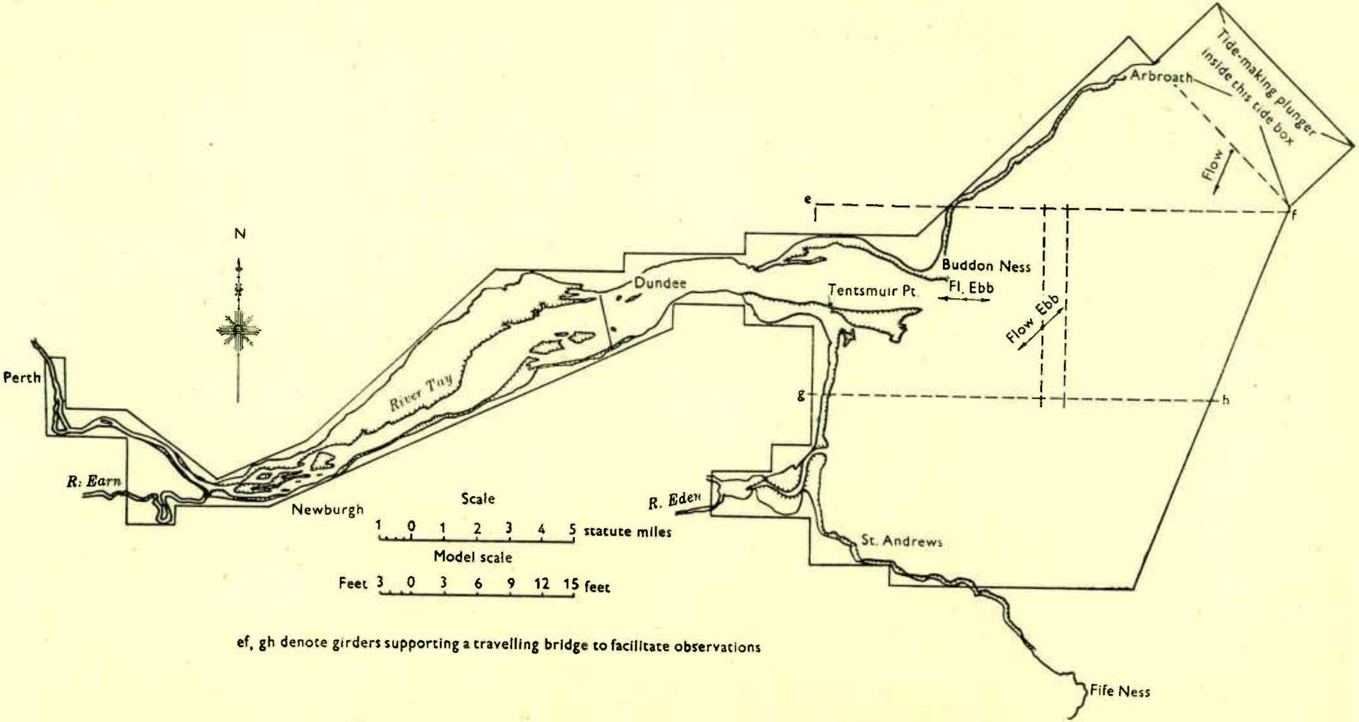
$$c^2 = \frac{g\lambda b}{2\pi a}$$

(*c*) Long waves, such as may occur when the water becomes still more shallow and for which:

$c^2 = gH$ approximately, where H denotes the depth of water.

Type (*a*) has a velocity proportional only to the square root of a horizontal dimension, whereas type (*b*) is influenced by both horizontal and vertical dimensions, and type (*c*) involves only a depth H.

Fig. 24.



ef, gh denote girders supporting a travelling bridge to facilitate observations

Scope of the Firth of Tay Model (Dundee Harbour Trust).

In order to cater strictly for all these possibilities in one and the same model, it is necessary to make the vertical scale the same as the horizontal, each being say 1 : x. The velocities in the model are then $\frac{1}{\sqrt{x}}$ of those in Nature.

Scales ranging between 1 : 25 and 1 : 200 have been successful in such investigations. It remains to be discovered how far it is permissible to use different horizontal and vertical scales and yet obtain results of sufficient qualitative accuracy.

(2) It is not yet practicable to affirm, with confidence, how nearly the pressures exerted by waves on sea walls may be predicted from scale models. Much depends upon the character of the wave itself and the sensitivity of the shock-pressure to small changes in the wave, including the volume of air present in the breaking wave. The piezo-electric method of measuring the short-duration pressures has, however, greatly enhanced the technical possibilities of measurement, and we may look forward to further developments along the lines initiated by Brigadier Bagnold before the war.[23]

(3) The tidal bore is another fascinating example of wave phenomena. The velocity of propagation c of such a wave having a height k above the surface of the stream of depth h into which it is advancing, is given [24] by

$$c^2 = \frac{2g(h + k)^2}{2h + k}$$

Here, c denotes the wave-velocity relative to the stream, and if k is small in comparison with h, $c = \sqrt{gh}$. Experiment has shown conclusively that the bore can be reproduced in a model having small channel dimensions and a distorted scale (or vertical exaggeration) with quite amazing accuracy.

In a model of the Dee estuary [25] (horizontal scale 1 : 5,000 ; vertical scale 1 : 200), a bore was seen to form some distance below Connah's Quay and to travel upstream to Chester weir. When the bed and the training walls were adjusted so as to represent existing conditions in the estuary itself, the time elapsing between the first sharp rise of the flood tide at Connah's Quay and that at a point near Chester weir was 9·8 seconds. The distance between these two points in the model was 247 centimetres (97·2 inches) so that the speed of propagation averaged 25·2 centimetres (0·827 foot) per second. The vertical scale being 1 : 200, this was equivalent to $25{\cdot}2 \times \sqrt{200}$ centimetres per second, or 7·94 miles per hour, in comparison with the figure of 8 miles per hour supplied by responsible eye-witnesses. Moreover, it was observed that the surface current followed the bore in the model with a speed of about three-quarters of that of the wave-crest—again a close agreement with Nature. Yet the average width of this channel in the model was only 2·5 centimetres (about 1 inch) and the vertical exaggeration was 25 : 1.

The classical investigation of the bore in tidal models was, however, made by Professor Gibson on his models of the Severn.[26] Even with a vertical exaggeration of 85 : 1, the bore made its appearance and travelled with a height and at a speed which may be said to agree with natural phenomena within the limits of observation; nor was any measurable discrepancy discernible as a result of changing the water temperature so much as to alter the kinematic viscosity by 72 per cent. and the surface tension by 4 per cent.

OTHER MODELS.

It is important that our knowledge of the rate of flow of rivers should be greatly extended. In many cases there are weirs, the head over which has been measured from time to time and under various conditions; it would be readily practicable also to install a continuously-recording instrument for such heads in the future. But the weirs are of many shapes and sizes; consequently their coefficients of discharge vary from one to another, whilst these coefficients also vary according to the head over the weir in any particular case.

It is a comparatively simple matter to find these coefficients (that is, to calibrate the weirs) by means of a scale model embracing a reasonable length of the approach channel and of the channel downstream. For this purpose, a scale of 1 : 25 or 1 : 50 is often quite satisfactory. If the scale is 1 : 50, then the flow of the actual river is predicted for a head H by multiplying the discharge as measured in the model under a head of $\frac{H}{50}$ by $50^{\frac{5}{2}}$. The only serious discrepancy, or scale-effect, likely to arise is when the head in the model falls below about $\frac{1}{4}$ inch, the surface tension of the nappe flowing over the weir then becoming, in some cases, disproportionately important.

THE COST OF MODELS.

It is quite impossible to generalize as to the cost of any particular model-investigation. A great deal depends upon whether the information which has to be gathered "from the field," before the model can be constructed or operated, is already available. If a special survey has to be made, the expense entailed may become a serious item in the overall cost of the investigation.

Again, in a climate such as ours, a model must be protected from the weather by some form of covering; in this respect, the Dundee Harbour Trust has been fortunate in being able to house its 125-foot long model of the Tay in a disused shed, with water- and electricity-supplies close at hand.

Sir Leopold Savile, Past-President I.C.E.,[27] has stated that the cost of the Rangoon tidal model investigation was £10,000. The much simpler examples of, say, weir-calibration may often be undertaken at a capital outlay on apparatus of £50 or less.

In relation to the information gained or the cost of the works under consideration, however, the laboratory costs are comparatively negligible and it may well be that the annual running charges of a well-equipped and adequately staffed national hydraulics experiment station will amount to less than 1 per cent. of the total yearly expenditure in this country on river and harbour and similar works.

CONCLUSION.

I hope that Sir Claude Inglis, M.I.C.E., who has had such vast experience of model experiments in his recent capacity as Director of the Indian Waterways Experiment Station at Poona, will not mind my quoting certain remarks from one of his Papers [28]:

> "It may, perhaps, be argued that with wide experience it should be possible to decide the best way to train a river, without the help of a model. To a certain extent this is true, but no two rivers seem to have identical conditions, and in the numerous recommendations made by the Poona Station no two have been the same. . . . Thus, despite the known limitations of mobile river models . . . such models are very helpful and indeed essential in solving complex river problems. It must not, however, be assumed that all that is required to obtain the correct answer is to have model experiments carried out. Success will be achieved only if those controlling the experiments have the experience, the knowledge, and the skill to diagnose the past and predict future changes in conditions in the model-length, as conditioned by changes upstream and downstream; and then, making allowances for model limitations, to evolve the best practical design in consultation with those responsible for the work.
>
> "To sum up: river models are a valuable aid to engineering skill which, however, they can never replace."

That, in my view, is a modest and reasonable claim.

ACKNOWLEDGEMENTS.

I am deeply indebted to the Director of the U.S. Waterways Experiment Station, Vicksburg, Mississippi; the Director of the Netherlands Hydraulic Laboratory at Delft; Mr. J. Dalgleish Easton, M.I.C.E.; Professor A. H. Gibson, D.Sc., LL.D., M.I.C.E.; Mr. Leopold Leighton, M.I.C.E.; and Mr. N. A. Matheson, M.I.C.E., for permission to use data, diagrams, and photographs. Mr. Leighton very kindly supplied the information concerning the behaviour of the Liverpool Bay training walls.

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Mr. F. E. Wentworth-Sheilds, in proposing a vote of thanks to Professor Allen, said that members had heard from the Lecturer and from the President that the Department of Scientific and Industrial Research had undertaken to build a station in Great Britain. They had appointed a Board of experts under the Chairmanship of the President, and a distinguished Director had been appointed.

Professor Allen should congratulate himself on belonging to the Manchester University Engineering School, where very fine work had been done in connexion with river models. If England were behind, Manchester was not! Professor Reynolds had been a pioneer in that work, in days when there was no Department of Scientific Research and no Government aid, financial or otherwise, and it had been carried out by the enthusiasm of such men as Reynolds, Gibson, and Professor Allen himself, who had converted the hard-headed Manchester men and others to the idea that the work was not merely a question of scientific interest, but also was of immense practical value.

It was very appropriate that the James Forrest Lecture should have been delivered by Professor Allen, who had been closely identified with that work and who, from his experience, knew what good results it could bring about.

Sir Claude Inglis seconded the motion and the vote of thanks was accorded by acclamation.